Abstract

In particle physics there exist two regions: the Standard Model which is fairly complete and the new physics sector which is completely unknown. In between and overlapping with both of these is neutrino physics. In this colloquium I will discuss some algorithmic and theoretical advances to improve our understanding of three-flavor oscillations. I will also discuss how new physics can complicate the oscillation picture. Finally, I will show how astrophysical environments can tell us more about neutrinos and how neutrinos can tell us more about astrophysical environments.

Nu Theory

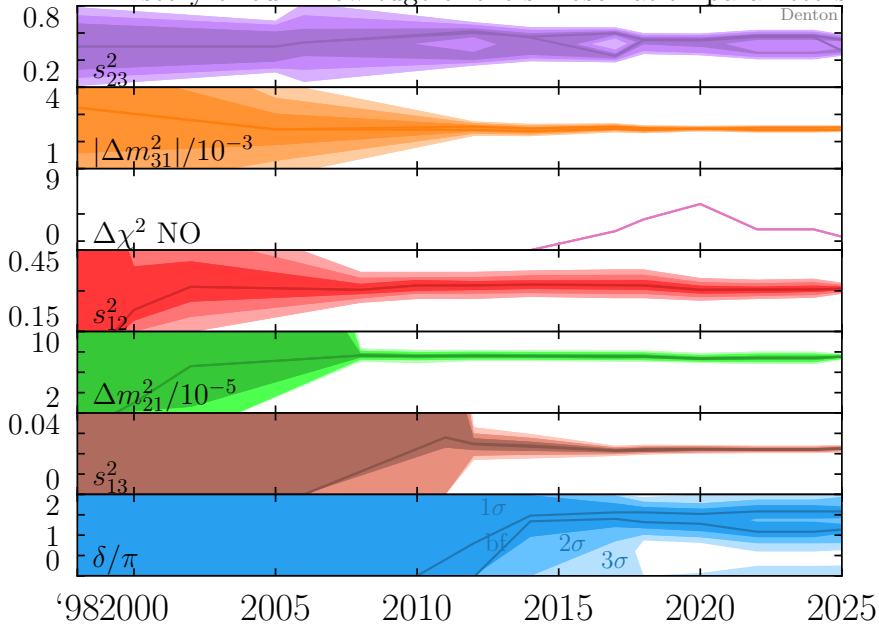
Peter B. Denton

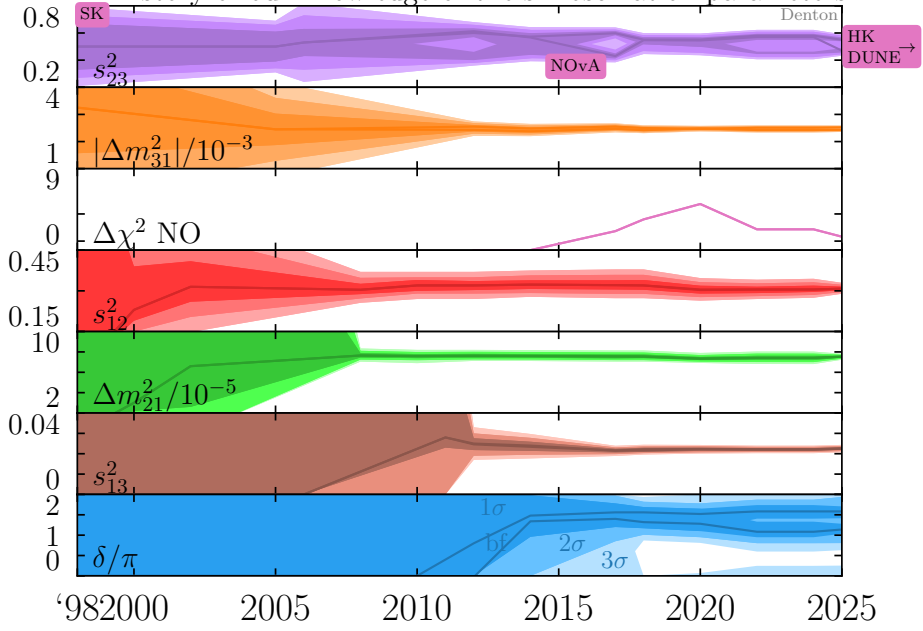
Wayne State University

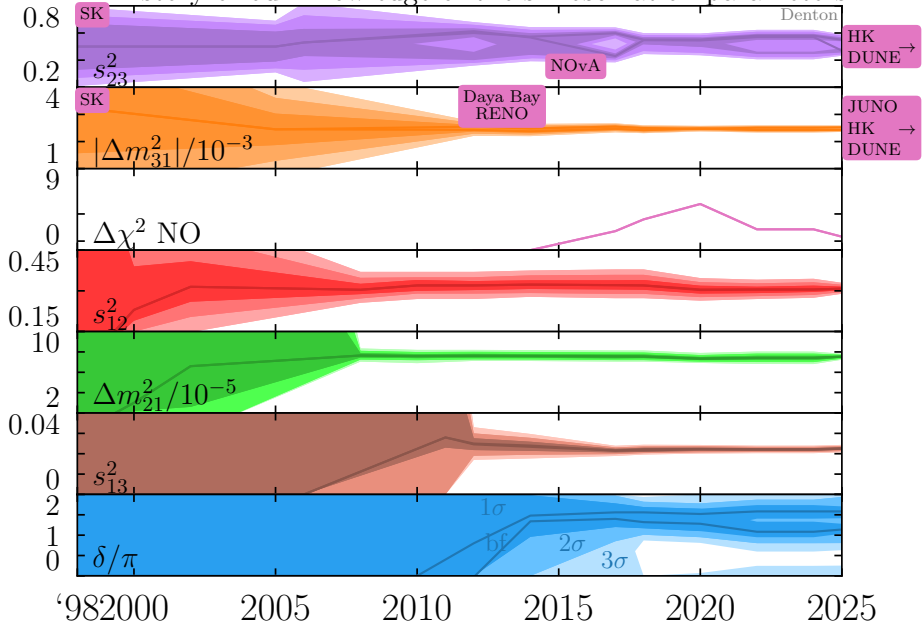
October 29, 2026

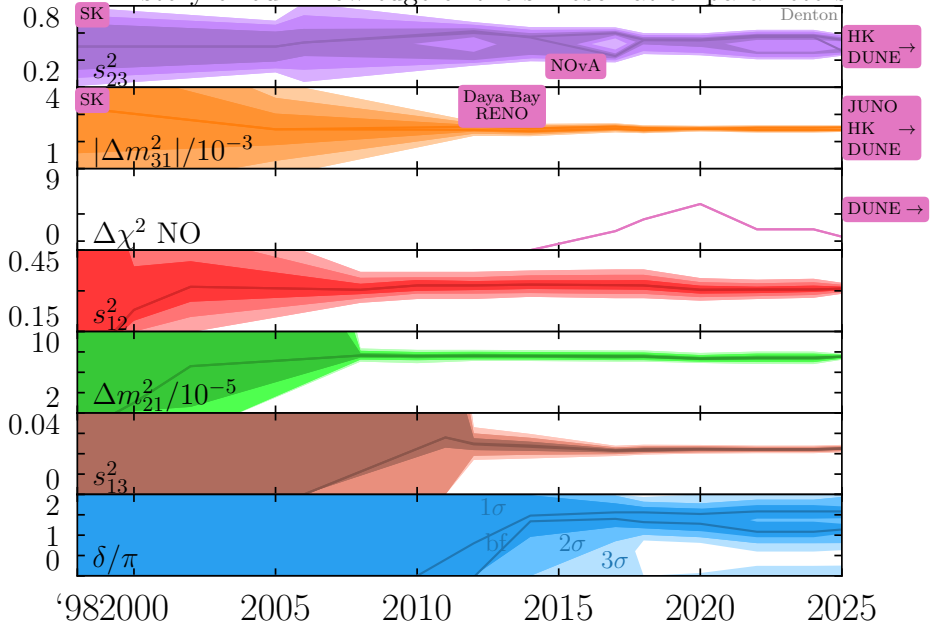


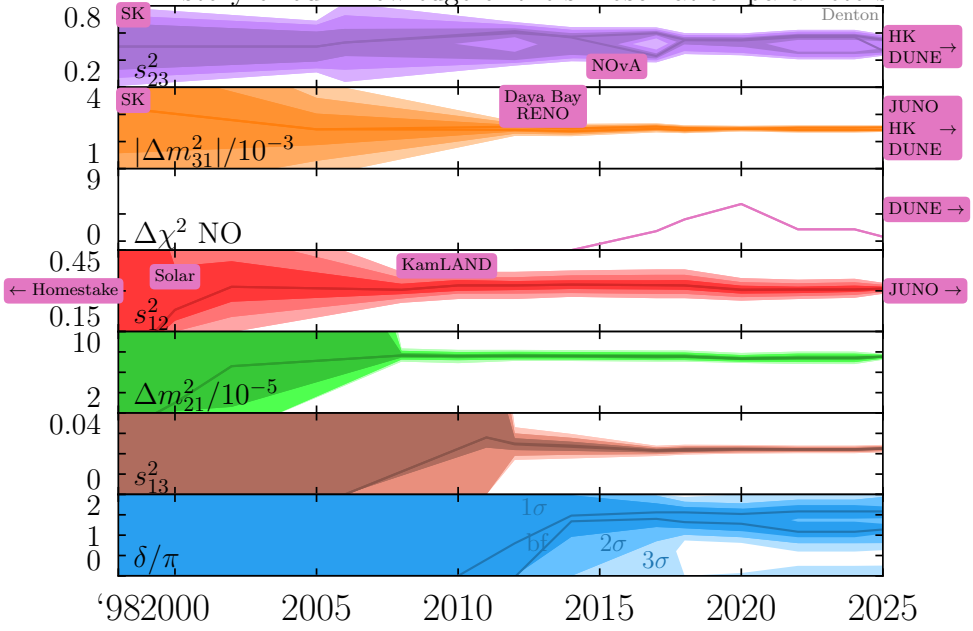

The BSM
Physicists

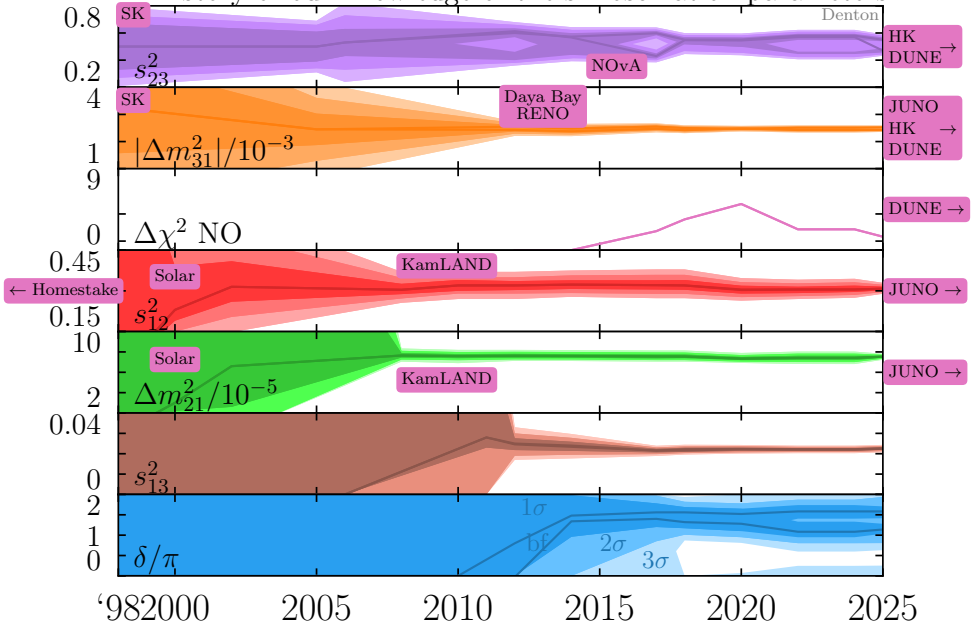

History of our knowledge of the six oscillation parameters
Denton
0.8
0.2
s^2_{23}
4
1
$|\Delta m^2_{31}|/10^{-3}$
9
0
$\Delta\chi^2$ NO
0.45
0.15
s^2_{12}
10
2
$\Delta m^2_{21}/10^{-5}$
0.04
0
s^2_{13}
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δ/π
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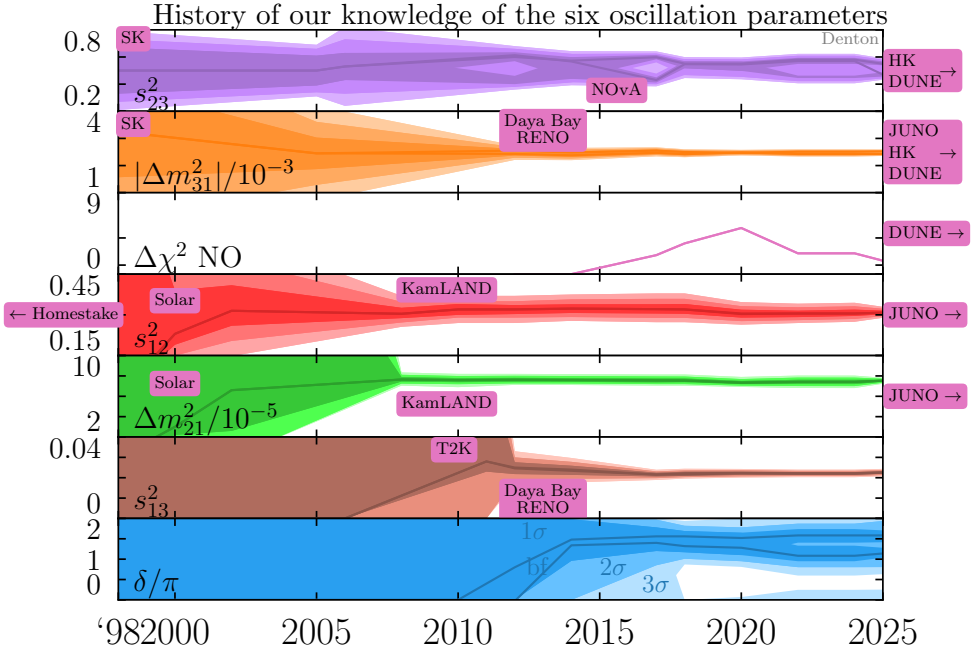

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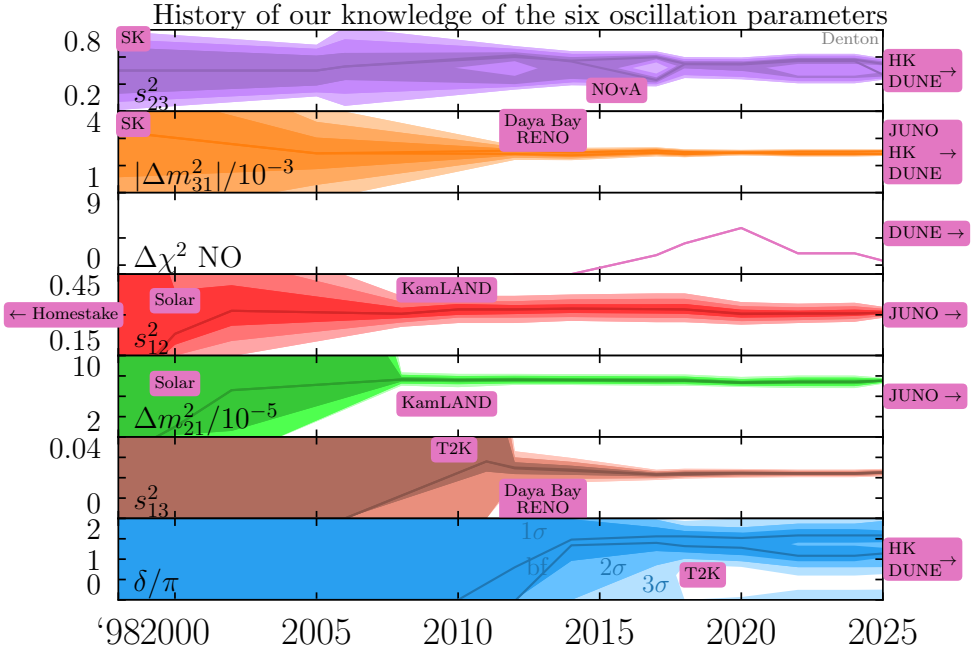

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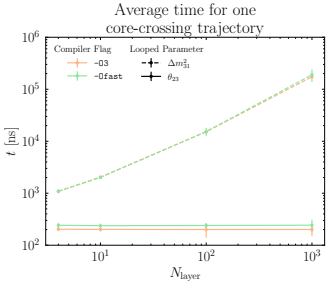

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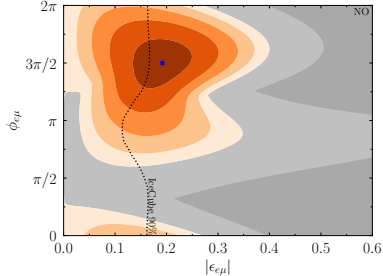

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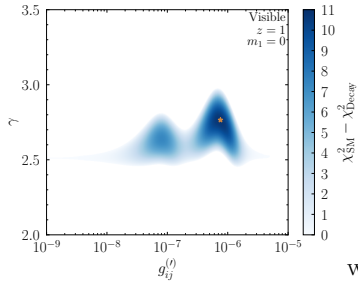
Part 1:
Standard Three-Flavor Oscillations



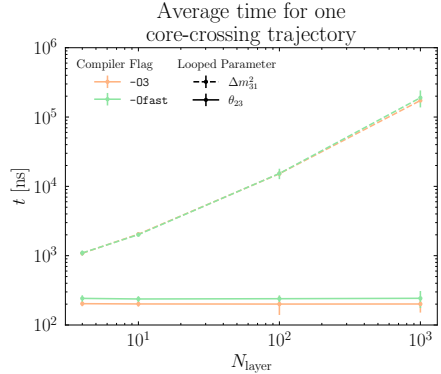
Part 2:
New Physics in Neutrinos



Part 3:
Neutrinos in Space



Part 1: Standard Three-Flavor Oscillations



Neutrino Oscillation Probabilities, Fast

`NuFast`

Constant density: github.com/PeterDenton/NuFast-LBL
Varying density: github.com/PeterDenton/NuFast-Earth

Fermilab computing experts bolster NOvA evidence, 1 million cores consumed

July 3, 2018 | Marcia Teckenbrock

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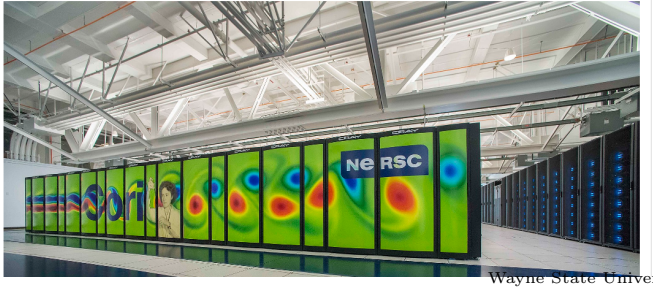
Array

How do you arrive at the physical laws of the universe when you're given experimental data on a renegade particle that interacts so rarely with matter, it can cruise through light-years of lead? You call on the power of advanced computing.

The NOvA neutrino experiment, in collaboration with the Department of Energy's Scientific Discovery through Advanced Computing (SciDAC-4) program and the HEPCloud program at DOE's Fermi National Accelerator Laboratory, was able to perform the largest-scale analysis ever to support the recent evidence of antineutrino oscillation, a phenomenon that may hold clues to how our universe evolved.

Using Cori, the newest supercomputer at the National Energy Research Scientific Computing Center (NERSC), located at Lawrence Berkeley National Laboratory, NOvA used over 1 million computing cores, or CPUs, between May 14 and 15 and over a short timeframe one week later. This is the largest number of CPUs ever used concurrently over this duration — about 54 hours — for a single high-energy physics experiment. This unprecedented amount of computing enabled scientists to carry out some of the most complicated techniques used in neutrino physics, allowing them to dig deeper into the seldom seen interactions of neutrinos. This Cori allocation was more than 400 times the amount of Fermilab computing allocated to the NOvA experiment and 50 times the total computing capacity at Fermilab allocated for all of its rare-physics experiments. A continuation of the analysis was performed on NERSC's Cori and Edison supercomputers one week later. In total, nearly 35 million core-hours were consumed by NOvA in the 54-hour period. Executing the same analysis on a single desktop computer would take 4,000 years.

FNAL Newsroom



Fermilab computing experts bolster NOvA evidence, 1 million cores consumed

July 3, 2018 | Marcia Teckenbrock

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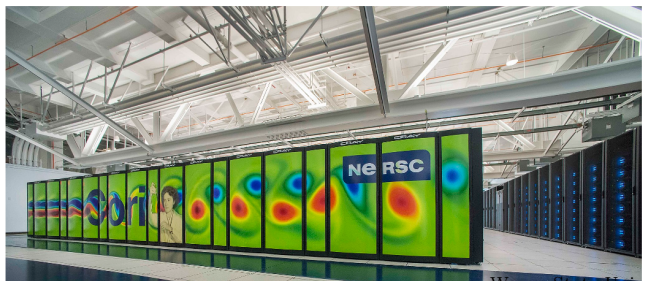
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Benchmarking showed that most of the time was spent computing probabilities

Fermilab allocated for all of its rare-physics experiments. A continuation of the analysis was performed on NERSC's Cori and Edison supercomputers one week later. In total, nearly 35 million core-hours were consumed by NOvA in the 54-hour period. Executing the same analysis on a single desktop computer would take 4,000 years.

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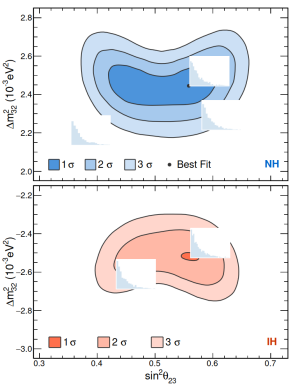


Monte-Carlo Estimates of Statistical Significances

Wilks' theorem is often wrong

At each point in parameter space, simulate the experiment many times

"many" means $\gg 1/p$ for a desired p-value
This is sometimes called Feldman-Cousins
G. Feldman, R. Cousins physics/9711021



NOvA/T2K are $\sim 3\sigma$ experiments,
but DUNE/HK will be $\gtrsim 5\sigma$ experiments!

NOvA: compute the probabilities "a gazillion times"
DUNE: compute the probabilities "a zillion times"

Algorithm for Constant Density (Long-Baseline Accelerator)

Need:

1. Eigenvalues
2. Eigenvectors
3. Construct the probability

Algorithm for Constant Density (Long-Baseline Accelerator)

Need:

1. Eigenvalues
2. Eigenvectors
3. Construct the probability

$$\lambda_3 \approx \Delta m^2_{31} + \frac{1}{2}\Delta m^2_{ee}\left(x - 1 + \sqrt{(1-x)^2 + 4x s^2_{13}}\right)$$

$$x \equiv \frac{a}{\Delta m^2_{ee}} \qquad \Delta m^2_{ee} \equiv \Delta m^2_{31} - s^2_{12}\Delta m^2_{21}$$

H. Minakata, S. Parke 1505.01826
PBD, H. Minakata, S. Parke 1604.08167

`NuFast-LBL` approach:

1. Approximate one eigenvalue: λ_3
   - Can iteratively improve precision rapdily with Newton's method
2. Get other two via two of the Cayley-Hamilton conditions
3. Actually only need $|V_{\alpha i}|^2$ of the eigenvectors

PBD, S. Parke, X. Zhang 1907.02534

   - Leverages NHS identity for the CP violating part

V. Naumov IJMP 1992
P. Harrison, W. Scott hep-ph/9912435

4. Use *Eigenvector-Eigenvalue Identity* to get $|V_{\alpha i}|^2$ from eigenvalues
5. Combine to probabilities

Algorithm for Constant Density (Long-Baseline Accelerator)

Need:

1. Eigenvalues
2. Eigenvectors
3. Construct the probability

$$|V_{\alpha i}|^2 = \frac{\prod_{k=1}^{n-1}(\lambda_i - \xi_k^\alpha)}{\prod_{k=1;k\neq i}^{n}(\lambda_i - \lambda_k)}$$

PBD, S. Parke, T. Tao, X. Zhang 1908.03795
ξ_k^α are submatrix eigenvalues: easy to compute

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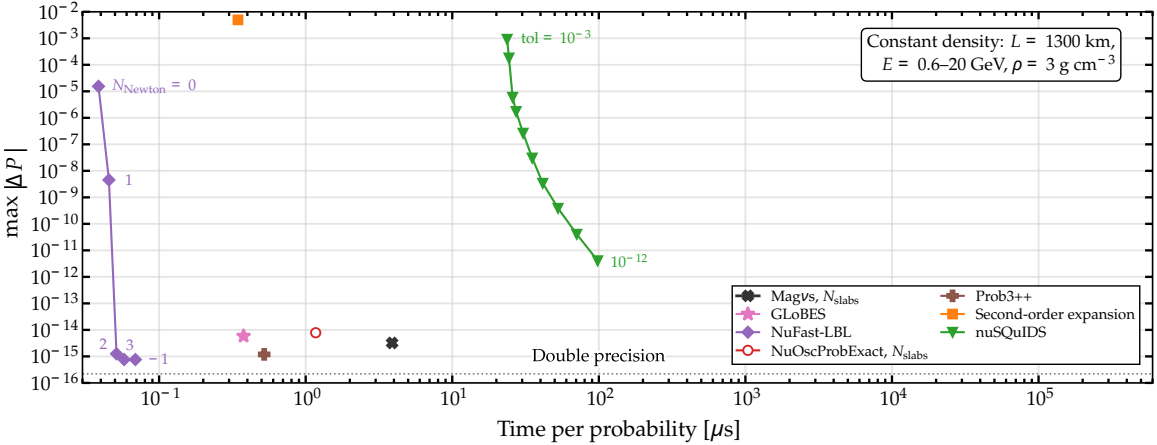
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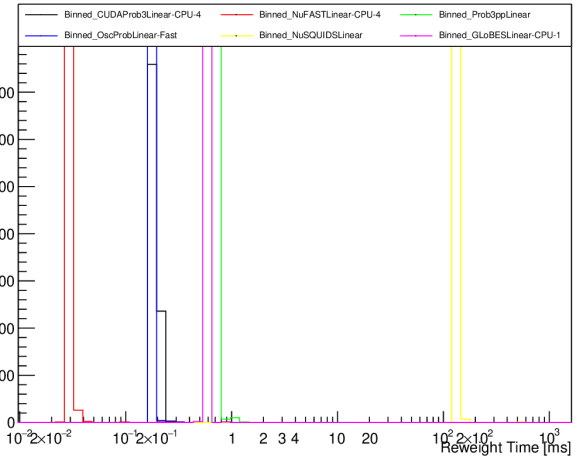
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NuFast-LBL is Fast: Theorist Comparison



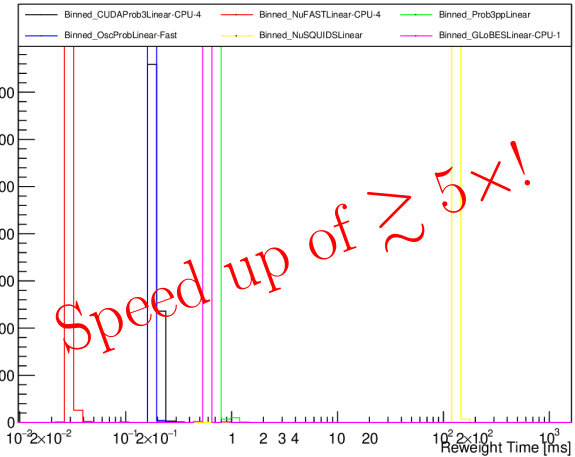
M. Bustamante 2610.07159

NuFast-LBL is Fast: Experimental Comparison



D. Barrow, et al. NuOscillator

NuFast-LBL is Fast: Experimental Comparison

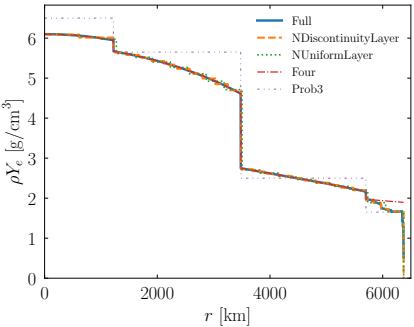


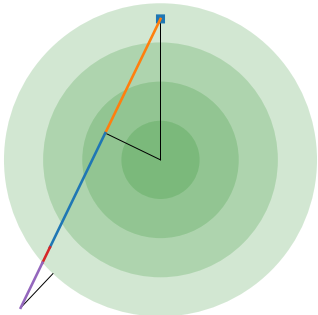
D. Barrow, et al. NuOscillator

Atmospherics and Solar are Harder

1. Propagate through the sharply varying Earth
   - Cannot rewrite probability in terms of $|V_{\alpha i}|^2$
2. Need many more computations than LBL
   - Many Earth layers necessary to accurately model Earth
   - Broader energy range
   - Varying $\cos\theta_z$ instead of fixed L
3. High precision necessary to avoid aliasing in solar

Earth Trajectories





- Various Earth density profiles
- Varying or constant density in each shell

Trajectory goes from:

1. Production height to surface
2. Surface to detector depth
3. Detector depth to deepest point
4. Deepest point to detector depth: this is the transpose of #3

New Innovations for Atmospherics

1. Same approach for eigenvalues
2. New approach for eigenvectors: advanced Eigenvector-Eigenvalue identity

   PBD, S. Parke, T. Tao, X. Zhang 1908.03795
   A. Abdullahi, S. Parke 2212.12565
3. Pull out $U_{23}(\theta_{23}, \delta)$ from computation
   - Hamiltonian is real in new basis
4. Cache computations
   - Trajectories
   - Eigenvalue and eigenvector information
   - Amplitude through the Earth
5. Apply θ_{23}, δ, production height in atmosphere at end

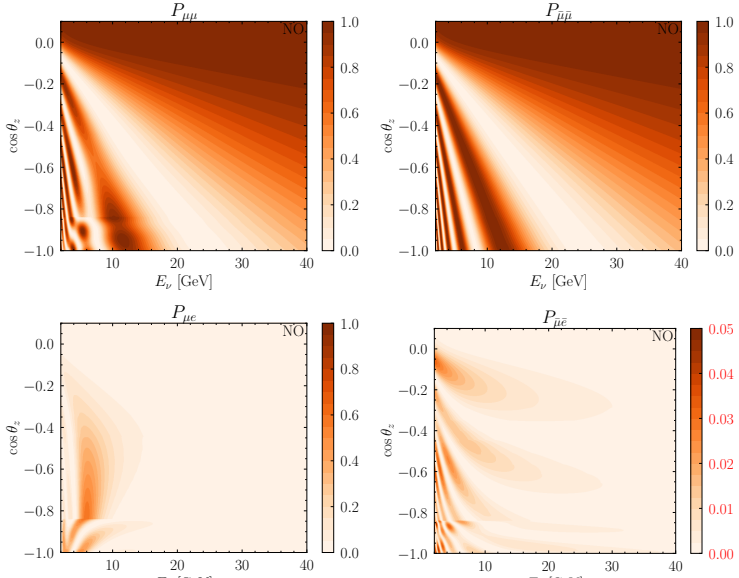
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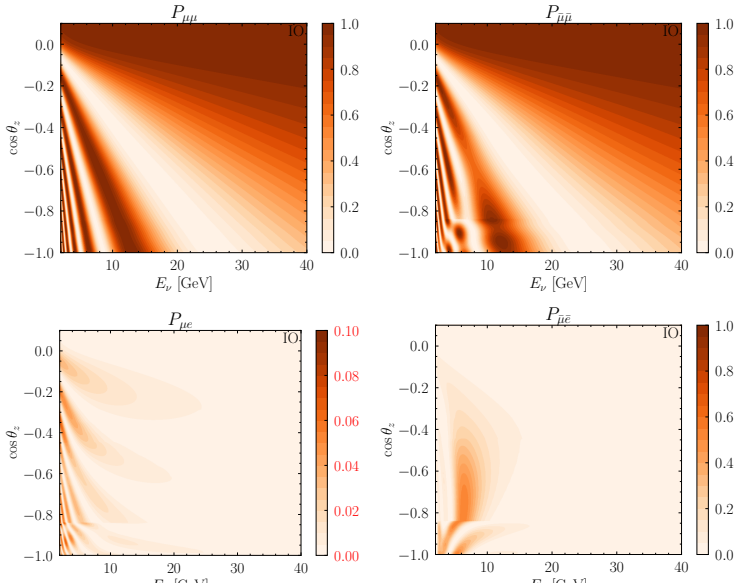
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Varying θ_{23}, δ, and production height is essentially free:
Put on inside of loops

Atmospheric Results: NO



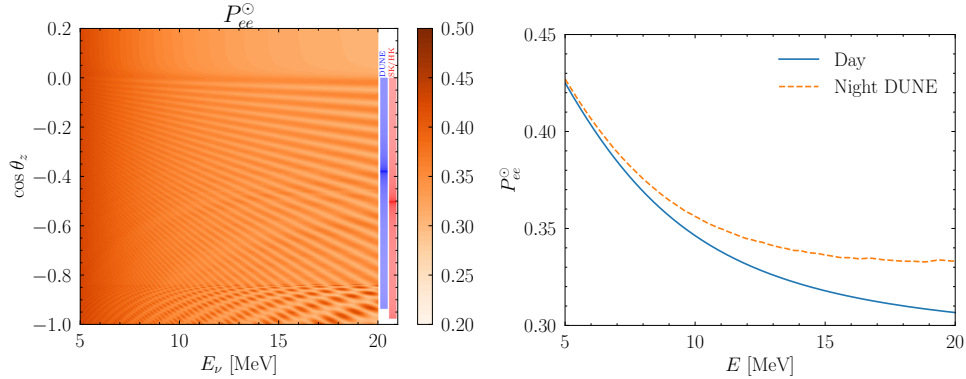
Atmospheric Results: IO



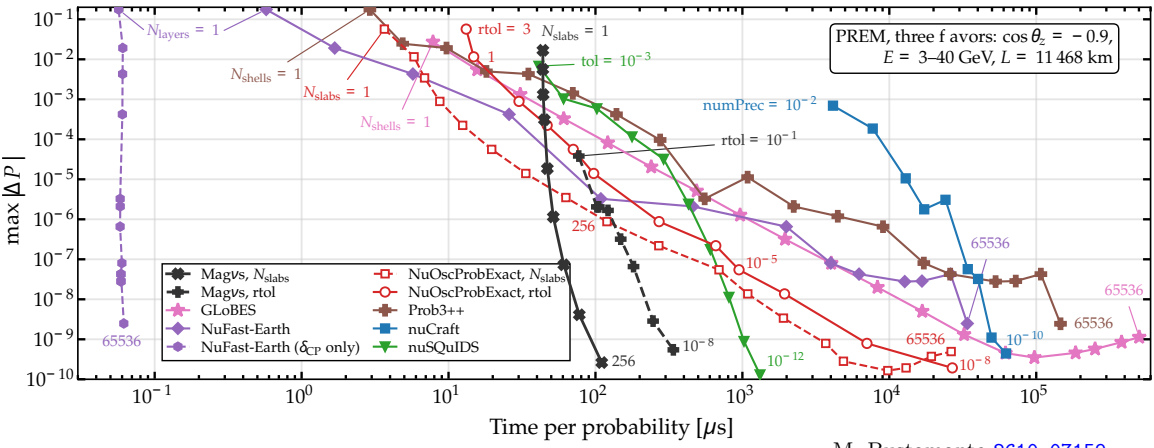
Nighttime Solar Neutrinos

At night solar neutrinos experience regeneration:
There are more ν_e's from the Sun at night than during the day!

SuperK has $\sim 2\sigma$ evidence for this effect; DUNE and HK aim to measure it well

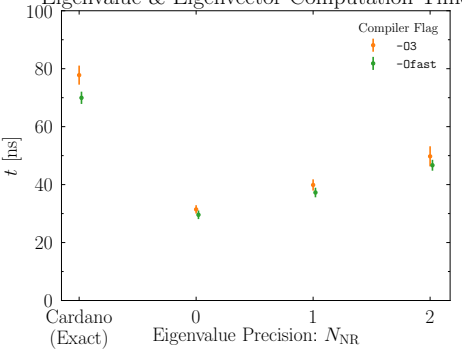


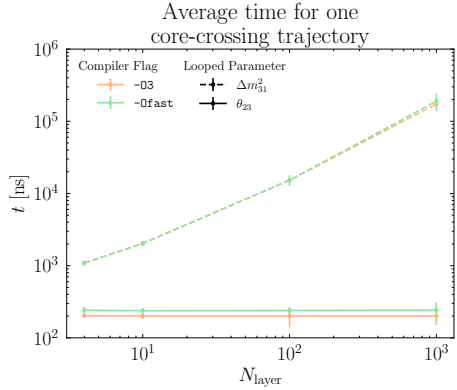
NuFast-Earth is Fast: Theorist Comparison



M. Bustamante 2610.07159

NuFast-Earth is Fast: Our Tests





`OscProb` takes 40 000 ns per trajectory at 44 layers (compare to 6000 ns or 200 ns)

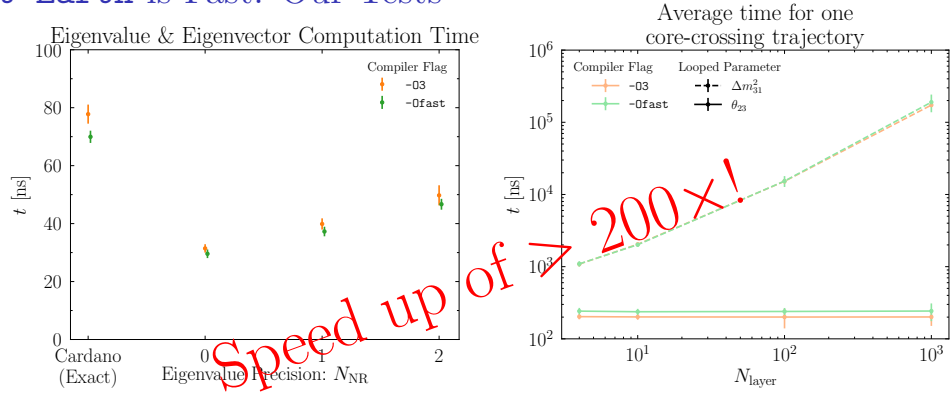
J. Coelho, R. Pestes, et al. github.com/joaoabcoelho/OscProb

`nuSQuIDS` takes >1M ns per trajectory

C. Argüelles, J. Salvado, C. Weaver 2112.13804

Other codes are designed with different goals in mind

NuFast-Earth is Fast: Our Tests



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CP violation in neutrinos

δ and CP Violation

$$J_{CP} = s_{12}c_{12}s_{13}c_{13}^2 s_{23}c_{23}\sin\delta$$

C. Jarlskog PRL 55, 1039 (1985)

δ and CP Violation

$$J_{CP} = s_{12}c_{12}s_{13}c_{13}^2 s_{23}c_{23}\sin\delta$$

C. Jarlskog PRL 55, 1039 (1985)

1. Strong interaction: no observed EDM $\Rightarrow$ CP (nearly) **conserved**

$$\frac{\bar{\theta}}{2\pi} < 10^{-11}$$

J. Pendlebury, et al. 1509.04411

2. Quark mass matrix: non-zero but **small** CP violation

$$\frac{|J_{\rm CKM}|}{J_{\rm max}} = 3\times 10^{-4}$$

J. Christenson, J. Cronin, V. Fitch, R. Turlay PRL 1964
CKMfitter 1501.05013

3. Lepton mass matrix: **?**

$$\frac{|J_{\rm PMNS}|}{J_{\rm max}} < 0.34$$

PBD, J. Gehrlein, R. Pestes 2008.01110

$$J_{\rm max} = \tfrac{1}{6\sqrt{3}} \approx 0.096$$

The Role of Appearance and Disappearance

- Disappearance ($\nu_\alpha \to \nu_\alpha$) are CP even ← nearly all experiments
- Appearance ($\nu_\alpha \to \nu_\beta$) have CP even and CP odd parts ← only two

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  ... given known oscillation parameters, systematics, and realistic experiments

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The Role of Appearance and Disappearance

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- **Disappearance can measure δ and determine if CP is violated or not**
  - CPV can be discovered with only disappearance measurements
  - Probes mostly $\cos\delta$ not $\sin\delta$
  - Requires measurements of two flavors
  - "Works through unitarity" (as do nearly all oscillation measurements)

Direct Analytic Calculation

Disappearance experiments measure various $|U_{\alpha i}|^2$ terms
Suppose 4 are measured: $|U_{e2}|^2$, $|U_{e3}|^2$, $|U_{\mu 2}|^2$, $|U_{\mu 3}|^2$

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$$J_{CP}^2 = |U_{e2}|^2|U_{\mu 2}|^2|U_{e3}|^2|U_{\mu 3}|^2 - \frac{1}{4}\left(1 - |U_{e2}|^2 - |U_{\mu 2}|^2 - |U_{e3}|^2 - |U_{\mu 3}|^2 + |U_{e2}|^2|U_{\mu 3}|^2 + |U_{e3}|^2|U_{\mu 2}|^2\right)^2$$

Can show that if any one $|U_{\alpha i}|^2 = 0 \Rightarrow J = 0$

Direct Analytic Calculation

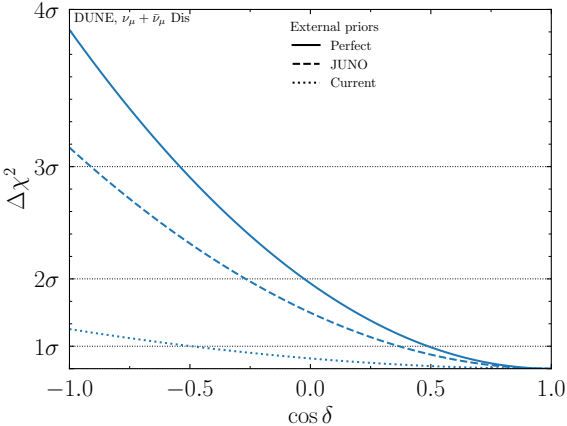
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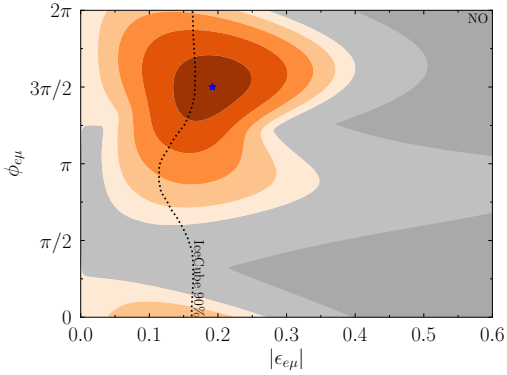
Disappearance can tell us if CP is violated,
but not if Nature prefers ν's or $\bar{\nu}$'s

JUNO and DUNE Disappearance Combined Sensitivities



Different systematics $\Rightarrow$ crucial cross check of δ

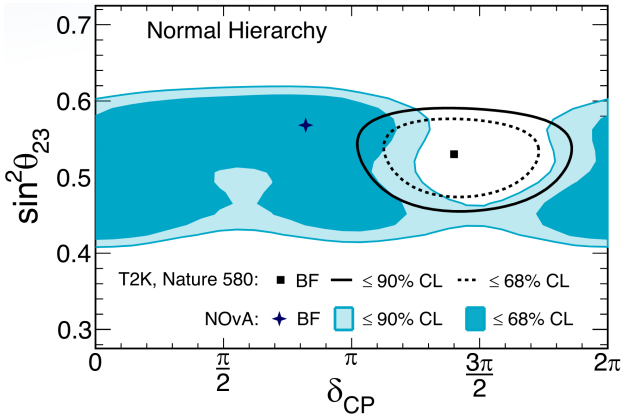
Part 2: New Physics in Neutrinos



CP Violation at NOvA and T2K?

T2K wants $\delta \sim 3\pi/2$ and $\sin^2\theta_{23}$ large

NOvA wants anything else



A. Himmel for NOvA 10.5281/zenodo.3959581

NSI Review

$$\mathcal{L}_{\text{NSI}}^{\text{NC}} = -2\sqrt{2}G_F \sum_{\alpha,\beta,f,P} \epsilon_{\alpha\beta}^{f,P} (\bar{\nu}_\alpha \gamma^\mu \nu_\beta)(\bar{f}\gamma_\mu f)$$

Models with large NSIs consistent with CLFV:

Y. Farzan, I. Shoemaker 1512.09147 Y. Farzan, J. Heeck 1607.07616 D. Forero and W. Huang 1608.04719
K. Babu, A. Friedland, P. Machado, I. Mocioiu 1705.01822 PBD, Y. Farzan, I. Shoemaker 1804.03660
U. Dey, N. Nath, S. Sadhukhan 1804.05808 Y. Farzan 1912.09408 N. Bernal, Y. Farzan 2211.15686
S. Abbaslu, Y. Farzan 2407.13834

Affects oscillations via new matter effect

$$H = \frac{1}{2E}\left[UM^2U^\dagger + a\begin{pmatrix} 1+\epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix}\right]$$

Matter potential $a \propto G_F \rho E$

Estimate Size of Effect: Magnitude

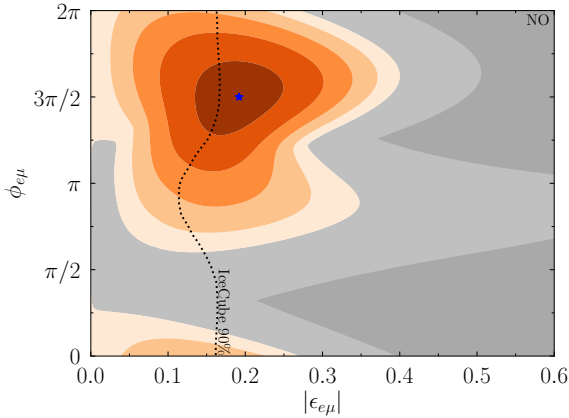
$$|\epsilon_{e\beta}| \approx \frac{s_{12}c_{12}c_{23}\pi\Delta m^2_{21}}{2s_{23}w_\beta} \left| \frac{\sin\delta_{\mathrm{T2K}} - \sin\delta_{\mathrm{NOvA}}}{a_{\mathrm{NOvA}} - a_{\mathrm{T2K}}} \right| \approx \begin{cases} 0.22 & \text{for } \beta = \mu \\ 0.24 & \text{for } \beta = \tau \end{cases}$$

$a \propto \rho E$
Requires $\phi_{e\beta} \sim \pm\pi/2$
$w_\beta = s_{23}$, c_{23} for $\beta = \mu, \tau$
Assumed upper octant $\theta_{23} > 45°$

Consistency checks:

- $\sin\delta_{\mathrm{NOvA}} = \sin\delta_{\mathrm{T2K}} \Rightarrow |\epsilon| = 0$
- $\sin\delta_{\mathrm{NOvA}} \neq \sin\delta_{\mathrm{T2K}}$ and $a_{\mathrm{NOvA}} = a_{\mathrm{T2K}} \Rightarrow |\epsilon| \to \infty$
- Octant:
  1. LBL is governed by ν_3
  2. Upper octant $\Rightarrow \nu_3$ is more ν_μ
  3. More $\nu_\mu \Rightarrow$ need less new physics coupling to ν_μ to produce a given effect

NSI Parameters



Orange is preferred over SM at integer values of $\Delta\chi^2$, dark gray is disfavored at 4.61

T. Ehrhardt, IceCube PPNT (2019)
IceCube 2106.07755

New physics, degeneracies, and the impossibility of determining the mass ordering

Mass Ordering: New Physics Degeneracies

In the presence of new physics such as NSI we have:

$$[\text{NO}] + [\epsilon = 0] \quad \equiv \quad [\text{IO}] + [\epsilon_{ee} = -2]$$

$$[\text{IO}] + [\epsilon = 0] \quad \equiv \quad [\text{NO}] + [\epsilon_{ee} = -2]$$

Equivalences hold even if all oscillation probabilities are *perfectly* measured

P. Bakhti, Y. Farzan 1403.0744
P. Coloma, T. Schwetz 1604.05772
P. Coloma, PBD, et al. 1701.04828
PBD, Y. Farzan, I. Shoemaker 1804.03660
PBD, S. Parke 2106.12436
PBD, J. Gehrlein 2204.09060



This is known as the **LMA-Dark** solution

Is the Mass Ordering Robust?

Need **scattering** to break



Can probe same NC $\epsilon = -2$ process in scattering, but...

1. CHARM and NuTeV for $M_{Z'} \gtrsim 10$ GeV

   P. Coloma, PBD, et al. 1701.04828

2. COHERENT for $M_{Z'} \gtrsim 50$ MeV and cosmology for $M_{Z'} \lesssim 5$ MeV

   PBD, Y. Farzan, I. Shoemaker 1804.03660

3. Dresden-II for ϵ_{ee} for any mediator mass

   PBD, J. Gehrlein 2204.09060

4. Can still evade with specific flavor structures

   $\epsilon_{\mu\mu} = \epsilon_{\tau\tau} = 2$ or certain u / d combinations

5. CCM, COHERENT, and DM direct detection can close all loopholes

Many Interesting New Physics Scenarios in Oscillations

1. Sterile neutrinos

2. Non-standard neutrino interactions (NSI)
   with any Lorentz structure: SPVAT

3. Non-standard neutrino self interactions
4. Neutrino decay
   with visible or invisible final states
5. Unitarity violation
6. Many others: neutrino – dark matter interactions, environmental decoherence, and Lorentz invariance or CPT violation

Many Interesting New Physics Scenarios in Oscillations

1. Sterile neutrinos

   PBD, Y. Farzan, I. Shoemaker 1811.01310
   PBD 2111.05793
   H. Davoudiasl, PBD 2301.09651

2. Non-standard neutrino interactions (NSI)

   with any Lorentz structure: SPVAT

   PBD, Y. Farzan, I. Shoemaker 1804.03660
   P. Coloma, PBD, et al. 1701.04828
   PBD, J. Gehrlein, R. Pestes 2008.01110
   PBD, J. Gehrlein 2008.06062, 2204.09060
   PBD, A. Giarnetti, D. Meloni 2210.00109, 2409.15411

3. Non-standard neutrino self interactions

   Barenboim, PBD, Oldengott 1903.02036

4. Neutrino decay

   with visible or invisible final states

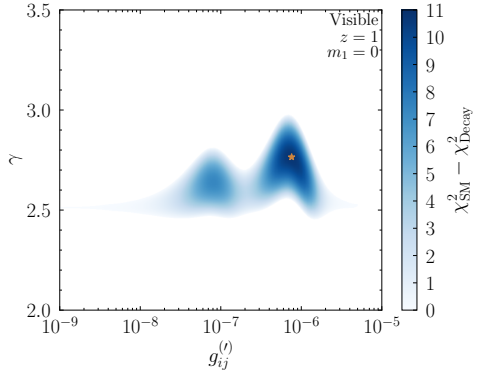
   PBD, I. Tamborra 1805.05950
   PBD, A. Abdullahi 2005.07200

5. Unitarity violation

   PBD 2109.14576
   PBD, J. Gehrlein 2109.14575

6. Many others: neutrino – dark matter interactions, environmental decoherence, and Lorentz invariance or CPT violation

Part 3: Neutrinos in Space



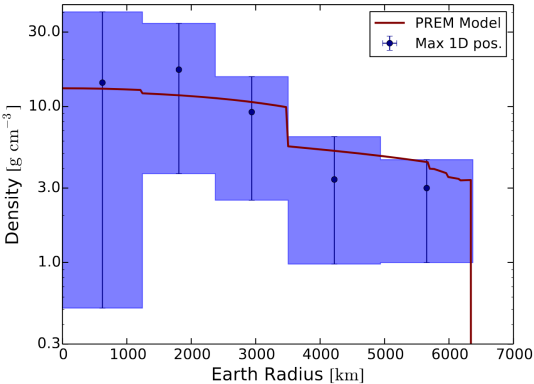
"Ask not what your country can do for you –
ask what you can do for your country"

What can neutrinos do for you
and
what you can do for neutrinos

Neutrinos shine a flashlight in opaque environments

Terrestrial Tomography

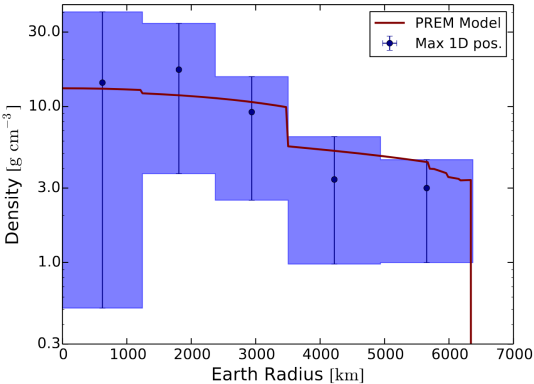
HE neutrinos are absorbed in the Earth
Assumes isotropic flux



A. Donini, S. Palomares-Ruiz, J. Salvado 1803.05901

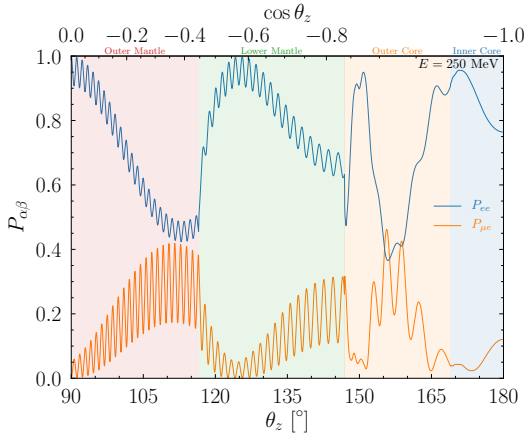
Terrestrial Tomography

HE neutrinos are absorbed in the Earth
Assumes isotropic flux



A. Donini, S. Palomares-Ruiz, J. Salvado 1803.05901

Can also do this with oscillations

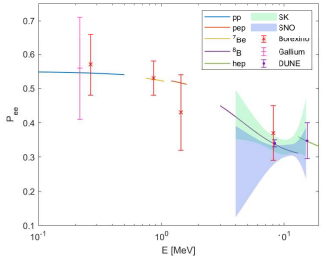


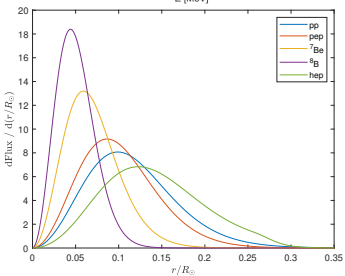
PBD, R. Pestes 2110.01148

DUNE-atm can measure $r_{\rm core}^{\oplus}$ to 9%

See also K. Kelly, et al. 2110.00003

Solar Tomography

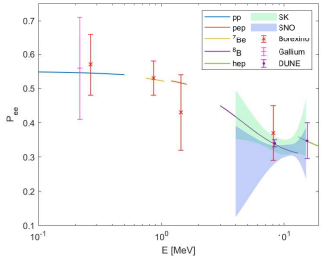


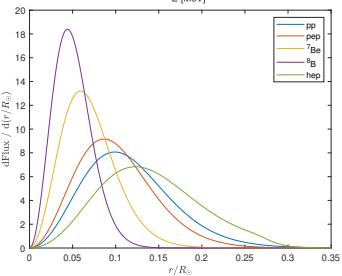


Solar neutrinos constrain Sun's density at different radii

Solar Tomography

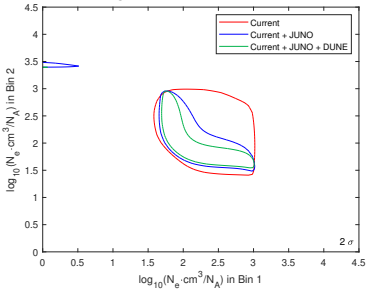
Solar neutrinos constrain Sun's density at different radii



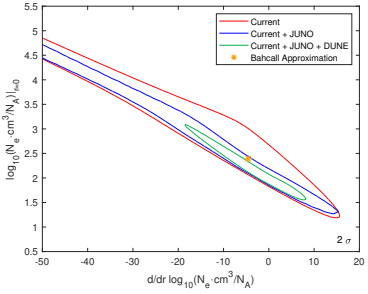


Bin 1: $\frac{r}{R_\odot} \in [0, 0.05]$

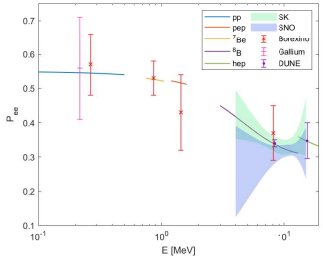
Bin 2: $\frac{r}{R_\odot} \in [0.05, 0.1]$

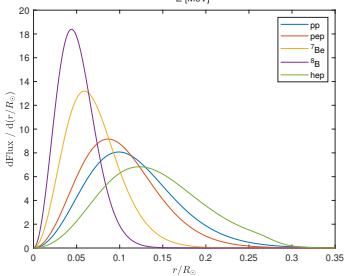


$$\log_{10} N_e = Ar + B$$



Solar Tomography

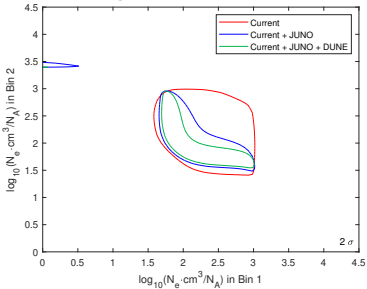




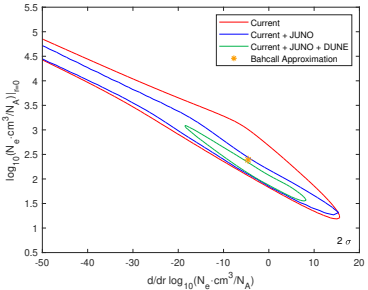
Solar neutrinos constrain Sun's density at different radii

Bin 1: $\frac{r}{R_\odot} \in [0, 0.05]$

Bin 2: $\frac{r}{R_\odot} \in [0.05, 0.1]$



$$\log_{10} N_e = Ar + B$$

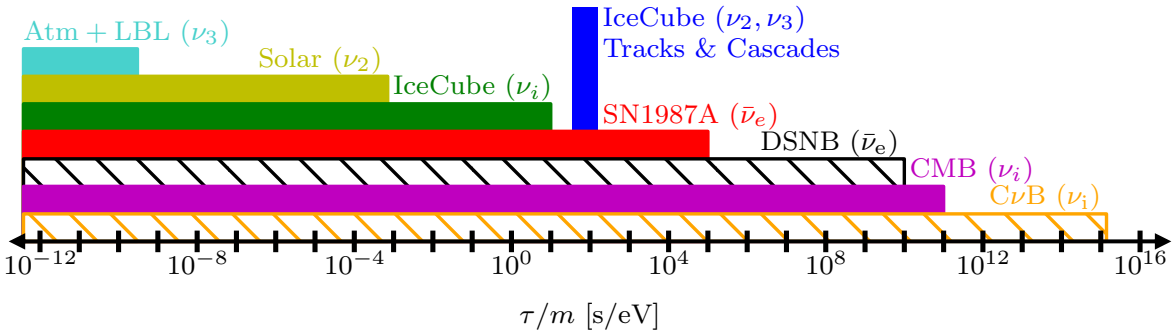


First determination of Sun's density with neutrinos

Only way to probe innermost 5% of the Sun

New particle physics in neutrinos from astrophysics

Invisible ν Decay Constraints and Evidence

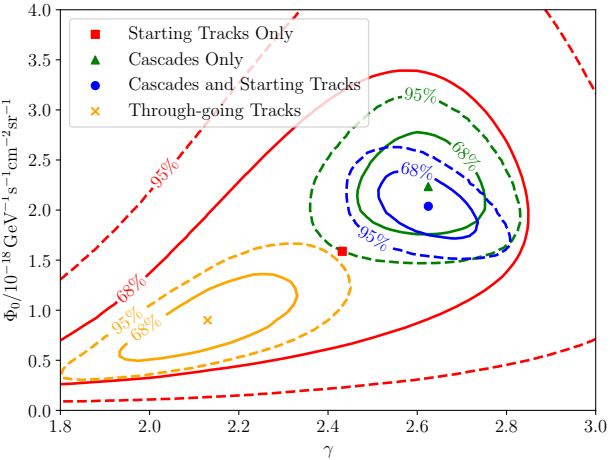


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J. Berryman, A. de Gouvea, D. Hernandez 1411.0308
G. Pagliaroli, et al. 1506.02624
PBD, I. Tamborra 1805.05950

Kamiokande-II, PRL 58 1490 (1987)
S. Ando hep-ph/0307169
S. Hannestad, G. Raffelt hep-ph/0509278
A. Long, C. Lunardini, E. Sabancila 1405.7654

Tension

$$\Phi(E) = \Phi_0 E^{-\gamma}$$



$$\Delta\gamma = +0.54$$

"The p-value for obtaining the combined fit result and the result reported here from an unbroken powerlaw flux is 3.3σ, and is therefore in significant **tension**."

IC 1607.08006

"This [cascade] fit [is] in **tension** with previous results based on through-going muons"

IC 1808.07629

"[with cascades] we reject spectral indices $\gamma \leq 2.28$ [latest track index] at $\geq 3\sigma$ level."

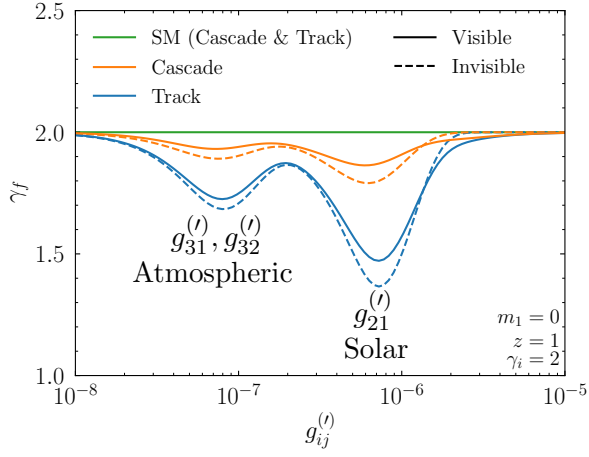
IC 2001.09520

Spectral Indices at IceCube

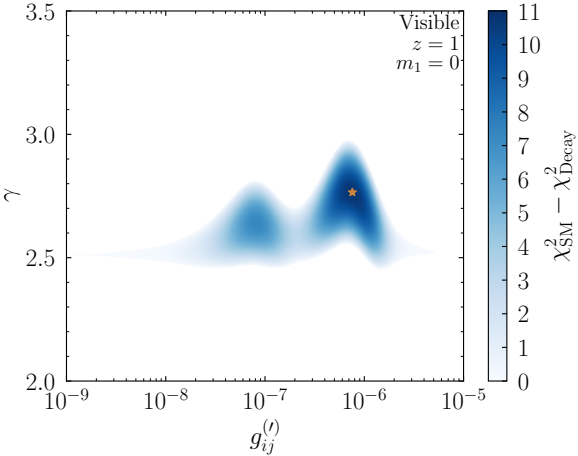
1. Neutrino decay completely removes tension
2. Is preferred over SM at $> 3\sigma$
3. Either visible or invisible works
4. Requires normal mass ordering
5. Decay only shifts the spectra one way:

$$\gamma_c > \gamma_t$$

this is the direction of the tension



Preferred Region: Visible



- Consistent with astro constraints including SN1987A
- Terrestrial constraints at $g \sim 10^{-2}$

$$\pi \to \ell\nu\phi, \; K \to \ell\nu\phi$$

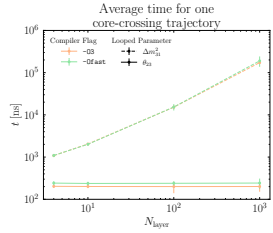
B. Dev, et al. 2407.12738

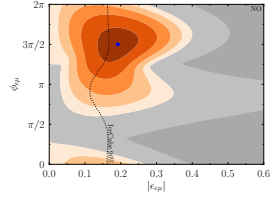
- Some tension with cosmology; possible to evade

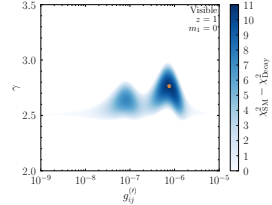
github.com/PeterDenton/Astro-Nu-Decay

Neutrino Theory Summary

- Fast probabilities are important; `NuFast` is fast
- More ways to get at CPV in neutrinos; systematics
- Data may be pointing to flavor-changing CPV NSI; can understand analytically
- Parameter space for LMA-Dark is closing fast; mass ordering robustness
- Neutrinos can probe opaque environments
- IceCube can probe particle BSM, $> 3\sigma$ for ν decay



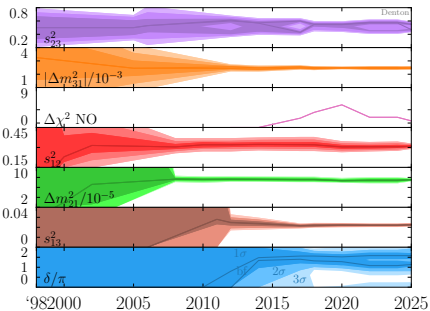




Thanks!

Backups

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M. Gonzalez-Garcia, M. Maltoni, J. Salvado 1001.4524

T2K 1106.2822

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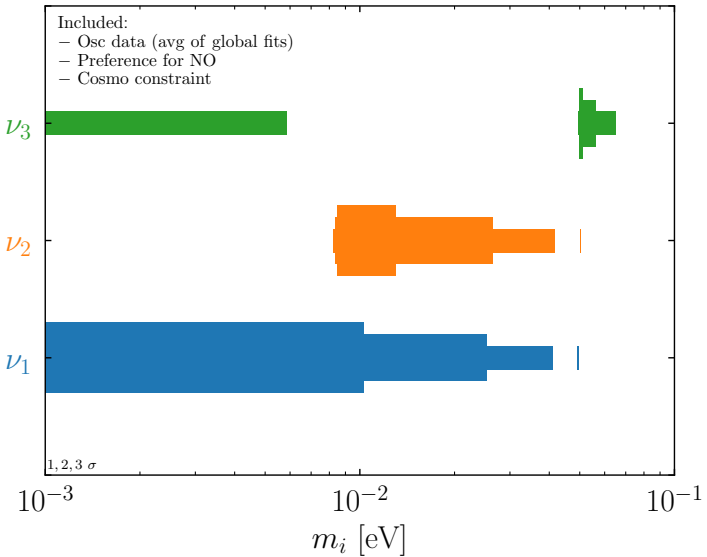
D. Forero, M. Tortola, J. Valle 1405.7540

P. de Salas, et al. 1708.01186

F. Capozzi et al. 2003.08511

I. Esteban, et al. 2007.14792

Absolute Masses



Optimal Structure of the Probability for Long-Baseline

1. Amplitude requires four trig functions of kinematic variables ($\Delta m^2_{ij} L/4E$) ×
2. Writing the probabilities out requires three trig functions ✓
3. Disappearance structure is straightforward:

$$P_{\alpha\alpha} = 1 - 4\sum_{i>j} |V_{\alpha i}|^2 |V_{\alpha j}|^2 \sin^2 \frac{\Delta\lambda_{ij} L}{4E}$$

H in matter has eigenvalues λ_i and eigenvectors $V_{\alpha i}$

Optimal Structure of the Probability for Long-Baseline

4. Appearance structure:
   T conserving:

$$P_{\mu e}^{TC} = 2\sum_{i>j}(|V_{\tau k}|^2 - |V_{\mu i}|^2|V_{ej}|^2 - |V_{\mu j}|^2|V_{ei}|^2)\sin^2\frac{\Delta\lambda_{ij}L}{4E}$$

Fun fact:
$2\Re(V_{\alpha i}V_{\beta j}^*V_{\alpha j}^*V_{\beta i})$
$= |V_{\alpha k}|^2|V_{\beta k}|^2 - |V_{\alpha i}|^2|V_{\beta i}|^2 - |V_{\alpha j}|^2|V_{\beta j}|^2$
$= |V_{\gamma k}|^2 - |V_{\alpha i}|^2|V_{\beta j}|^2 - |V_{\alpha j}|^2|V_{\beta i}|^2$

   T violating:

$$P_{\mu e}^{TV} = -8J\frac{\Delta m_{21}^2\Delta m_{31}^2\Delta m_{32}^2}{\Delta\lambda_{21}\Delta\lambda_{31}\Delta\lambda_{32}}\sin\frac{\Delta\lambda_{21}L}{4E}\sin\frac{\Delta\lambda_{31}L}{4E}\sin\frac{\Delta\lambda_{32}L}{4E}$$

Leverages NHS identity:
V. Naumov IJMP 1992
P. Harrison, W. Scott hep-ph/9912435

Account for Matter in Long-Baseline

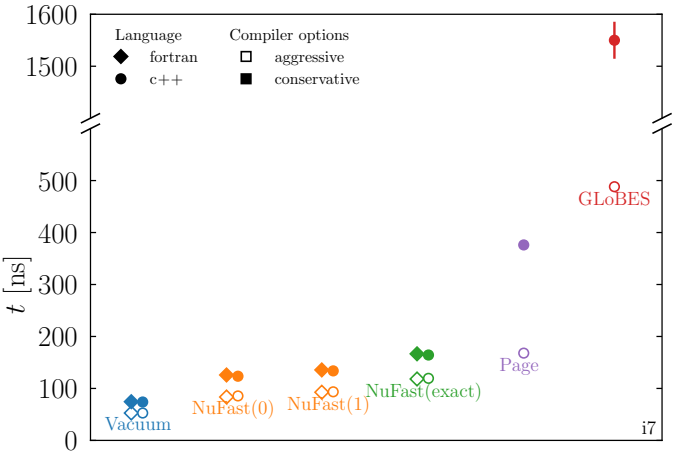
1. Need the eigenvalues λ_i
2. For eigenvectors, naively need $\Re(V_{\alpha i} V^*_{\beta j} V^*_{\alpha j} V_{\beta i})$
3. Given our form, need only the $|V_{\alpha i}|^2$ and J
   - Don't need any phase information of the eigenvectors!

   Leverages PBD, S. Parke, X. Zhang 1907.02534
4. Can compute the $|V_{\alpha i}|^2$ from the λ_i and submatrix eigenvalues (requires only a square root) using Eigenvector-Eigenvalue Identity

$$|V_{\alpha i}|^2 = \frac{\prod_{k=1}^{n-1}(\lambda_i - \xi_k^\alpha)}{\prod_{k=1;k\neq i}^{n}(\lambda_i - \lambda_k)}$$

See e.g. PBD, S. Parke, T. Tao, X. Zhang 1908.03795
Can actually avoid the $\sqrt{\ }$ in practice

NuFast-LBL is Fast: Our Comparison

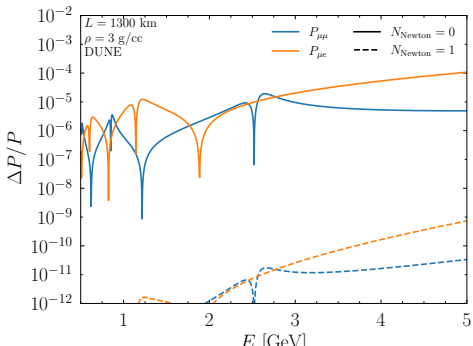


See also J. Page 2309.06900
and P. Huber, et al. hep-ph/0701187

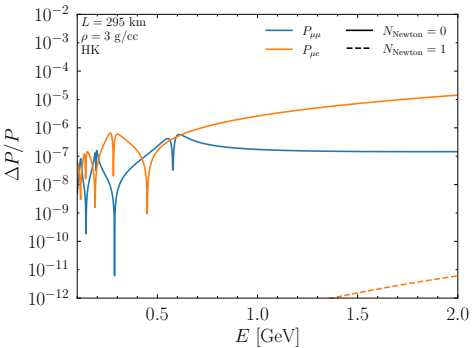
NuFast-LBL Precision

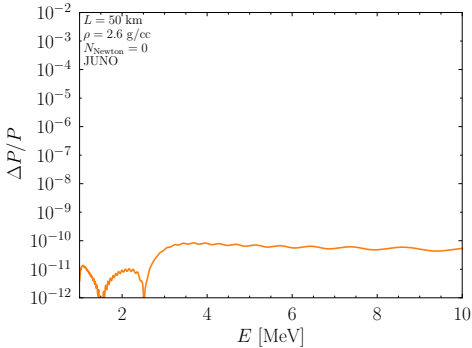
Is this approximation okay?

DUNE requires $\lesssim 1\%$ level precision



Precision


$L = 295$ km
$\rho = 3$ g/cc
HK
$P_{\mu\mu}$
$P_{\mu e}$
$N_{\text{Newton}} = 0$
$N_{\text{Newton}} = 1$
$\Delta P/P$
E [GeV]


$L = 50$ km
$\rho = 2.6$ g/cc
$N_{\text{Newton}} = 0$
JUNO
$\Delta P/P$
E [MeV]

NuFast-LBL: The Code

- Code is on github: github.com/PeterDenton/NuFast-LBL
- Implementations in c++ and f90

```
// NuFast.cpp
// Calculate the probabilities:
Probability_Matter_LBL(s12sq, s13sq, s23sq, delta,
    Dmsq21, Dmsq31, L, E, rho, Ye, N_Newton,
    &probs_returned);
// Print out the probabilities to terminal
for (int alpha = 0; alpha < 3; alpha++)
{
   for (int beta = 0; beta < 3; beta++)
   {
       printf("%d %d %g\n", alpha, beta,
           probs_returned[alpha][beta]);
   } // beta, 3
} // alpha, 3
```

- $\bar{\nu}$: $E < 0$; IO: $\Delta m^2_{31} < 0$
- Folder called `Benchmarks` to make the plots and speed tests in the paper
- Used in `NuOscillator`, `MaCh3`, `GUNDAM`, and theory papers
- Speed up of $\gtrsim 5\times$ over state-of-the-art in realistic usage

Optimal Structure of the Probability for Atmospherics

1. Cannot apply the previous tricks through the Earth
2. Must compute the amplitude in Each layer

$$\prod_j \mathcal{A}_j$$

Optimal Structure of the Probability for Atmospherics

3. Work in the tilde basis with θ_{23} and δ pulled out
   - Can shift δ from θ_{13} to θ_{23}

$$\begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} R_{12} \to \begin{pmatrix} 1 & & \\ & c_{23} & s_{23}e^{i\delta} \\ & -s_{23}e^{-i\delta} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13} \\ & 1 & \\ -s_{13} & & c_{13} \end{pmatrix} R_{12}$$

Optimal Structure of the Probability for Atmospherics

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   - U_{23} commutes with the matter potential

$$(2E)H_f = U_{23}(\theta_{23}, \delta) \left[R_{13} R_{12} \begin{pmatrix} 0 & & \\ & \Delta m^2_{21} & \\ & & \Delta m^2_{31} \end{pmatrix} R_{12}^T R_{13}^T + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right] U_{23}^\dagger(\theta_{23}, \delta)$$

$$H_f = U_{23}(\theta_{23}, \delta) \tilde{H} U^\dagger(\theta_{23}, \delta)$$

Optimal Structure of the Probability for Atmospherics

3. Work in the tilde basis with θ_{23} and δ pulled out
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$$H_f = U_{23}(\theta_{23},\delta)\tilde{H}U^\dagger(\theta_{23},\delta)$$

$$\mathcal{A} = \prod_j \mathcal{A}_j = \prod_j e^{-iH_{f,j}L_j} = U_{23}(\theta_{23},\delta)\left(\prod_j e^{-i\tilde{H}_j L_j}\right) U_{23}^\dagger(\theta_{23},\delta)$$

$$U_{23}U_{23}^\dagger = \mathbb{1}$$

Optimal Structure of the Probability for Atmospherics

3. Work in the tilde basis with θ_{23} and δ pulled out
   - Can shift δ from θ_{13} to θ_{23}

$$\begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} R_{12} \to \begin{pmatrix} 1 & & \\ & c_{23} & s_{23}e^{i\delta} \\ & -s_{23}e^{-i\delta} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13} \\ & 1 & \\ -s_{13} & & c_{13} \end{pmatrix} R_{12}$$

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$$H_f = U_{23}(\theta_{23},\delta)\tilde{H}U^\dagger(\theta_{23},\delta)$$

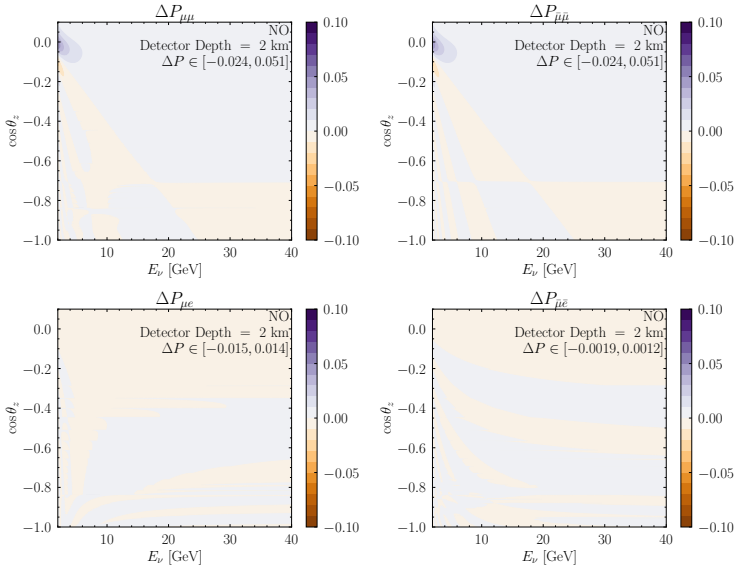
$$\mathcal{A} = \prod_j^{\sim} \mathcal{A}_j = \prod_j e^{-iH_{f,j}L_j} = U_{23}(\theta_{23},\delta)\left(\prod_j e^{-i\tilde{H}_j L_j}\right) U_{23}^\dagger(\theta_{23},\delta)$$

$\tilde{H}$ is real! And doesn't depend on θ_{23} or δ

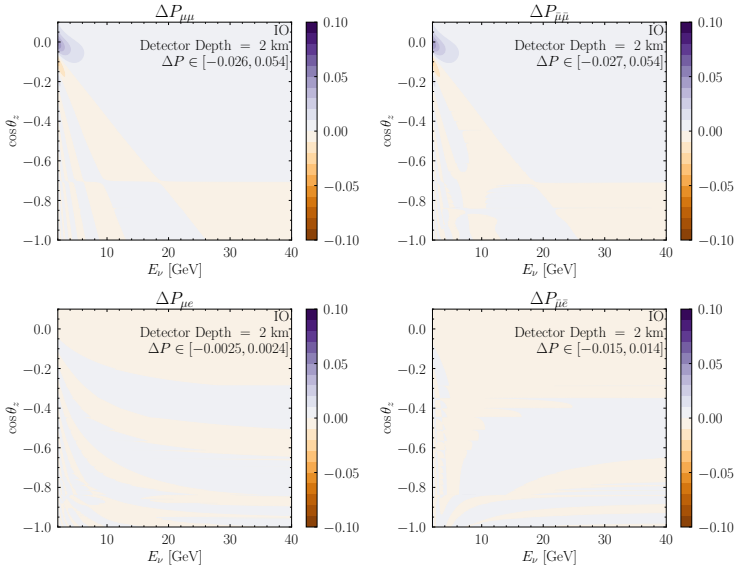
Reduces many unnecessary computations

$U_{23}U_{23}^\dagger = \mathbb{1}$

Atmospheric Results: Detector Depth: NO



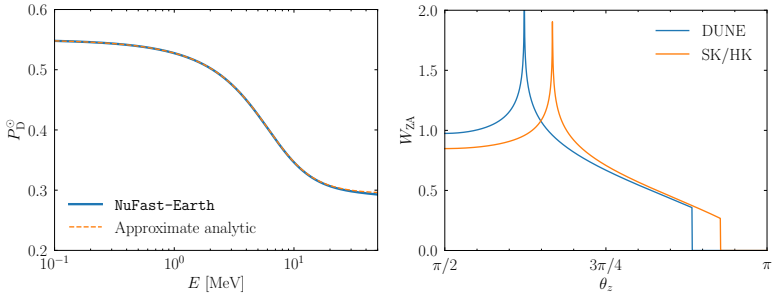
Atmospheric Results: Detector Depth: IO



Atmospheric Results: Detector Depth: Summary

$\max \lvert\Delta P\rvert$	$\nu_\mu \to \nu_\mu$		$\nu_\mu \to \nu_e$	
	ν	$\bar{\nu}$	ν	$\bar{\nu}$
NO	5.1%	5.1%	1.5%	0.2%
IO	5.4%	5.4%	0.3%	1.5%

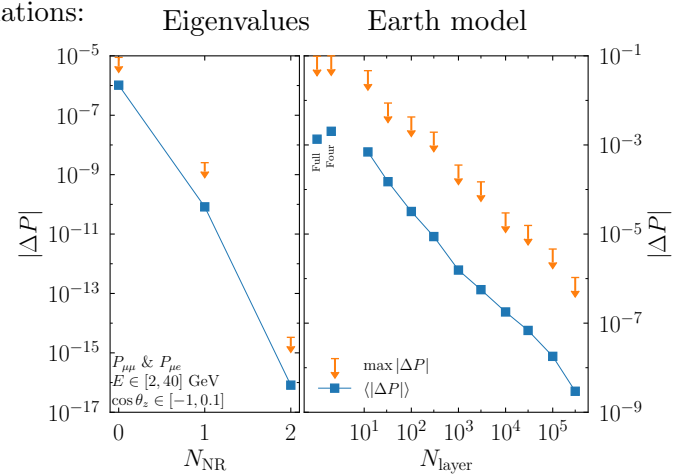
NuFast-Earth Solar Neutrino Validation



Analytically understand the small deviation at high energy

NuFast-Earth Precision

Two approximations:



"True" has exact eigenvalues and 1M layers

Calculate $|\Delta P|$ across 100×100 grid in energy and angle

NuFast-Earth Speed Example of Caching

Ingredients

1. 44 layer Earth
2. Single core-crossing trajectory: $\cos\theta_z = -1$

   Does not take advantage of reused eigen- calculations
3. 100 Δm^2_{31} points and 100 θ_{23} points

Run time:

1. `OscProb`:

$$100 \times 100 \times (4 \times 10^4 \text{ ns}) = 0.4 \text{ s}$$

2. `NuFast-Earth` wrong order (loop Δm^2_{31} first):

$$100 \times 99 \times (6 \times 10^3 \text{ ns}) + 100 \times (2 \times 10^2 \text{ ns}) = 0.06 \text{ s}$$

   $6.7\times$

3. `NuFast-Earth` right order (loop θ_{23} first):

$$100 \times (6 \times 10^3 \text{ ns}) + 100 \times 99 \times (2 \times 10^2 \text{ ns}) = 0.002 \text{ s}$$

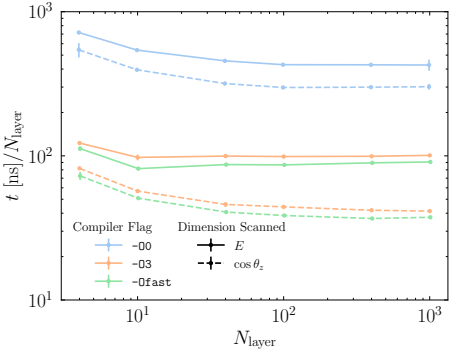
   $200\times$

NuFast-Earth Speed: Grid

Number of eigenvalue and eigenvector computations: $N_{\text{layer}} \times N_E$

Time per Earth layer for one core-crossing trajectory:

- 100 energy by 1 zenith grid (solid)
- 100 zenith by 1 energy grid (dashed)



Zenith angle bins $\sim 2\times$ faster than energy bins

Cost of computing eigenvalues and eigenvectors comparable to converting to amplitudes and multiplying matrices (which also depend on L and thus $\cos\theta_z$)

NuFast-Earth Fast Parameters

Four fast parameters for insides of loops:

- θ_{23}: `Set_s23sq`
- δ: `Set_delta`
- Production height: `Set_Production_Height`
- Density at production in Sun/supernova: `Set_rhoYe_Sun`

Also zenith grid points are $\sim 2\times$ faster than energy grid points

NuFast-Earth Flow

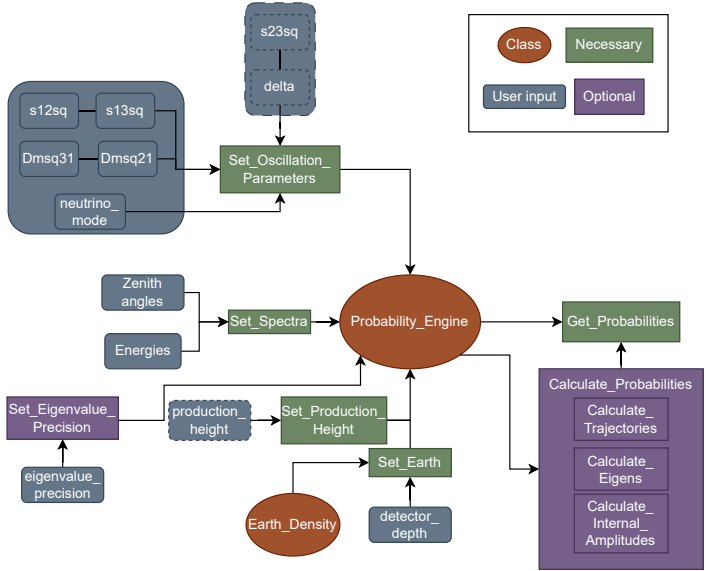
```
// Initialize the engine
Probability_Engine probability_engine;

// Set the oscillation parameters to nu-fit 6 best fit
    values:
probability_engine.Set_Oscillation_Parameters(0.307,
    0.02195, 0.561, 177 * M_PI / 180, 7.49e-5, 2.534e-3,
    true); //(s12sq, s13sq, s23sq, delta, Dmsq21, Dmsq31,
     neutrino_mode)

// Set energy and zenith angle arrays
std::vector<double> energies = {1, 2, 3, 4, 5}; // GeV
std::vector<double> coszs = {-1, -0.5, 0, 1}; // core-
    crossing to horizontal to down-going
probability_engine.Set_Spectra(energies, coszs);

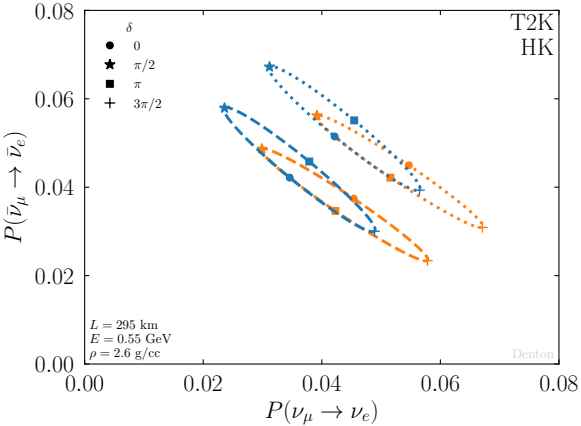
// Create Earth model instance
PREM_NDiscontinuityLayer earth_density(2, 10, 10, 5); //
    2 layers in the inner core, 10 layers in the outer
    core, 10 layers in the inner mantle, and 5 layers in
    the outer mantle
// Set Earth details
probability_engine.Set_Earth(2, &earth_density); //
    detector depth in km
// Set production height (optional)
probability_enginge.Set_Production_Height(10); // km,
    recalculations after changing this are fast

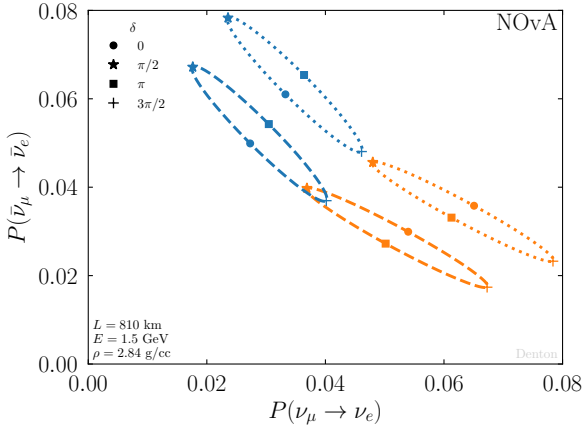
// Do the calculations
std::vector<std::vector<Matrix3r>> probabilities =
    probabilitie_engine.Get_Probabilities();
```



Dashed boxes = fast!

δ: What is it Really?





$\delta \not\Rightarrow$ Baryogenesis

The amount of leptogenesis is a function of:

1. δ
2. the heavy mass scale
3. α, β (Majorana phases)
4. CP phases in the RH neutrinos
5. ...

C. Hagedorn, et al. 1711.02866

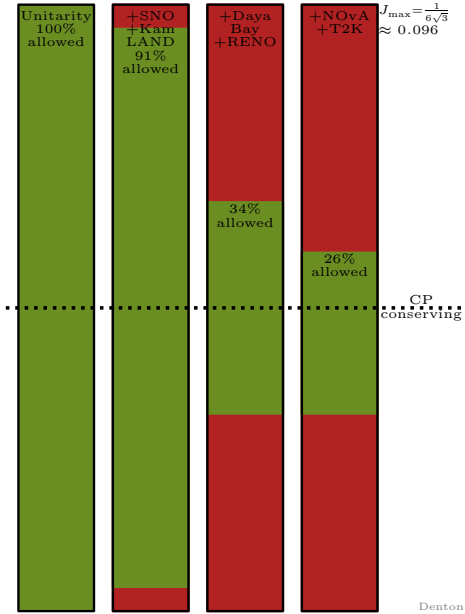
K. Moffat, et al. 1809.08251

$$\begin{aligned} \text{Measuring } \delta = 0, \pi \quad &\not\Rightarrow \quad \text{no leptogenesis} \\ \text{Measuring } \delta \neq 0, \pi \quad &\not\Rightarrow \quad \text{leptogenesis} \end{aligned}$$

δ, J: Current Status

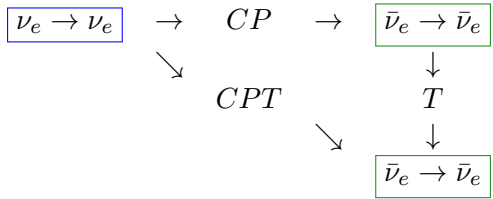
Maximal CP violation is already ruled out:

1. $\theta_{12} \neq 45^\circ$ at $\sim 15\sigma$
2. $\theta_{13} \neq \tan^{-1} \frac{1}{\sqrt{2}} \approx 35^\circ$ at many (100) σ
3. $\theta_{23} = 45^\circ$ allowed at $\sim 1\sigma$
4. $|\sin\delta| = 1$ allowed



Denton
2020

CP, T: Disappearance

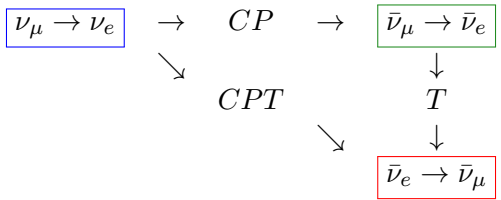


Disappearance measurements are even eigenstates of CP

$$CP[P(\nu_e \to \nu_e)] = P(\bar{\nu}_e \to \bar{\nu}_e) \overset{CPT}{=} P(\nu_e \to \nu_e)$$

Assume that CPT is a good symmetry

CP, T: Appearance



Appearance measurements are not eigenstates of CP

Appearance and Disappearance, CP Even and CP Odd Terms

Disappearance:

$$\begin{aligned} P(\nu_\alpha \to \nu_\alpha) = 1 &- 4|U_{\alpha1}|^2|U_{\alpha2}|^2 \sin^2 \Delta_{21} \\ &- 4|U_{\alpha1}|^2|U_{\alpha3}|^2 \sin^2 \Delta_{31} \\ &- 4|U_{\alpha2}|^2|U_{\alpha3}|^2 \sin^2 \Delta_{32} \\ &= P^{CP+}_{\alpha\alpha} \end{aligned}$$

Appearance:

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) = &- 4\Re[U_{\alpha1}U^*_{\beta1}U^*_{\alpha2}U_{\beta2}] \sin^2 \Delta_{21} \\ &- 4\Re[U_{\alpha1}U^*_{\beta1}U^*_{\alpha3}U_{\beta3}] \sin^2 \Delta_{31} \\ &- 4\Re[U_{\alpha3}U^*_{\beta3}U^*_{\alpha2}U_{\beta2}] \sin^2 \Delta_{32} \\ &\pm 8J_{CP} \sin \Delta_{21} \sin \Delta_{31} \sin \Delta_{32} \\ = P^{CP+}_{\alpha\beta} &+ P^{CP-}_{\alpha\beta} \end{aligned}$$

$\Delta_{ij} \equiv \Delta m^2_{ij} L/4E$

Sign depends on α, β

Parameter Counting

1. Four parameters in the PMNS matrix

Majorana phases are irrelevant in oscillations

Parameter Counting

1. Four parameters in the PMNS matrix

   Majorana phases are irrelevant in oscillations

2. Disappearance experiments of one flavor can measure up to three amplitudes
   - Electron neutrino row:
     - KamLAND/SNO+/JUNO measured one
     - Daya Bay/RENO measured a different one
     - JUNO will measure all three
   - Muon neutrino row:
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$$P(\nu_\alpha \to \nu_\alpha) = 1 - 4\sum_{i>j} C_{ij}^{\alpha} \sin^2 \Delta_{ij}$$

$\Delta_{ij} \equiv \Delta m^2_{ij} L/4E$

$$C_{ij}^{\alpha} = |U_{\alpha i}|^2 |U_{\alpha j}|^2$$

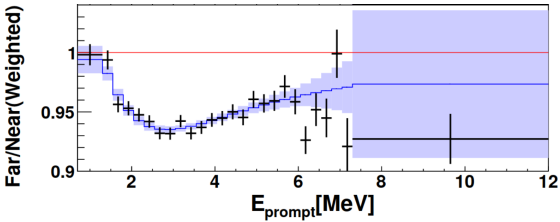
$$|U_{\alpha i}| = \left(\frac{C_{ij}^{\alpha} C_{ik}^{\alpha}}{C_{jk}^{\alpha}} \right)^{1/4}$$

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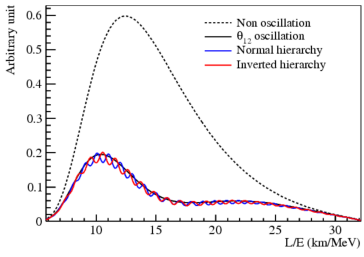
Daya Bay 1809.02261

Parameter Counting

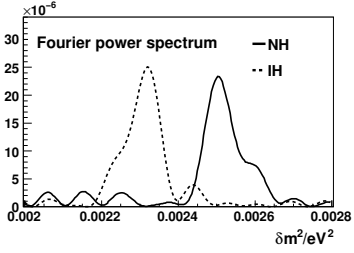
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JUNO 1507.05613



L. Zhan, et al. 0807.3203

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3. Only two parameters for one flavor are independent

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3. Only two parameters for one flavor are independent
4. Given good measurements of the ν_e and ν_μ disappearance, 4 independent parameters will be measured
   - Any row can be "simple" (e.g. $c_{12}c_{13}$, $s_{12}c_{13}$, ...) $\Rightarrow$ no one row is ever enough
   - That is, CPV is physical and cannot depend on parameterization

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   - That is, CPV is physical and cannot depend on parameterization
5. This is sufficient to constrain $\cos\delta$ and three mixing angles

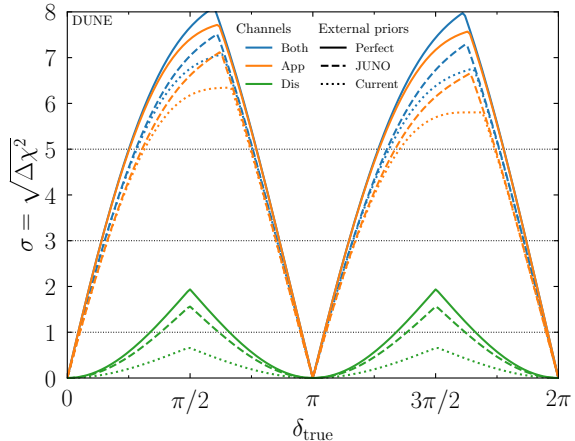
Parameter Counting

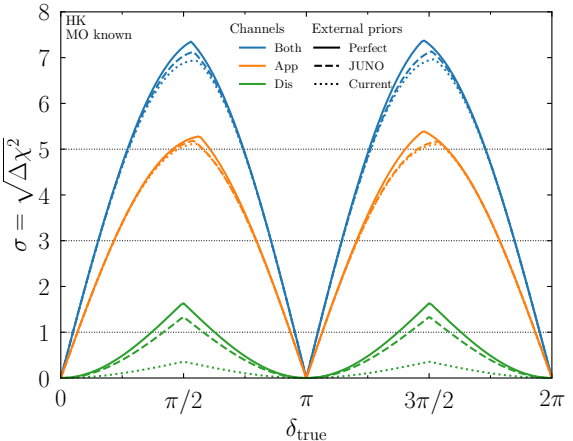
1. Four parameters in the PMNS matrix

   Majorana phases are irrelevant in oscillations

2. Disappearance experiments of one flavor can measure up to three amplitudes
   - Electron neutrino row:
     - KamLAND/SNO+/JUNO measured one
     - Daya Bay/RENO measured a different one
     - JUNO will measure all three
   - Muon neutrino row:
     - MINOS/T2K/NOνA measured one
3. Only two parameters for one flavor are independent
4. Given good measurements of the ν_e and ν_μ disappearance, 4 independent parameters will be measured
   - Any row can be "simple" (e.g. $c_{12}c_{13}$, $s_{12}c_{13}$, ...) $\Rightarrow$ no one row is ever enough
   - That is, CPV is physical and cannot depend on parameterization
5. This is sufficient to constrain $\cos\delta$ and three mixing angles
6. If we determine $\cos\delta \neq \pm 1 \quad \Rightarrow \quad$ CP is violated!

JUNO and HK Disappearance Sensitivities





δ: Future Sensitivities

Possible to get at CPV with CPC processes

Disappearance probability:

$$P(\nu_\alpha \to \nu_\alpha) = 1 - 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2 \Delta_{21} - 4|U_{\alpha 1}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{31} - 4|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{32}\,,$$

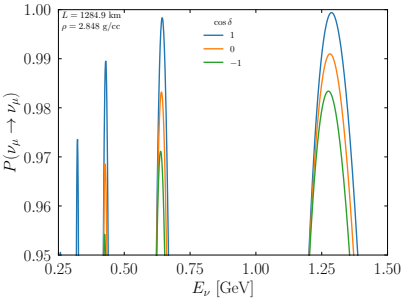
$$\Delta_{ij} = \Delta m^2_{ij} L/4E$$

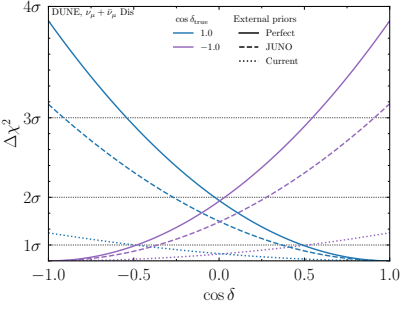
Can measure all three coeffs of each frequency $\Rightarrow$ 2 dofs

δ (and CPV) needs 4 dofs $\Rightarrow$ two dis measurements

ν_e: Daya Bay and KamLAND/JUNO

ν_μ: precision at DUNE/HK





Important cross check

Different and cleaner systematics than appearance

When δ and When J?

If the goal is **CP violation** the Jarlskog invariant should be used

however

If the goal is **measuring the parameters** one must use δ

Given θ_{12}, θ_{13}, θ_{23}, and J, I can't determine the sign of $\cos\delta$ which is physical

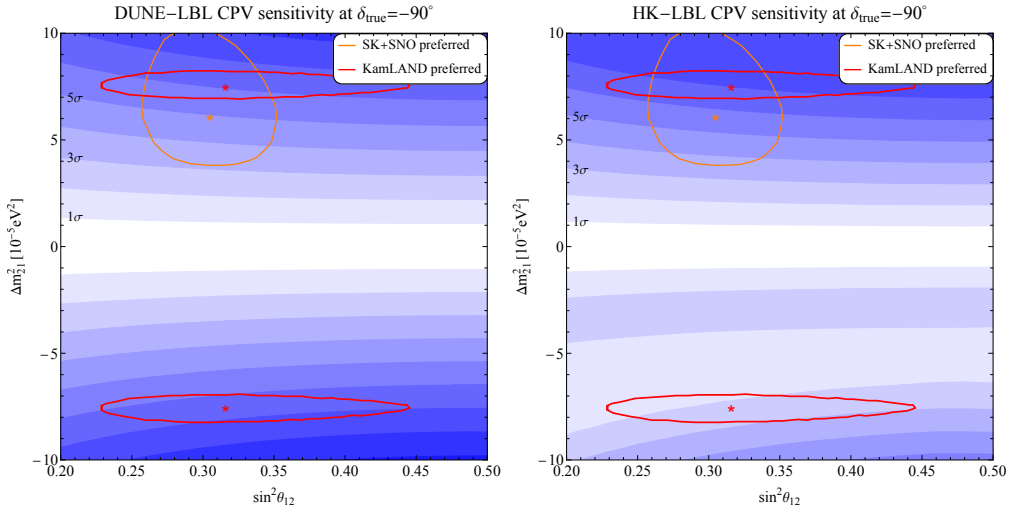
e.g. $P(\nu_\mu \to \nu_\mu)$ depends on $\cos\delta$ a tiny bit
PBD 2309.03262

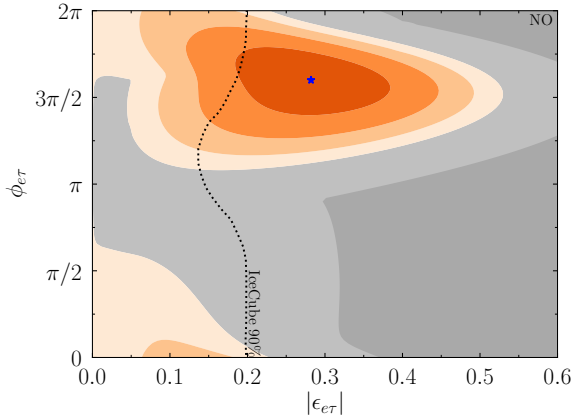
- T2K/HK are mostly sensitivity to $\sin\delta$; they should focus on J

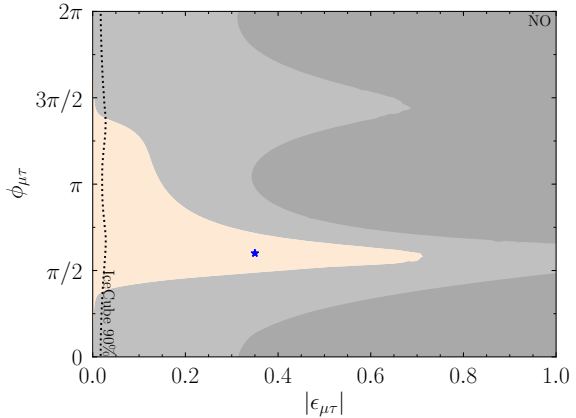
T2K does this now!

- NOvA/DUNE has modest $\cos\delta$ sensitivity; both J and δ should be reported

Impact of the True Solar Parameters on δ




NO
IceCube 90%
$\phi_{e\tau}$
$|\epsilon_{e\tau}|$
2π
3π/2
π
π/2
0
0.0
0.1
0.2
0.3
0.4
0.5
0.6


NO
IceCube 90%
$\phi_{\mu\tau}$
$|\epsilon_{\mu\tau}|$
2π
3π/2
π
π/2
0
0.0
0.2
0.4
0.6
0.8
1.0

Other CP Violating NSI Constraints

NSI effects grow with energy, density, and distance

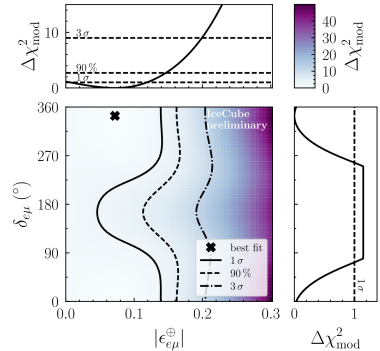
Other CP Violating NSI Constraints

NSI effects grow with energy, density, and distance

Best probes:

- $\epsilon_{\mu\tau}$: atmospheric
- $\epsilon_{e\mu}$, $\epsilon_{e\tau}$: LBL appearance, atmospheric
- IceCube
  - Constraint is at LBL best fit with 3 yrs

    10 yrs of data in the bank
  - Prefers non-zero $|\epsilon_{e\mu}|$ at $\sim 1\sigma$



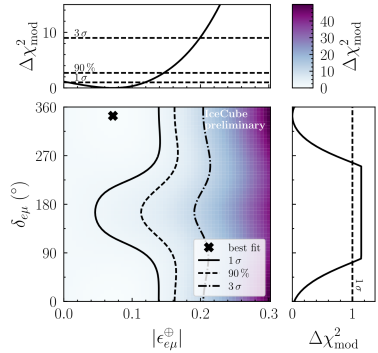
T. Ehrhardt, IceCube PPNT (2019)
IceCube 2106.07755

Other CP Violating NSI Constraints

NSI effects grow with energy, density, and distance

Best probes:

- $\epsilon_{\mu\tau}$: atmospheric
- $\epsilon_{e\mu}$, $\epsilon_{e\tau}$: LBL appearance, atmospheric
- IceCube
  - Constraint is at LBL best fit with 3 yrs
    10 yrs of data in the bank
  - Prefers non-zero $|\epsilon_{e\mu}|$ at $\sim 1\sigma$
- Super-K
  - Only consider real NSI
  - Comparable sensitivity as IceCube
- COHERENT
  - Only applies to NSI models with $M_{Z'} \gtrsim 10$ MeV
  - NSI u, d, e configuration matters
  - Comparable constraints



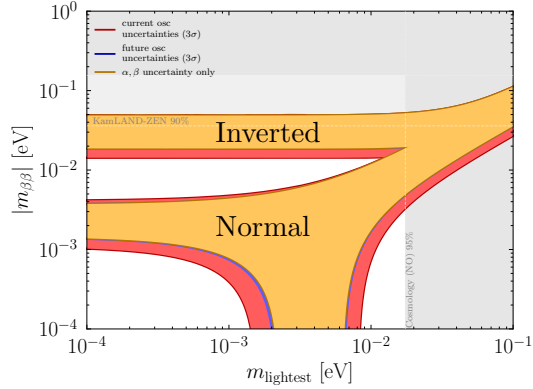
T. Ehrhardt, IceCube PPNT (2019)
IceCube 2106.07755

Super-K 1109.1889

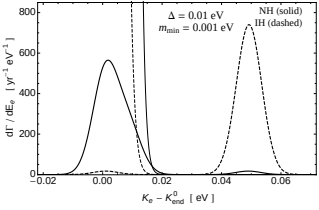
COHERENT 1708.01294
PBD, Y. Farzan, I. Shoemaker 1804.03660
PBD, J. Gehrlein 2008.06062

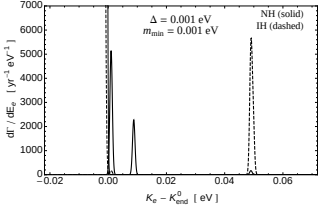
Mass Ordering: Broad Implications

- Affects cosmology
- Affects $0\nu\beta\beta$
- Affects end point measurements
- Affects CνB



PBD, J. Gehrlein 2308.09737





A. Long, C. Lunardini, E. Sabancilar 1405.7654

Neutrino Decay

Since neutrinos have different masses, they decay

- Loop suppressed
- Long lifetime: $\tau \gtrsim 10^{35}$ years

Test this!

Typical Lagrangian for $\nu_i \to \nu_j + \phi$ with $m_i > m_j$

$$\mathcal{L} \supset \frac{g_{ij}}{2}\bar{\nu}_j \nu_i \phi + \frac{g'_{ij}}{2}\bar{\nu}_j i\gamma_5 \nu_i \phi$$

Neutrino Decay Phenomenology

Neutrino decay is phenomenologically classified into:

- **Invisible decay**:
  - The decay products are sterile or too low energy to be detected
  - Results in a *depletion* of the flux below the relevant energy
- **Visible decay**:
  - Decay products are detected
  - In addition to depletion, there is *regeneration*
  - Regeneration happens at a lower energy than depletion

Why IceCube for Neutrino Decay

- DSNB and CνB are still some time off
- The next galactic supernova could come tomorrow, or in fifty years
- If ν_1 is stable SN1987A isn't too relevant (25 events + theory uncertainties)
  - Mass ordering looks to be normal at $\sim 3 - 3.5\ \sigma$

    Less now: PBD, J. Gehrlein, R. Pestes 2008.01110
  - Texture in the $\nu - \phi$ mixing matrix
- Early universe constraints mostly constrain the typical decay diagram

  G. Dvali and L. Funcke 1602.03191

  M. Escudero and M. Fairbairn 1907.05425
- IceCube measures **all three flavors** over > 1 decade in energy
- Astrophysical uncertainties seem like a problem, aren't really

How to Calculate Visible Neutrino Decay



Ingredients:

1. Oscillation averaged/decohered SM contribution
2. Depletion component
   - This takes us to invisible decay
3. Regeneration component at lower energies
   - This takes us to visible decay

Steps:

1. Integrate over decay location
2. Integrate over initial energy spectrum due to regeneration
3. Include multiple decays
4. Include cosmology
5. Mix thoroughly, let bake for an hour

M. Lindner, T. Ohlsson, W. Winter astro-ph/0105309

J. Beacom et al. hep-ph/0211305

P. Baerwald, M. Bustamante, W. Winter 1208.4600

SM Contribution: How to Calculate

First we define a "probability"

$$P_{\alpha\beta}(E_f) \equiv \frac{\Phi^E_{\alpha\beta}(E_f)}{\Phi^S_{\alpha}(E_f)}$$

Not actually a probability as it can be more than 1, but is probability-like and is useful

Over large distances the mass states decohere and/or the wave packets separate

$$\frac{\Delta m^2 L}{E} \gg 1$$

This is easily satisfied for extragalactic sources

Wave packet separation results in an identical flux to oscillation averaging:

$$\sin^2\left(\frac{\Delta m^2 L}{4E}\right) \to \frac{1}{2}$$

SM Contribution: The Probability

Given the usual Hamiltonian,

$$H = U_{\text{PMNS}} \begin{pmatrix} 0 & & \\ & \frac{\Delta m^2_{21}}{2E} & \\ & & \frac{\Delta m^2_{31}}{2E} \end{pmatrix} U^\dagger_{\text{PMNS}}$$

The SM oscillation probability is:

$$P^{\text{SM}}_{\alpha\beta} = \left| U^*_{\alpha 1} U_{\beta 1} + U^*_{\alpha 2} U_{\beta 2} e^{-i\frac{\Delta m^2_{21} L}{2E}} + U^*_{\alpha 3} U_{\beta 3} e^{-i\frac{\Delta m^2_{31} L}{2E}} \right|^2$$

When averaged/decohered:

$$\bar{P}^{\text{SM}}_{\alpha\beta} = \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2$$

No interference terms.

Depletion Component: How to Calculate

$$H = U_{\rm PMNS} \begin{pmatrix} 0 & & \\ & \frac{\Delta m^2_{21}}{2E} - \frac{i}{2}\Gamma_2 & \\ & & \frac{\Delta m^2_{31}}{2E} - \frac{i}{2}\Gamma_3 \end{pmatrix} U^\dagger_{\rm PMNS}$$

Assume here and throughout that ν_1 is stable (no lighter sterile neutrino) and the normal ordering

Depletion Component: How to Calculate

$$H = U_{\rm PMNS} \begin{pmatrix} 0 & & \\ & \frac{\Delta m^2_{21}}{2E} - \frac{i}{2}\Gamma_2 & \\ & & \frac{\Delta m^2_{31}}{2E} - \frac{i}{2}\Gamma_3 \end{pmatrix} U^\dagger_{\rm PMNS}$$

Assume here and throughout that ν_1 is stable (no lighter sterile neutrino) and the normal ordering

The partial width including scalar and pseudo-scalar as well as $\nu \to \nu$ and $\nu \to \bar{\nu}$

$$\Gamma_{ij} = \frac{m_i m_j}{16\pi E_i} \left\{ g_{ij}^2 \left[f(x_{ij}) + k(x_{ij}) \right] + g_{ij}'^2 \left[h(x_{ij}) + k(x_{ij}) \right] \right\}$$

$$f(x) = \frac{x}{2} + 2 + \frac{2}{x}\log x - \frac{2}{x^2} - \frac{1}{2x^3}$$

$$h(x) = \frac{x}{2} - 2 + \frac{2}{x}\log x + \frac{2}{x^2} - \frac{1}{2x^3}$$

$$k(x) = \frac{x}{2} - \frac{2}{x}\log x - \frac{1}{2x^3}$$

$\Gamma_i = \sum_j \Gamma_{ij}$
$\tau_i = m_i/E_i\Gamma_i$
$x_{ij} \equiv m_i/m_j$
See slide 97 on $\nu/\bar{\nu}$

Depletion Component: The Probability

$$\bar{P}^{\rm dep}_{\alpha\beta}(E,L) = -|U_{\alpha 2}|^2|U_{\beta 2}|^2(1-e^{-\Gamma_2 L}) - |U_{\alpha 3}|^2|U_{\beta 3}|^2(1-e^{-\Gamma_3 L})$$

The invisible probability is:

$$\bar{P}^{\rm inv}_{\alpha\beta} = \bar{P}^{\rm SM}_{\alpha\beta} + \bar{P}^{\rm dep}_{\alpha\beta}$$

$$\bar{P}^{\rm inv}_{\alpha\beta} = |U_{\alpha 1}|^2|U_{\beta 1}|^2 + |U_{\alpha 2}|^2|U_{\beta 2}|^2 e^{-\Gamma_2 L} + |U_{\alpha 3}|^2|U_{\beta 3}|^2 e^{-\Gamma_3 L}$$

Further Details

Full calculation is much more involved:

1. Regeneration: requires integrating over decay point and initial energy
2. Multiple decays possible: $\nu_3 \to \nu_2 \to \nu_1$
3. Cosmology affects energy at production, decay point(s), and to the Earth

Summary of Parameters

More important

1. γ: harder spectra $\Rightarrow$ large regeneration component
2. m_1: higher mass scale $\gtrsim 0.1$ eV $\Rightarrow$ large regeneration component
3. g_{ij}: different features depending on the texture

Less important

4. Redshift evolution $\Rightarrow$ small effect
5. Scalar/Pseudo-scalar $\Rightarrow$ small effect
6. $\nu \to \nu$, $\nu \to \bar{\nu} \Rightarrow$ small effect

Regeneration Component: Analytic Limits

- Verify that as $E_f \to \infty$, $\bar{P}_{ij}^{\text{reg}} \to 0$ as expected

SM

Regeneration Component: Analytic Limits

- Verify that as $E_f \to \infty$, $\bar{P}_{ij}^{\text{reg}} \to 0$ as expected

SM

- As $E_f \to 0$:

Full decay

$$\lim_{E_f \to 0} \bar{P}_{ij}^{\text{reg}}(E_f, L) = \frac{z(x)}{\gamma y(x)} \left(1 - \frac{1}{x^{2\gamma}}\right)$$

Regeneration Component: Analytic Limits

- Verify that as $E_f \to \infty$, $\bar{P}_{ij}^{\text{reg}} \to 0$ as expected

SM

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  - As $m_1 \to \infty$

Degenerate masses

$$\lim_{\substack{E_f \to 0 \\ m_1 \to \infty}} \bar{P}_{ij}^{\text{reg}}(E_f, L) = 1$$

Regeneration Component: Analytic Limits

- Verify that as $E_f \to \infty$, $\bar{P}_{ij}^{\text{reg}} \to 0$ as expected

SM

- As $E_f \to 0$:

Full decay

$$\lim_{E_f \to 0} \bar{P}_{ij}^{\text{reg}}(E_f, L) = \frac{z(x)}{\gamma y(x)} \left(1 - \frac{1}{x^{2\gamma}}\right)$$

  - As $m_1 \to \infty$

Degenerate masses

$$\lim_{\substack{E_f \to 0 \\ m_1 \to \infty}} \bar{P}_{ij}^{\text{reg}}(E_f, L) = 1$$

  - As $m_1 \to 0$

Depends on nature of interaction!

$$\lim_{\substack{E_f \to 0 \\ m_1 \to 0}} \bar{P}_{ij}^{\text{reg}}(E_f, L) = \frac{1}{\gamma}$$

Regeneration Component: Analytic Limits with Cosmology

Full decay:

- As $m_1 \to \infty$

$$\lim_{\substack{E_f \to 0 \\ m_1 \to \infty}} \bar{P}^{\rm reg}_{ij}(E_f, L) = (1+z)^{-2\gamma}$$

$$\lim_{\substack{E_f \to 0 \\ m_1 \to \infty}} \bar{P}^{\rm vis}_{\alpha\beta} = (1+z)^{-\gamma} \left[|U_{\alpha 1}|^2 |U_{\beta 1}|^2 + |U_{\alpha 2}|^2 |U_{\beta 2}|^2 + (1+z)^{-\gamma} |U_{\alpha 3}|^2 |U_{\beta 1}|^2 \right]$$

Regeneration Component: Analytic Limits with Cosmology

Full decay:

- As $m_1 \to \infty$

$$\lim_{\substack{E_f \to 0 \\ m_1 \to \infty}} \bar{P}^{\rm reg}_{ij}(E_f, L) = (1+z)^{-2\gamma}$$

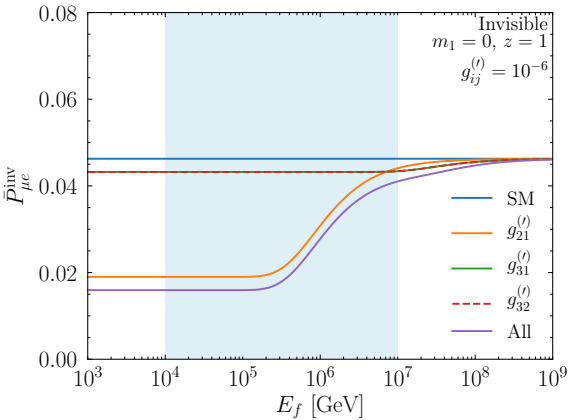
$$\lim_{\substack{E_f \to 0 \\ m_1 \to \infty}} \bar{P}^{\rm vis}_{\alpha\beta} = (1+z)^{-\gamma} \left[|U_{\alpha 1}|^2 |U_{\beta 1}|^2 + |U_{\alpha 2}|^2 |U_{\beta 2}|^2 + (1+z)^{-\gamma} |U_{\alpha 3}|^2 |U_{\beta 1}|^2 \right]$$

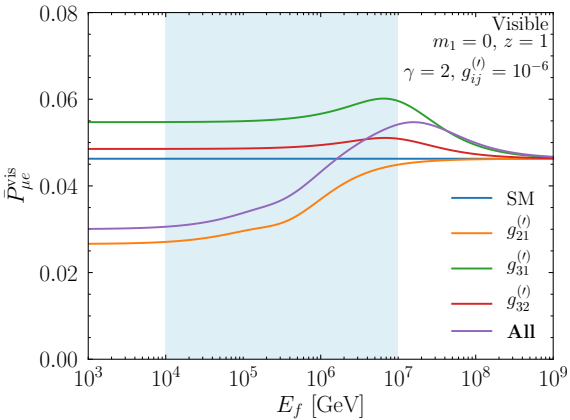
- As $m_1 \to 0$

$$\lim_{\substack{E_f \to 0 \\ m_1 \to 0}} \bar{P}^{\rm reg}_{ij}(E_f, L) = \frac{(1+z)^{-2\gamma}}{\gamma}$$

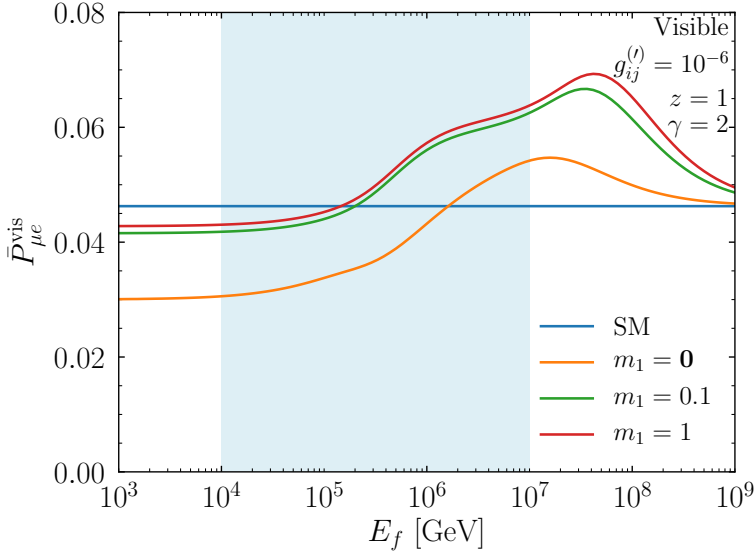
$$\lim_{\substack{E_f \to 0 \\ m_1 \to 0}} \bar{P}^{\rm vis}_{\alpha\beta} = (1+z)^{-\gamma} \left[|U_{\alpha 1}|^2 |U_{\beta 1}|^2 + |U_{\alpha 2}|^2 |U_{\beta 2}|^2 + \frac{(1+z)^{-\gamma}}{\gamma} |U_{\alpha 3}|^2 |U_{\beta 1}|^2 \right]$$

Invisible vs. Visible Decay

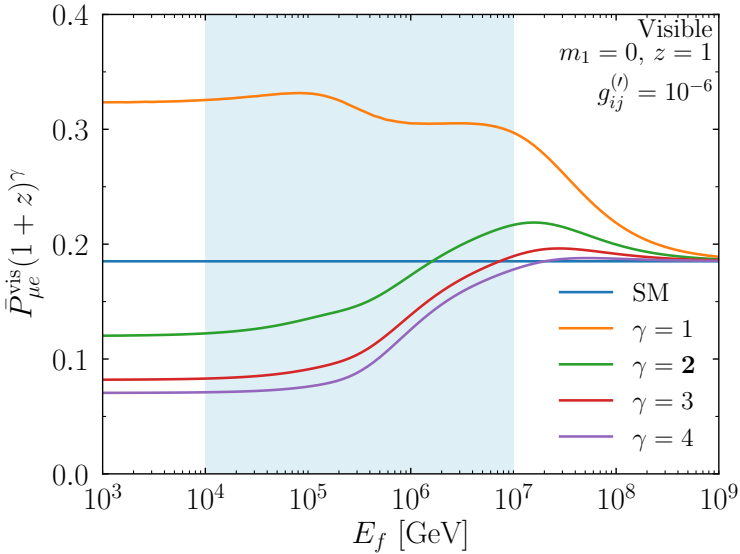




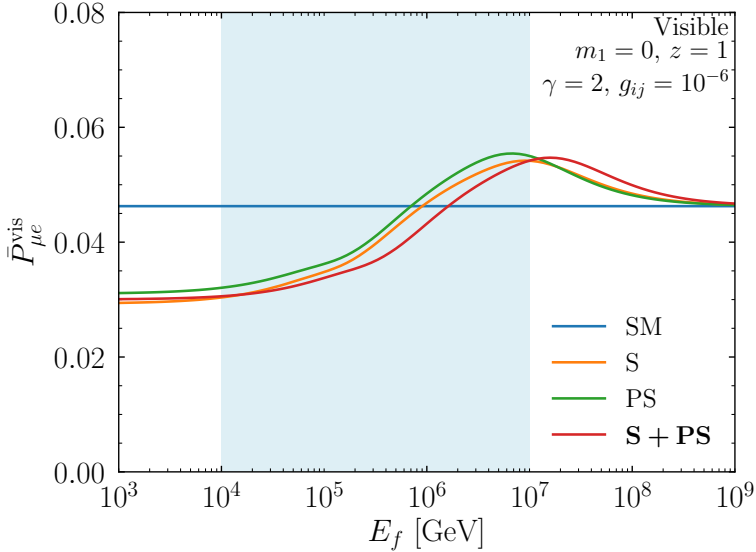
Results: Visible Decay: Neutrino mass scale



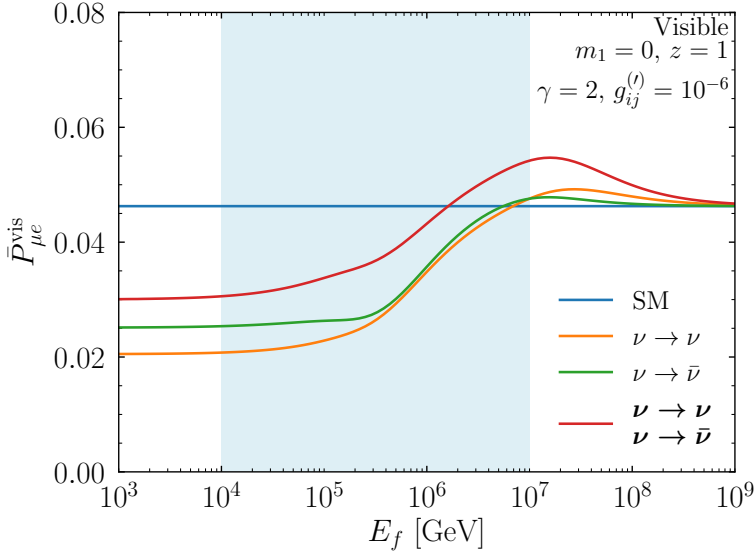
Results: Visible Decay: Spectral Index



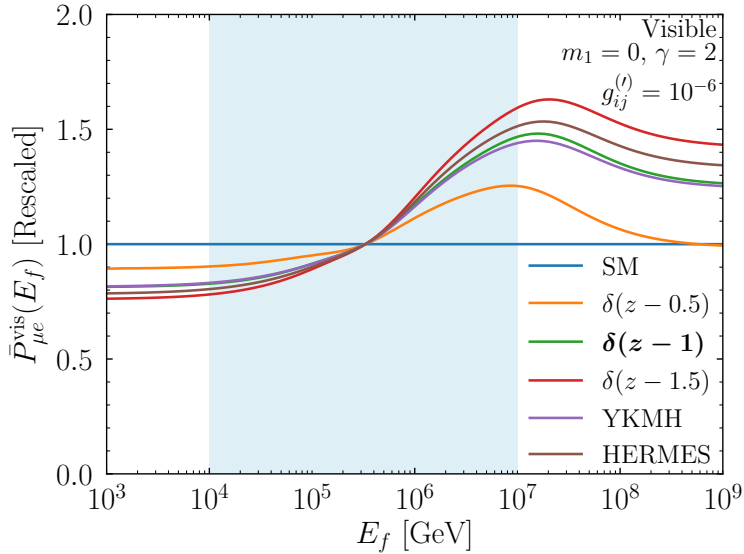
Results: Visible Decay: Scalar vs. Pseudo-scalar



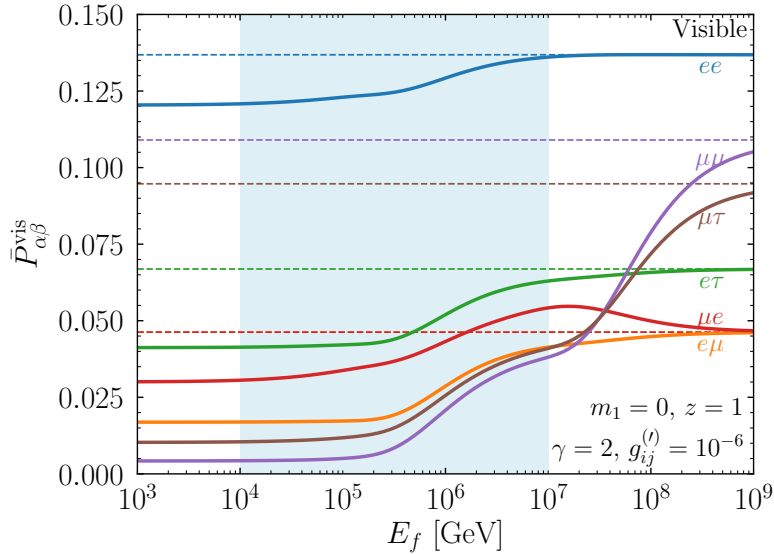
Results: Visible Decay: Neutrinos vs. Anti-neutrinos



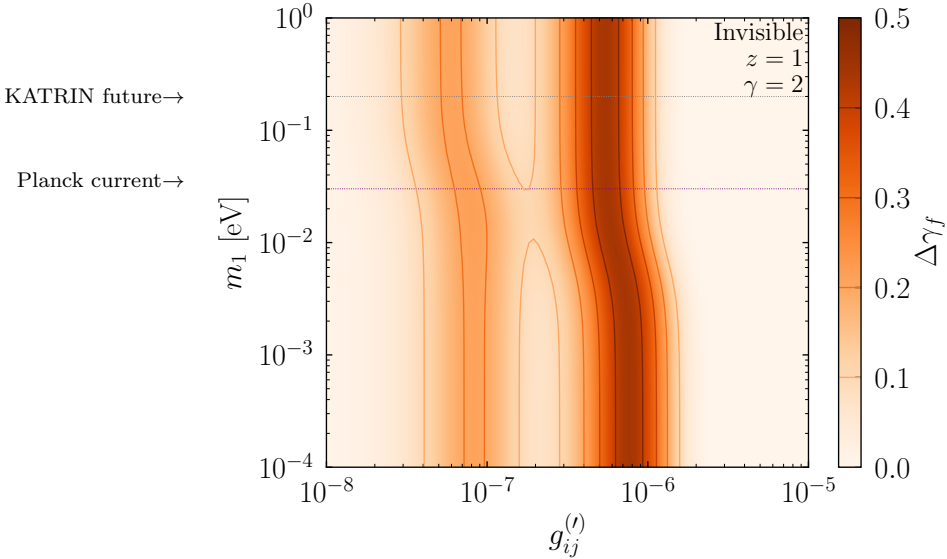
Visible Decay for Different Redshift Evolution Functions



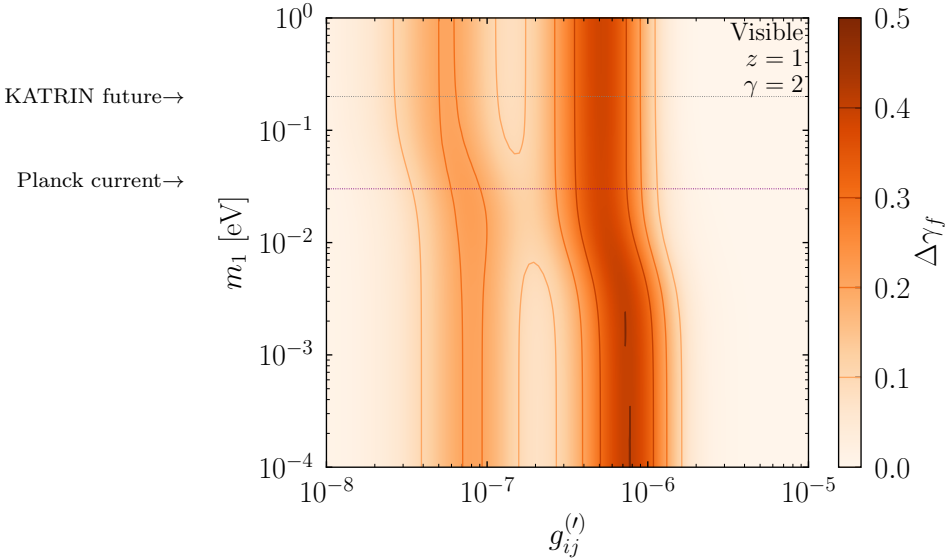
Results: Visible Decay: Flavors



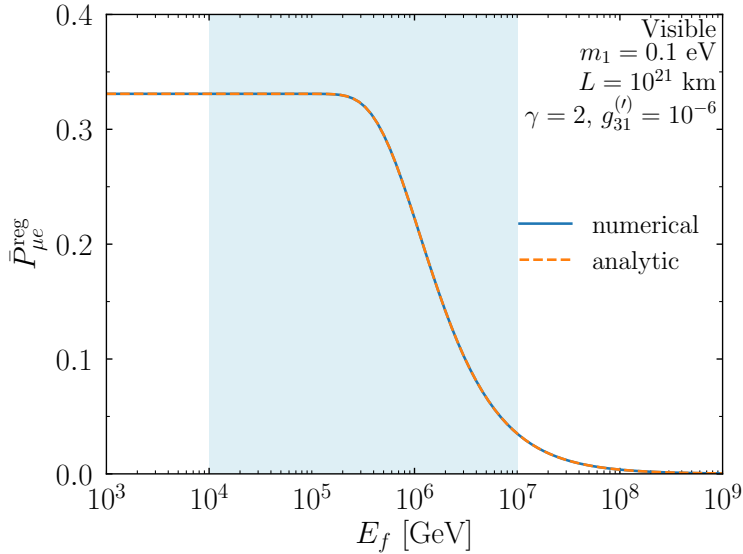
IceCube Track and Cascade Spectral Index Difference: Invisible Decay



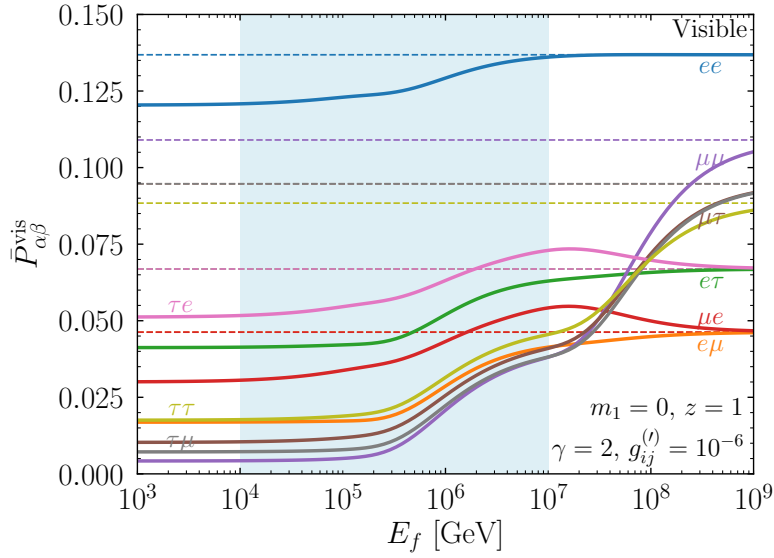
IceCube Track and Cascade Spectral Index Difference: Visible Decay



Analytic Validation



Results: Visible Decay: Flavors with ν_τ



IceCube Measures:

- **Energy**
- **Flavor(ish)**
- Direction
- Time

Some Track to Cascade with Decay Observations

- Decay usually hardens the spectrum
  - Only $\bar{P}^{\rm vis}_{\mu e} > \bar{P}^{\rm SM}_{\mu e}$ for $m_1 \sim 0$ and $\gamma \sim 2$
  - While $\bar{P}^{\rm vis}_{\tau\beta} > \bar{P}^{\rm SM}_{\tau\beta}$, no ν_τ's are produced at the sources
- The effect is larger for tracks than cascades

$\max \Delta\gamma$	$g^{(\prime)}_{21}$	$g^{(\prime)}_{31}$	$g^{(\prime)}_{32}$	All
Invisible	0.006	0.200	0.200	0.438
Visible	0.042	0.227	0.172	0.400

$\min \Delta\gamma = -0.01$

$\Delta\gamma \equiv \gamma_c - \gamma_t$

- This is the same direction of the IceCube data!
- The other sign (cascades harder than tracks) requires the inverted ordering

Uncertainties

or "How to muck it all up with astrophysics"

What doesn't work:

- Multiple classes of sources with different spectra
- pp vs. $p\gamma$ sources
- Different redshift evolution $\Rightarrow$ shift the g_{ij}
- Neutron decay sources
- Varying the oscillation parameters
- IceCube track or cascade normalization

What could work: (other than neutrino decay)

- Muon damped $\Rightarrow \Delta\gamma \sim +0.2$
- Track and cascade spectra are fit over slightly different energy ranges $\Rightarrow$ broken power law can help
- Energy misreconstruction (tracks could be susceptible to this)
- Dark matter?