The best way to probe CP violation in the lepton sector is with long-baseline accelerator neutrino experiments in the appearance mode. I will show that it is possible to discover CP violation with disappearance experiments only, by combining JUNO for electron neutrinos and DUNE or Hyper-Kamiokande for muon neutrinos. While the maximum sensitivity to discover CP is quite modest, some values of δ may be disfavored by $> 3\sigma$ depending on the true value of δ.

Neutrino oscillation experiments will be entering the precision era in the next decade. Correctly estimating the confidence intervals from data for the oscillation parameters requires very large Monte Carlo data sets involving calculating the oscillation probabilities in matter many, many times. In this talk, I will leverage past work to present a new, fast, precise technique for calculating neutrino oscillation probabilities in matter optimized for long-baseline neutrino oscillations called NuFast. I will also present recent results for atmospheric and solar neutrinos.

CP-Violation with Neutrino Disappearance and NuFast

Peter B. Denton

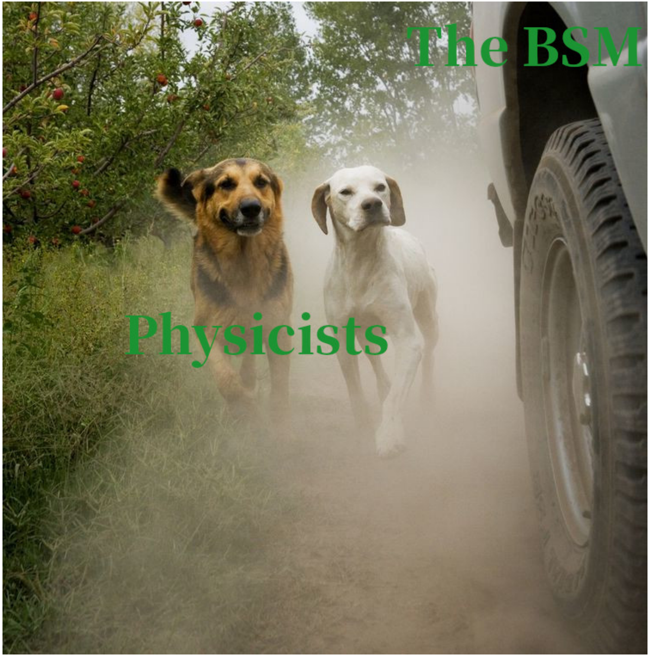
KCL

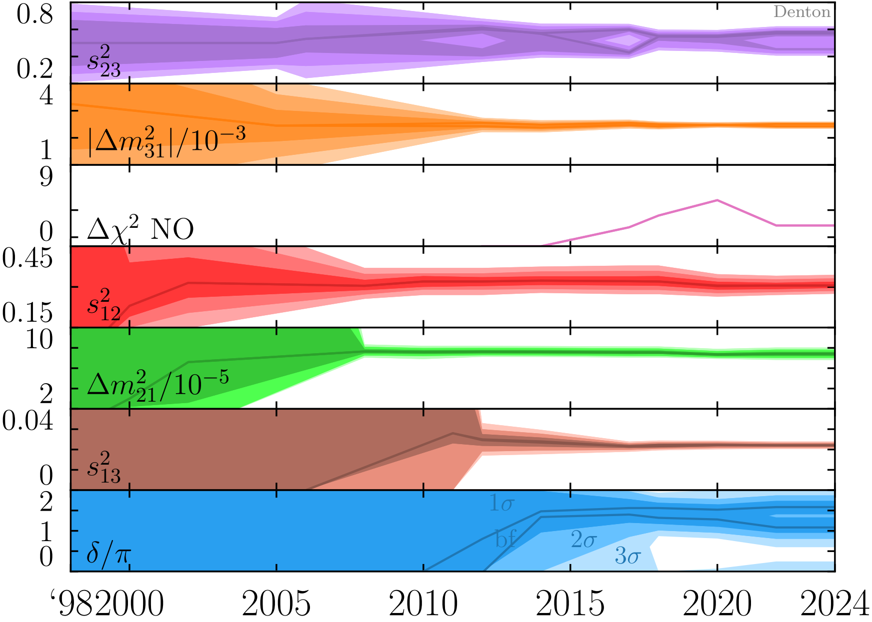
January 12, 2026

2309.03262 PRL

2405.02400 and 2511.04735 with S. Parke

github.com/PeterDenton/NuFast-LBL


The BSM
Physicists


Denton
0.8
0.2
s^2_{23}
4
1
$|\Delta m^2_{31}|/10^{-3}$
9
0
$\Delta\chi^2$ NO
0.45
0.15
s^2_{12}
10
2
$\Delta m^2_{21}/10^{-5}$
0.04
0
s^2_{13}
2
1
0
δ/π
1σ
bf
2σ
3σ
'98
2000
2005
2010
2015
2020
2024

Four Known Unknown in Particle Physics: All Neutrinos

Atmospheric mass ordering

θ_{23} octant

Complex phase

Absolute mass scale

Atmospheric mass ordering

θ_{23} octant

$\boxed{\text{Complex phase}}$

Absolute mass scale

Why is CPV interesting?

δ and CP Violation

$$J_{CP} = s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23}\sin\delta$$

C. Jarlskog PRL 55, 1039 (1985)

δ and CP Violation

$$J_{CP} = s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23}\sin\delta$$

C. Jarlskog PRL 55, 1039 (1985)

1. Strong interaction: no observed EDM $\Rightarrow$ CP (nearly) **conserved**

$$\frac{\bar{\theta}}{2\pi} < 10^{-11}$$

J. Pendlebury, et al. 1509.04411

2. Quark mass matrix: non-zero but **small** CP violation

$$\frac{|J_{\rm CKM}|}{J_{\rm max}} = 3\times10^{-4}$$

CKMfitter 1501.05013

3. Lepton mass matrix: **?**

$$\frac{|J_{\rm PMNS}|}{J_{\rm max}} < 0.34$$

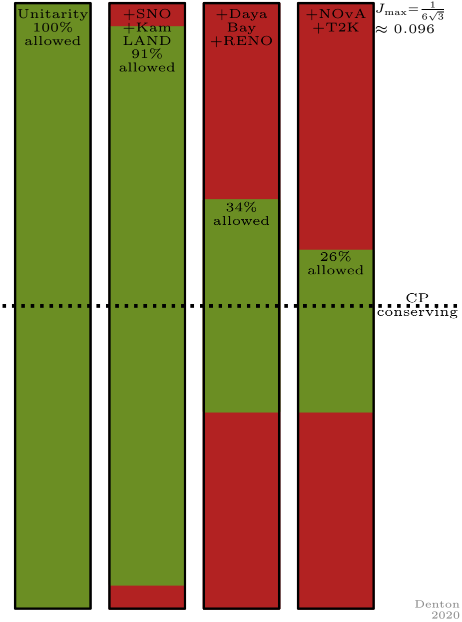
PBD, J. Gehrlein, R. Pestes 2008.01110

$J_{\rm max} = \frac{1}{6\sqrt{3}} \approx 0.096$

δ, J: Current Status

Maximal CP violation is already ruled out:

1. $\theta_{12} \neq 45^\circ$ at $\sim 15\sigma$
2. $\theta_{13} \neq \tan^{-1}\frac{1}{\sqrt{2}} \approx 35^\circ$ at many (100) σ
3. $\theta_{23} = 45^\circ$ allowed at $\sim 1\sigma$
4. $|\sin\delta| = 1$ allowed



When δ and When J?

If the goal is **CP violation** the Jarlskog invariant should be used

however

If the goal is **measuring the parameters** one must use δ

Given θ_{12}, θ_{13}, θ_{23}, and J, I can't determine the sign of $\cos\delta$ which is physical

e.g. $P(\nu_\mu \to \nu_\mu)$ depends on $\cos\delta$

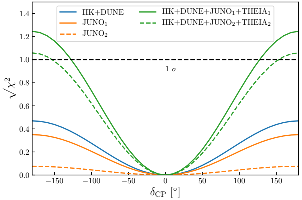
Other Non-standard CPV Probes

1. Some information in solar due to loops in elastic scattering

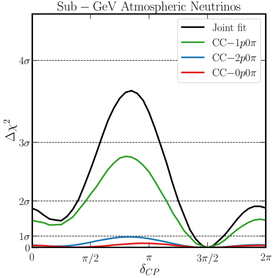
V. Brdar, X-J. Xu 2306.03160
K. Kelly, et al. 2407.03174 requires 3k Borexinos

2. Sub-GeV → sub-100 MeV atmospherics

K. Kelly, et al. 1904.02751
See also e.g. A. Suliga, J. Beacom 2306.11090



Solar (no systematics)



Atmospherics at DUNE

CP, T: Disappearance

$$
\begin{array}{ccccc}
\boxed{\nu_e \to \nu_e} & \to & CP & \to & \boxed{\bar{\nu}_e \to \bar{\nu}_e} \\
 & \searrow & & & \downarrow \\
 & & CPT & & T \\
 & & & \searrow & \downarrow \\
 & & & & \boxed{\bar{\nu}_e \to \bar{\nu}_e}
\end{array}
$$

Disappearance measurements are even eigenstates of CP

$$CP[P(\nu_e \to \nu_e)] = P(\bar{\nu}_e \to \bar{\nu}_e) \stackrel{CPT}{=} P(\nu_e \to \nu_e)$$

Assume that CPT is a good symmetry

CP, T: Appearance

$$\begin{array}{ccccc}
\boxed{\nu_\mu \to \nu_e} & \to & CP & \to & \boxed{\bar{\nu}_\mu \to \bar{\nu}_e} \\
 & \searrow & & & \downarrow \\
 & & CPT & & T \\
 & & & \searrow & \downarrow \\
 & & & & \boxed{\bar{\nu}_e \to \bar{\nu}_\mu}
\end{array}$$

Appearance measurements are not eigenstates of CP

Appearance, Disappearance, and CP

Appearance and Disappearance, CP Even and CP Odd Terms

Disappearance:

$$\begin{aligned}P(\nu_\alpha \to \nu_\alpha) = 1 &- 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2 \Delta_{21}\\ &- 4|U_{\alpha 1}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{31}\\ &- 4|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{32}\\ = P_{\alpha\alpha}^{CP+}\end{aligned}$$

$\Delta_{ij} \equiv \Delta m_{ij}^2 L/4E$

Appearance and Disappearance, CP Even and CP Odd Terms

Disappearance:

$$\begin{aligned} P(\nu_\alpha \to \nu_\alpha) = 1 &- 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2 \Delta_{21} \\ &- 4|U_{\alpha 1}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{31} \\ &- 4|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{32} \\ &= P_{\alpha\alpha}^{CP+} \end{aligned}$$

Appearance:

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) = &- 4\Re[U_{\alpha 1}U_{\beta 1}^*U_{\alpha 2}^*U_{\beta 2}] \sin^2 \Delta_{21} \\ &- 4\Re[U_{\alpha 1}U_{\beta 1}^*U_{\alpha 3}^*U_{\beta 3}] \sin^2 \Delta_{31} \\ &- 4\Re[U_{\alpha 3}U_{\beta 3}^*U_{\alpha 2}^*U_{\beta 2}] \sin^2 \Delta_{32} \\ &\pm 8J_{CP} \sin \Delta_{21} \sin \Delta_{31} \sin \Delta_{32} \\ &= P_{\alpha\beta}^{CP+} + P_{\alpha\beta}^{CP-} \end{aligned}$$

$\Delta_{ij} \equiv \Delta m_{ij}^2 L/4E$

Sign depends on α, β

Conventional Wisdom

1. Appearance is sensitive to CPV [True]

Conventional Wisdom

1. Appearance is sensitive to CPV [True]
2. Disappearance has no CPV sensitivity [False]

Conventional Wisdom

1. Appearance is sensitive to CPV [True]
2. Disappearance has no CPV sensitivity [False]
3. Any CPV or δ dependence in disappearance is in ν_μ not ν_e [Confusing/False]

$$U_{\rm PDG}=\begin{pmatrix}c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta}\\ -c_{23}s_{12}-s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12}-s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13}\\ s_{23}s_{12}-c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12}-c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13}\end{pmatrix}$$

Can reparameterize so that any row/column is "simple"

PBD, R. Pestes 2006.09384

Correct Statements

- Appearance is the best way to measure δ and CPV

  …given known oscillation parameters, systematics, and realistic experiments

  - Probes mostly $\sin\delta$ not $\cos\delta$
  - Don't need both ν and $\bar{\nu}$ (but systematics)
- **Disappearance can measure δ**
  - CPV can be discovered with only disappearance measurements
  - Probes mostly $\cos\delta$ not $\sin\delta$
  - Requires measurements of two flavors
  - "Works through unitarity" (as do nearly all oscillation measurements)

Parameter Counting

1. Four parameters in the PMNS matrix

Majorana phases are irrelevant in oscillations

Parameter Counting

1. Four parameters in the PMNS matrix

   Majorana phases are irrelevant in oscillations

2. Disappearance experiments of one flavor can measure up to three amplitudes
   - Electron neutrino row:
     - KamLAND/SNO+/JUNO measured one
     - Daya Bay/RENO measured a different one
     - JUNO will measure all three
   - Muon neutrino row:
     - MINOS/T2K/NOνA measured one

Parameter Counting

1. Four parameters in the PMNS matrix

Majorana phases are irrelevant in oscillations

2. Disappearance experiments of one flavor can measure up to three amplitudes
   - Electron neutrino row:
     - KamLAND/SNO+/JUNO measured one
     - Daya Bay/RENO measured a different one
     - JUNO will measure all three
   - Muon neutrino row:
     - MINOS/T2K/NOνA measured one

$$P(\nu_\alpha \to \nu_\alpha) = 1 - 4\sum_{i>j} C_{ij}^\alpha \sin^2 \Delta_{ij}$$

$$\Delta_{ij} \equiv \Delta m_{ij}^2 L/4E$$

$$C_{ij}^\alpha = |U_{\alpha i}|^2 |U_{\alpha j}|^2$$

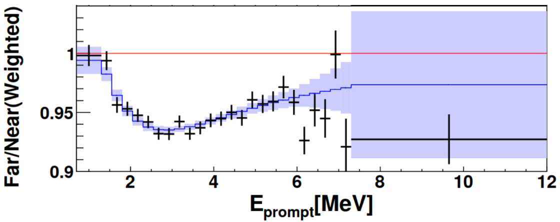
$$|U_{\alpha i}| = \left(\frac{C_{ij}^\alpha C_{ik}^\alpha}{C_{jk}^\alpha}\right)^{1/4}$$

Parameter Counting

1. Four parameters in the PMNS matrix

Majorana phases are irrelevant in oscillations

2. Disappearance experiments of one flavor can measure up to three amplitudes
   - Electron neutrino row:
     - KamLAND/SNO+/JUNO measured one
     - Daya Bay/RENO measured a different one
     - JUNO will measure all three
   - Muon neutrino row:
     - MINOS/T2K/NOνA measured one



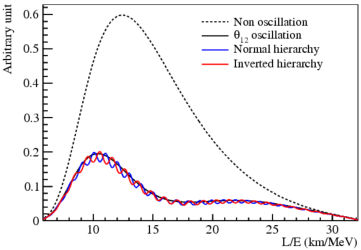
Daya Bay 1809.02261

Parameter Counting

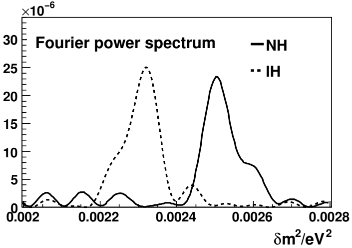
1. Four parameters in the PMNS matrix

   Majorana phases are irrelevant in oscillations

2. Disappearance experiments of one flavor can measure up to three amplitudes
   - Electron neutrino row:
     - KamLAND/SNO+/JUNO measured one
     - Daya Bay/RENO measured a different one
     - JUNO will measure all three
   - Muon neutrino row:
     - MINOS/T2K/NOνA measured one



JUNO 1507.05613



L. Zhan, et al. 0807.3203

Parameter Counting

1. Four parameters in the PMNS matrix

   Majorana phases are irrelevant in oscillations
2. Disappearance experiments of one flavor can measure up to three amplitudes
   - Electron neutrino row:
     - KamLAND/SNO+/JUNO measured one
     - Daya Bay/RENO measured a different one
     - JUNO will measure all three
   - Muon neutrino row:
     - MINOS/T2K/NOνA measured one
3. Only two parameters for one flavor are independent

Parameter Counting

1. Four parameters in the PMNS matrix

   Majorana phases are irrelevant in oscillations

2. Disappearance experiments of one flavor can measure up to three amplitudes
   - Electron neutrino row:
     - KamLAND/SNO+/JUNO measured one
     - Daya Bay/RENO measured a different one
     - JUNO will measure all three
   - Muon neutrino row:
     - MINOS/T2K/NOνA measured one
3. Only two parameters for one flavor are independent
4. Given good measurements of the ν_e and ν_μ disappearance, 4 independent parameters will be measured
   - Any row can be "simple" (e.g. $c_{12}c_{13}$, $s_{12}c_{13}$, ...) $\Rightarrow$ no one row is ever enough
   - That is, CPV is physical and cannot depend on parameterization

Parameter Counting

1. Four parameters in the PMNS matrix

   Majorana phases are irrelevant in oscillations
2. Disappearance experiments of one flavor can measure up to three amplitudes
   - Electron neutrino row:
     - KamLAND/SNO+/JUNO measured one
     - Daya Bay/RENO measured a different one
     - JUNO will measure all three
   - Muon neutrino row:
     - MINOS/T2K/NOνA measured one
3. Only two parameters for one flavor are independent
4. Given good measurements of the ν_e and ν_μ disappearance, 4 independent parameters will be measured
   - Any row can be "simple" (e.g. $c_{12}c_{13}$, $s_{12}c_{13}$, ...) $\Rightarrow$ no one row is ever enough
   - That is, CPV is physical and cannot depend on parameterization
5. This is sufficient to constrain $\cos\delta$ and three mixing angles

Parameter Counting

1. Four parameters in the PMNS matrix

   Majorana phases are irrelevant in oscillations
2. Disappearance experiments of one flavor can measure up to three amplitudes
   - Electron neutrino row:
     - KamLAND/SNO+/JUNO measured one
     - Daya Bay/RENO measured a different one
     - JUNO will measure all three
   - Muon neutrino row:
     - MINOS/T2K/NOνA measured one
3. Only two parameters for one flavor are independent
4. Given good measurements of the ν_e and ν_μ disappearance, 4 independent parameters will be measured
   - Any row can be "simple" (e.g. $c_{12}c_{13}$, $s_{12}c_{13}$, ...) $\Rightarrow$ no one row is ever enough
   - That is, CPV is physical and cannot depend on parameterization
5. This is sufficient to constrain $\cos\delta$ and three mixing angles
6. If we determine $\cos\delta \neq \pm 1 \quad \Rightarrow \quad$ CP is violated!

Direct Analytic Calculation

Disappearance experiments measure various $|U_{\alpha i}|^2$ terms
Suppose 4 are measured: $|U_{e2}|^2$, $|U_{e3}|^2$, $|U_{\mu 2}|^2$, $|U_{\mu 3}|^2$

Actually this gives all 9 magnitudes by unitarity

Direct Analytic Calculation

Disappearance experiments measure various $|U_{\alpha i}|^2$ terms
Suppose 4 are measured: $|U_{e2}|^2$, $|U_{e3}|^2$, $|U_{\mu 2}|^2$, $|U_{\mu 3}|^2$

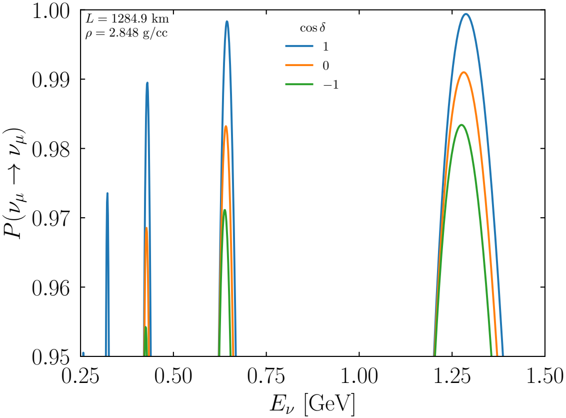
Actually this gives all 9 magnitudes by unitarity

$$J_{CP}^2 = |U_{e2}|^2|U_{\mu 2}|^2|U_{e3}|^2|U_{\mu 3}|^2 - \frac{1}{4}\left(1 - |U_{e2}|^2 - |U_{\mu 2}|^2 - |U_{e3}|^2 - |U_{\mu 3}|^2 + |U_{e2}|^2|U_{\mu 3}|^2 + |U_{e3}|^2|U_{\mu 2}|^2\right)^2$$

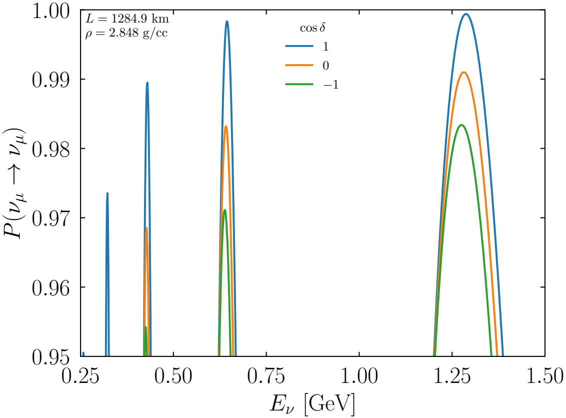
Disappearance can tell us if CP is violated,
but not if Nature prefers ν's or $\bar{\nu}$'s

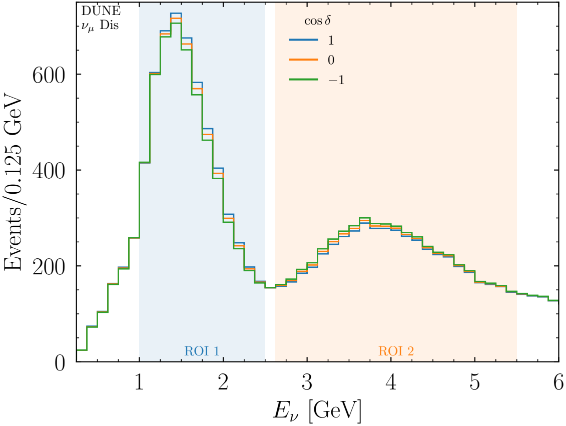
Can show that if any one $|U_{\alpha i}|^2 = 0 \Rightarrow J = 0$

Where is $|U_{\mu 2}|^2$?



Where is $|U_{\mu 2}|^2$?





6.5 yrs ν_μ rates

$\cos\delta$	ROI 1	ROI 2
1	5506	5038
0	5418	5115
-1	5334	5193

Approximate Size of $|U_{\mu 2}|^2$ Signal: (21) Sector

- There is no δ information in $|U_{\mu 1}|^2 + |U_{\mu 2}|^2$ (sum of Δ_{31} and Δ_{32} terms)

Approximate Size of $|U_{\mu 2}|^2$ Signal: (21) Sector

- There is no δ information in $|U_{\mu 1}|^2 + |U_{\mu 2}|^2$ (sum of Δ_{31} and Δ_{32} terms)
- There is δ information in $|U_{\mu 1}|^2 |U_{\mu 2}|^2$

Approximate Size of $|U_{\mu2}|^2$ Signal: (21) Sector

- There is no δ information in $|U_{\mu1}|^2+|U_{\mu2}|^2$ (sum of Δ_{31} and Δ_{32} terms)
- There is δ information in $|U_{\mu1}|^2|U_{\mu2}|^2$
- This comes from the Δm^2_{21} term

DUNE and HK can measure Δm^2_{21} somewhat
PBD, J. Gehrlein 2302.08513

Approximate Size of $|U_{\mu 2}|^2$ Signal: (21) Sector

- There is no δ information in $|U_{\mu1}|^2+|U_{\mu2}|^2$ (sum of Δ_{31} and Δ_{32} terms)
- There is δ information in $|U_{\mu1}|^2|U_{\mu2}|^2$
- This comes from the Δm^2_{21} term

DUNE and HK can measure Δm^2_{21} somewhat
PBD, J. Gehrlein 2302.08513

- This term is

$$P_{\mu\mu}\supset -4c^2_{23}\left(s^2_{12}c^2_{12}+s_{23}c_{23}s_{13}\sin 2\theta_{12}\cos 2\theta_{12}\cos\delta\right)\sin^2\Delta_{21}$$

$$\approx -2\quad(0.21\quad+\quad 0.03\cos\delta)\left(\frac{\pi}{33}\right)^2$$

$\Delta m^2_{21}/|\Delta m^2_{31}|\approx 33$

Approximate Size of $|U_{\mu2}|^2$ Signal: (21) Sector

- There is no δ information in $|U_{\mu1}|^2+|U_{\mu2}|^2$ (sum of Δ_{31} and Δ_{32} terms)
- There is δ information in $|U_{\mu1}|^2|U_{\mu2}|^2$
- This comes from the Δm^2_{21} term

DUNE and HK can measure Δm^2_{21} somewhat
PBD, J. Gehrlein 2302.08513

- This term is

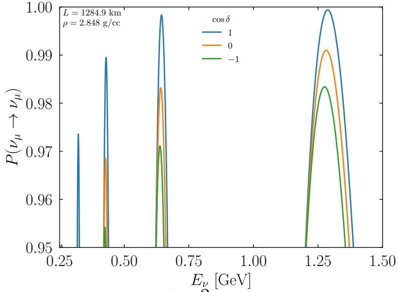
$$P_{\mu\mu} \supset -4c^2_{23}\left(s^2_{12}c^2_{12}+s_{23}c_{23}s_{13}\sin 2\theta_{12}\cos 2\theta_{12}\cos\delta\right)\sin^2\Delta_{21}$$

$$\approx -2 \quad (0.21 \quad + \quad 0.03\cos\delta)\left(\frac{\pi}{33}\right)^2$$

$\Delta m^2_{21}/|\Delta m^2_{31}| \approx 33$

- So the probability is large for $\cos\delta=-1$?
- So the effect is $\sim -0.0005\cos\delta$?

Approximate Size of $|U_{\mu 2}|^2$ Signal: (21) Sector



- There is no δ information in $|U_{\mu 1}|^2 + |U_{\mu 2}|^2$
- There is δ information in $|U_{\mu 1}|^2 |U_{\mu 2}|^2$
- This comes from the Δm^2_{21} term

DUNE and HK can measure Δm^2_{21} somewhat
PBD, J. Gehrlein 2302.08513

- This term is

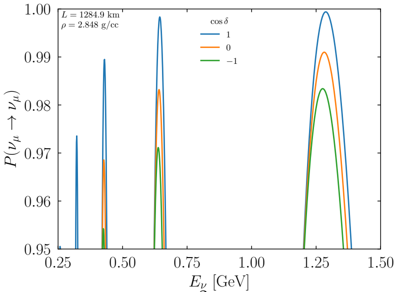
$$P_{\mu\mu} \supset -4c^2_{23}\left(s^2_{12}c^2_{12} + s_{23}c_{23}s_{13}\sin 2\theta_{12}\cos 2\theta_{12}\cos\delta\right)\sin^2\Delta_{21}$$

$$\approx -2 \quad (0.21 \quad + \quad 0.03\cos\delta)\left(\frac{\pi}{33}\right)^2$$

$\Delta m^2_{21}/|\Delta m^2_{31}| \approx 33$

- So the probability is large for $\cos\delta = -1$?
- So the effect is $\sim -0.0005\cos\delta$?

Approximate Size of $|U_{\mu2}|^2$ Signal: (21) Sector



DUNE and HK can measure Δm^2_{21} somewhat
PBD, J. Gehrlein 2302.08513

- There is no δ information in $|U_{\mu1}|^2 + |U_{\mu2}|^2$
- There is δ information in $|U_{\mu1}|^2|U_{\mu2}|^2$
- This comes from the Δm^2_{21} term

- This term is

$$P_{\mu\mu} \supset -4c_{23}^2\left(s_{12}^2c_{12}^2 + s_{23}c_{23}s_{13}\sin 2\theta_{12}\cos 2\theta_{12}\cos\delta\right)\sin^2\Delta_{21}$$

$$\approx -2 \quad (0.21 \quad + \quad 0.03\cos\delta)\left(\frac{\pi}{33}\right)^2$$

$\Delta m^2_{21}/|\Delta m^2_{31}| \approx 33$

- So the probability is large for $\cos\delta = -1$? Sign is wrong
- So the effect is $\sim -0.0005\cos\delta$? Magnitude is ~ 16 too small

Matter Effects Matter: (21) Sector

- Let's start at

$$P_{\mu\mu} \supset -4c_{23}^2\left(s_{12}^2c_{12}^2 + s_{23}c_{23}s_{13}\sin 2\theta_{12}\cos 2\theta_{12}\cos\delta\right)\sin^2\Delta_{21}$$

Matter Effects Matter: (21) Sector

- Let's start at

$$P_{\mu\mu} \supset -4c_{23}^2 \left(s_{12}^2 c_{12}^2 + s_{23} c_{23} s_{13} \sin 2\theta_{12} \cos 2\theta_{12} \cos\delta\right) \sin^2 \Delta_{21}$$

- Solar splitting in matter modified by

$$\Delta m_{21}^2 \to \Delta m_{21}^2 \mathcal{S}_\odot$$

$$\mathcal{S}_\odot \approx \sqrt{(\cos 2\theta_{12} - c_{13}^2 a/\Delta m_{21}^2)^2 + \sin^2 2\theta_{12}} \approx 3.4$$

at $E = 1.3$ GeV

PBD, S. Parke 1902.07185

Matter Effects Matter: (21) Sector

- Let's start at

$$P_{\mu\mu} \supset -4c_{23}^2 \left(s_{12}^2 c_{12}^2 + s_{23} c_{23} s_{13} \sin 2\theta_{12} \cos 2\theta_{12} \cos\delta\right) \sin^2 \Delta_{21}$$

- Solar splitting in matter modified by

$$\Delta m_{21}^2 \to \Delta m_{21}^2 \mathcal{S}_\odot$$

$$\mathcal{S}_\odot \approx \sqrt{(\cos 2\theta_{12} - c_{13}^2 a/\Delta m_{21}^2)^2 + \sin^2 2\theta_{12}} \approx 3.4$$

at $E = 1.3$ GeV

PBD, S. Parke 1902.07185

- Mixing angle is modified

$$\cos 2\theta_{12} = 0.37 \to \frac{\cos 2\theta_{12} - c_{13}^2 a/\Delta m_{21}^2}{\mathcal{S}_\odot} \approx -0.96 < 0$$

$a \propto \rho E$

Matter Effects Matter: (21) Sector

- Let's start at

$$P_{\mu\mu} \supset -4c_{23}^2 \left(s_{12}^2 c_{12}^2 + s_{23} c_{23} s_{13} \sin 2\theta_{12} \cos 2\theta_{12} \cos\delta\right) \sin^2 \Delta_{21}$$

- Solar splitting in matter modified by

$$\Delta m_{21}^2 \to \Delta m_{21}^2 \mathcal{S}_\odot$$

$$\mathcal{S}_\odot \approx \sqrt{(\cos 2\theta_{12} - c_{13}^2 a/\Delta m_{21}^2)^2 + \sin^2 2\theta_{12}} \approx 3.4$$

at $E = 1.3$ GeV

PBD, S. Parke 1902.07185

- Mixing angle is modified

$$\cos 2\theta_{12} = 0.37 \to \frac{\cos 2\theta_{12} - c_{13}^2 a/\Delta m_{21}^2}{\mathcal{S}_\odot} \approx -0.96 < 0$$

- So the sign is swapped

$a \propto \rho E$

$$\sin 2\theta_{12} \cos 2\theta_{12} = 0.35 \to -0.26$$

Matter Effects Matter: (21) Sector

- Let's start at

$$P_{\mu\mu} \supset -4c_{23}^2\left(s_{12}^2c_{12}^2 + s_{23}c_{23}s_{13}\sin 2\theta_{12}\cos 2\theta_{12}\cos\delta\right)\sin^2\Delta_{21}$$

- Solar splitting in matter modified by

$$\Delta m_{21}^2 \to \Delta m_{21}^2 \mathcal{S}_\odot$$

$$\mathcal{S}_\odot \approx \sqrt{(\cos 2\theta_{12} - c_{13}^2 a/\Delta m_{21}^2)^2 + \sin^2 2\theta_{12}} \approx 3.4$$

at $E = 1.3$ GeV
PBD, S. Parke 1902.07185

- Mixing angle is modified

$$\cos 2\theta_{12} = 0.37 \to \frac{\cos 2\theta_{12} - c_{13}^2 a/\Delta m_{21}^2}{\mathcal{S}_\odot} \approx -0.96 < 0$$

- So the sign is swapped

$a \propto \rho E$

$$\sin 2\theta_{12}\cos 2\theta_{12} = 0.35 \to -0.26$$

- Also s_{13} increases in matter $\sim 15\%$: total effect is $0.004\cos\delta$

Matter Effects Matter: (21) Sector

- Let's start at

$$P_{\mu\mu} \supset -4c_{23}^2\left(s_{12}^2c_{12}^2 + s_{23}c_{23}s_{13}\sin 2\theta_{12}\cos 2\theta_{12}\cos\delta\right)\sin^2\Delta_{21}$$

- Solar splitting in matter modified by

$$\Delta m_{21}^2 \to \Delta m_{21}^2 \mathcal{S}_\odot$$

$$\mathcal{S}_\odot \approx \sqrt{(\cos 2\theta_{12} - c_{13}^2 a/\Delta m_{21}^2)^2 + \sin^2 2\theta_{12}} \approx 3.4$$

at $E = 1.3$ GeV

PBD, S. Parke 1902.07185

- Mixing angle is modified

$$\cos 2\theta_{12} = 0.37 \to \frac{\cos 2\theta_{12} - c_{13}^2 a/\Delta m_{21}^2}{\mathcal{S}_\odot} \approx -0.96 < 0$$

- So the sign is swapped

$a \propto \rho E$

$$\sin 2\theta_{12}\cos 2\theta_{12} = 0.35 \to -0.26$$

- Also s_{13} increases in matter $\sim 15\%$: total effect is $0.004\cos\delta$
- This gets us **half** of the effect, and the correct sign

Matter Effects Matter: (32) Sector

- $\frac{\Delta m^2_{\mu\mu} L}{4E}$ in matter at $P_{\max}$ is $\sim \pi$

H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283
PBD, S. Parke 2401.10326

Matter Effects Matter: (32) Sector

- $\frac{\Delta m^2_{\mu\mu} L}{4E}$ in matter at $P_{\rm max}$ is $\sim \pi$

H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283
PBD, S. Parke 2401.10326

- The Δm^2_{32} component is a bit off π at $P_{\rm max}$

Matter Effects Matter: (32) Sector

- $\frac{\Delta m^2_{\mu\mu} L}{4E}$ in matter at $P_{\max}$ is $\sim \pi$

H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283
PBD, S. Parke 2401.10326

- The Δm^2_{32} component is a bit off π at $P_{\max}$
- Leading order in s_{13}:

$$\begin{aligned}
&\approx -4s^2_{23}(c^2_{12}c^2_{23} - 2s_{13}s_{12}c_{12}s_{23}c_{23}\cos\delta)\sin^2\Delta_{32} \\
&\approx -2 \quad (0.0094 \qquad\qquad -0.023\cos\delta)0.1 \qquad\qquad \text{(matter)}
\end{aligned}$$

Matter Effects Matter: (32) Sector

- $\frac{\Delta m^2_{\mu\mu} L}{4E}$ in matter at $P_{\rm max}$ is $\sim \pi$

H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283
PBD, S. Parke 2401.10326

- The Δm^2_{32} component is a bit off π at $P_{\rm max}$
- Leading order in s_{13}:

$$\begin{aligned}&\approx -4 s^2_{23} (c^2_{12} c^2_{23} - 2 s_{13} s_{12} c_{12} s_{23} c_{23} \cos\delta) \sin^2 \Delta_{32}\\ &\approx -2 \quad (0.0094 \qquad\qquad - 0.023 \cos\delta) 0.1 \qquad\qquad \text{(matter)}\end{aligned}$$

- Adds in another $\approx 0.004 \cos\delta$ effect

Matter Effects Matter: (32) Sector

- $\frac{\Delta m^2_{\mu\mu} L}{4E}$ in matter at $P_{\max}$ is $\sim \pi$

H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283
PBD, S. Parke 2401.10326

- The Δm^2_{32} component is a bit off π at $P_{\max}$
- Leading order in s_{13}:

$$\begin{aligned} &\approx -4 s_{23}^2 (c_{12}^2 c_{23}^2 - 2 s_{13} s_{12} c_{12} s_{23} c_{23} \cos\delta) \sin^2 \Delta_{32} \\ &\approx -2 \quad (0.0094 \qquad -0.023 \cos\delta) 0.1 \qquad \text{(matter)} \end{aligned}$$

- Adds in another $\approx 0.004 \cos\delta$ effect
- Total is $\approx 0.008 \cos\delta$ which agrees with numerical calculation

Matter Effects Matter: (32) Sector

- $\frac{\Delta m^2_{\mu\mu} L}{4E}$ in matter at $P_{\rm max}$ is $\sim\pi$

H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283
PBD, S. Parke 2401.10326

- The Δm^2_{32} component is a bit off π at $P_{\rm max}$
- Leading order in s_{13}:

$$\begin{aligned}&\approx -4s^2_{23}(c^2_{12}c^2_{23} - 2s_{13}s_{12}c_{12}s_{23}c_{23}\cos\delta)\sin^2\Delta_{32}\\ &\approx -2\quad (0.0094 \qquad -0.023\cos\delta)0.1 \qquad \text{(matter)}\end{aligned}$$

- Adds in another $\approx 0.004\cos\delta$ effect
- Total is $\approx 0.008\cos\delta$ which agrees with numerical calculation

$\sim 1\%$ effect

Matter effect shifts the numbers

Numerous subtle effects in play

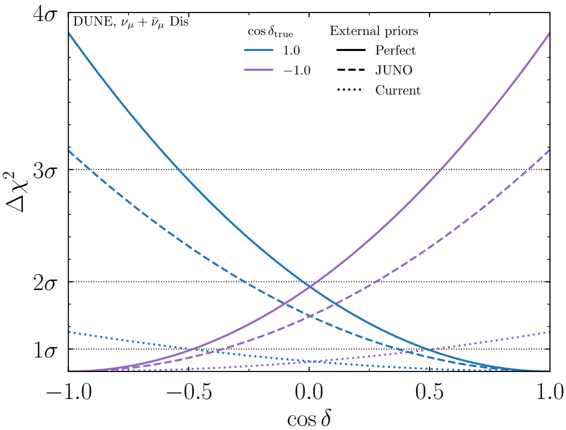
Numerical Studies

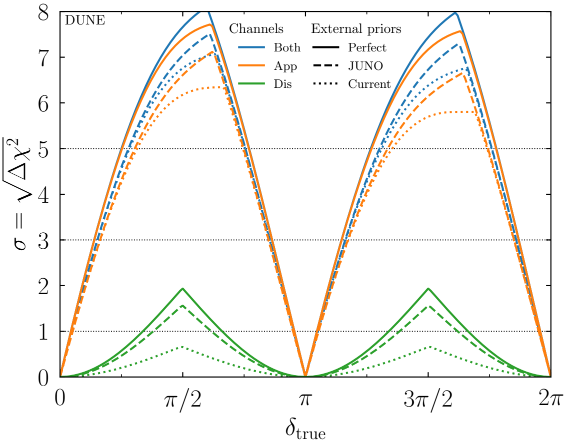
Inputs are *only*:

- Daya Bay data for θ_{13} 1809.02261
- KamLAND data for θ_{12} and Δm^2_{21} 1303.4667
- JUNO 6 yrs precision sensitivity on θ_{12}, Δm^2_{21}, Δm^2_{31} 2204.13249
- DUNE 6.5+6.5 yrs disappearance channels sensitivity only 2103.04797

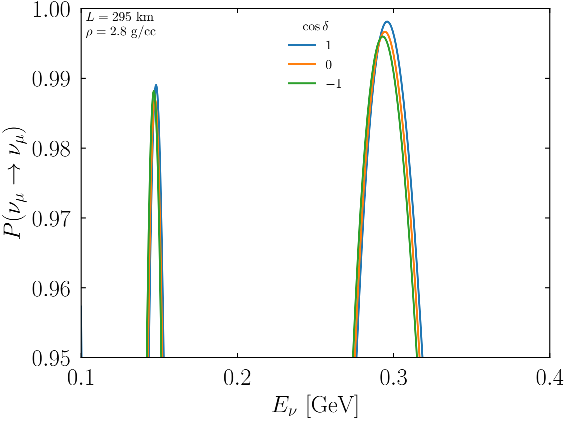
Also looked at varying JUNO's and DUNE's runtime, and at HK

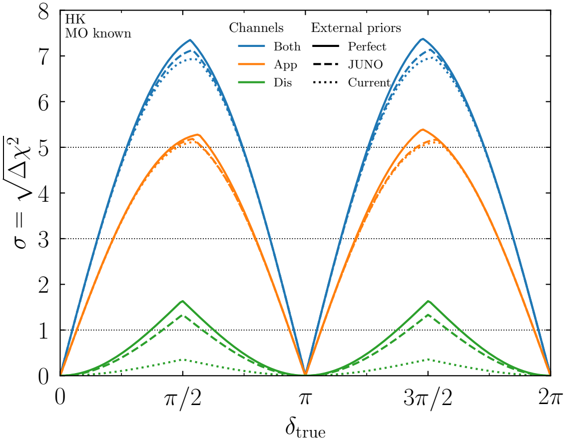
JUNO and DUNE Disappearance Sensitivities





JUNO and HK Disappearance Sensitivities





NuFast-LBL

A fast code for long-baseline
neutrino oscillation probabilities in matter

github.com/PeterDenton/NuFast-LBL

NuFast-Earth

A fast code for neutrino oscillations through the Earth from the atmosphere, the Sun, or a supernova

github.com/PeterDenton/NuFast-Earth

2405.02400 & 2511.04735 with S. Parke

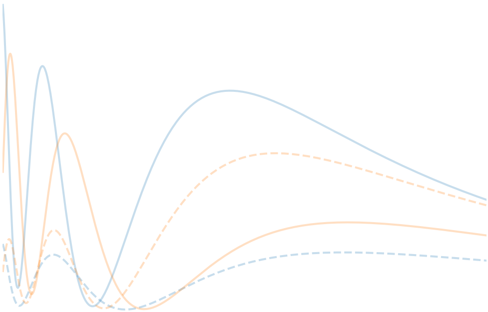
The Problem

Fundamental parameters

Physical observables

$$\Delta m^2_{21},\ \Delta m^2_{31}$$
$$s^2_{23},\ s^2_{13},\ s^2_{12}$$
$$\delta$$

$\boxed{\begin{array}{c}\Rightarrow\\ \text{in matter}\end{array}}$



Many Approaches

Solve the Schrödinger equation

$$i\frac{d}{dt}|\nu\rangle = H(t)|\nu\rangle$$

If $H(t) = H$ (constant density)

$$\mathcal{A}(\nu_\alpha \to \nu_\beta) = \left[e^{-iHL}\right]_{\beta\alpha} \qquad\qquad P = |\mathcal{A}|^2$$

Exponential requires computing eigenvalues and eigenvectors of H

Many Approaches

Modify vacuum probabilities

- Get the eigenvalues by solving the cubic

Cardano 1545
V. Barger, et al. PRD 22 (1980) 2718

- Get the eigenvectors

H. Zaglauer, K. Schwarzer Z.Phys. C40 (1988) 273
K. Kimura, A. Takamura, H. Yokomakura hep-ph/0205295
PBD, S. Parke, X. Zhang 1907.02534
A. Abdulahi, S. Parke 2212.12565

Fermilab computing experts bolster NOvA evidence, 1 million cores consumed

July 3, 2018 | Marcia Teckenbrock

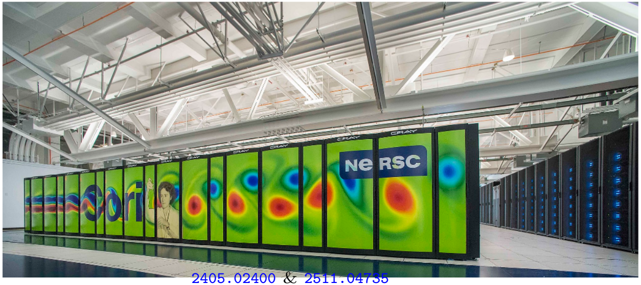
Share Tweet Email

Array

How do you arrive at the physical laws of the universe when you're given experimental data on a renegade particle that interacts so rarely with matter, it can cruise through light-years of lead? You call on the power of advanced computing.

The NOvA neutrino experiment, in collaboration with the Department of Energy's Scientific Discovery through Advanced Computing (SciDAC-4) program and the HEPCloud program at DOE's Fermi National Accelerator Laboratory, was able to perform the largest-scale analysis ever to support the recent evidence of antineutrino oscillation, a phenomenon that may hold clues to how our universe evolved.

Using Cori, the newest supercomputer at the National Energy Research Scientific Computing Center (NERSC), located at Lawrence Berkeley National Laboratory, NOvA used over 1 million computing cores, or CPUs, between May 14 and 15 and over a short timeframe one week later. This is the largest number of CPUs ever used concurrently over this duration — about 54 hours — for a single high-energy physics experiment. This unprecedented amount of computing enabled scientists to carry out some of the most complicated techniques used in neutrino physics, allowing them to dig deeper into the seldom seen interactions of neutrinos. This Cori allocation was more than 400 times the amount of Fermilab computing allocated to the NOvA experiment and 50 times the total computing capacity at Fermilab allocated for all of its rare-physics experiments. A continuation of the analysis was performed on NERSC's Cori and Edison supercomputers one week later. In total, nearly 35 million core-hours were consumed by NOvA in the 54-hour period. Executing the same analysis on a single desktop computer would take 4,000 years.



FNAL Newsroom

Monte-Carlo Estimates of Statistical Significances

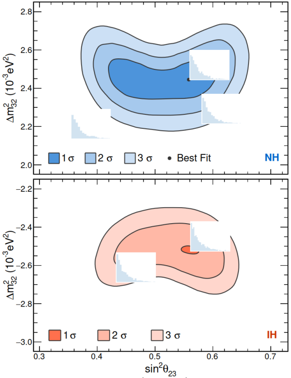
Wilks' theorem is often wrong

At each point in parameter space, simulate the experiment many times

"many" means $\gg 1/p$ for a desired p-value

This is sometimes called Feldman-Cousins

G. Feldman, R. Cousins physics/9711021
This isn't actually what was novel in the FC paper



Study found most of the time was spent computing probabilities

NOvA/T2K are $\sim 3\sigma$ experiments, but DUNE/HK will be $\gtrsim 5\sigma$ experiments!

DUNE sensitivities require computing the probabilities "a zillion times"

How to Achieve Speed

1. Avoid costly operations
   - `sin`, `cos` (and inverse functions) are very slow
   - `sqrt` is quite slow, but not as bad as trigs
   - Division is slower than multiplication ($0.2x$ may be faster than $x/5$)
2. Avoid complex numbers as possible
3. Reduce repeated calculations
   - Compute $\frac{L}{4E}$ in the correct units once
   - Compute each of the three $\sin\frac{\Delta m^2_{ij}L}{4E}$ once
4. Don't perform unnecessary linear algebra computations

All of these are compiler dependent

Optimal Structure of the Probability for Long-Baseline

1. Amplitude requires four trig functions of kinematic variables $(\Delta m^2_{ij}L/4E)$ ✗
2. Writing the probabilities out requires three trig functions ✓
3. Disappearance structure is straightforward:

$$P_{\alpha\alpha} = 1 - 4\sum_{i>j} |V_{\alpha i}|^2 |V_{\alpha j}|^2 \sin^2 \frac{\Delta\lambda_{ij} L}{4E}$$

H in matter has eigenvalues λ_i and eigenvectors $V_{\alpha i}$

Optimal Structure of the Probability for Long-Baseline

4. Appearance structure:
   T conserving:

$$P^{TC}_{\mu e}=2\sum_{i>j}(|V_{\tau k}|^2-|V_{\mu i}|^2|V_{ej}|^2-|V_{\mu j}|^2|V_{ei}|^2)\sin^2\frac{\Delta\lambda_{ij}L}{4E}$$

Fun fact:

$$2\Re(V_{\alpha i}V^*_{\beta j}V^*_{\alpha j}V_{\beta i})$$
$$=|V_{\alpha k}|^2|V_{\beta k}|^2-|V_{\alpha i}|^2|V_{\beta i}|^2-|V_{\alpha j}|^2|V_{\beta j}|^2$$
$$=|V_{\gamma k}|^2-|V_{\alpha i}|^2|V_{\beta j}|^2-|V_{\alpha j}|^2|V_{\beta i}|^2$$

T violating:

$$P^{TV}_{\mu e}=-8J\frac{\Delta m^2_{21}\Delta m^2_{31}\Delta m^2_{32}}{\Delta\lambda_{21}\Delta\lambda_{31}\Delta\lambda_{32}}\sin\frac{\Delta\lambda_{21}L}{4E}\sin\frac{\Delta\lambda_{31}L}{4E}\sin\frac{\Delta\lambda_{32}L}{4E}$$

Leverages NHS identity:
V. Naumov IJMP 1992
P. Harrison, W. Scott hep-ph/9912435

Account for Matter in Long-Baseline

1. Need the eigenvalues λ_i
2. For eigenvectors, naively need $\Re(V_{\alpha i}V^*_{\beta j}V^*_{\alpha j}V_{\beta i})$
3. Given our form, need only the $|V_{\alpha i}|^2$ and J
   - Don't need any phase information of the eigenvectors!

Leverages PBD, S. Parke, X. Zhang 1907.02534

4. Can compute the $|V_{\alpha i}|^2$ from the λ_i and submatrix eigenvalues (requires only a square root) using Eigenvector-Eigenvalue Identity

$$|V_{\alpha i}|^2 = \frac{\prod_{k=1}^{n-1}(\lambda_i - \xi_k^\alpha)}{\prod_{k=1;k\neq i}^{n}(\lambda_i - \lambda_k)}$$

See e.g. PBD, S. Parke, T. Tao, X. Zhang 1908.03795
Can actually avoid the $\sqrt{\ }$ in practice

Eigenvalues are Hard

The eigenvalues in matter λ_i depend on S:

$$S = \cos\left\{\frac{1}{3}\cos^{-1}\left[\frac{2A^3 - 9AB + 27C}{2\left(A^2 - 3B\right)^{3/2}}\right]\right\}$$

where

$$A = \sum \lambda_i = \Delta m^2_{21} + \Delta m^2_{31} + a$$
$$B = \sum_{i>j} \lambda_i \lambda_j = \Delta m^2_{21}\Delta m^2_{31} + a[\Delta m^2_{21}(1 - |U_{e2}|^2) + \Delta m^2_{31}(1 - |U_{e3}|^2)]$$
$$C = \prod \lambda_i = a\Delta m^2_{21}\Delta m^2_{31}|U_{e1}|^2$$

Approximate Eigenvalues

1. Instead, approximate one eigenvalue
   - λ_3 is best because is never parametrically small and easy to approximate
2. From DMP:

$$\lambda_3 \approx \Delta m^2_{31} + \frac{1}{2}\Delta m^2_{ee}\left(x - 1 + \sqrt{(1-x)^2 + 4xs^2_{13}}\right)$$

$$x \equiv \frac{a}{\Delta m^2_{ee}} \qquad \Delta m^2_{ee} \equiv \Delta m^2_{31} - s^2_{12}\Delta m^2_{21}$$

H. Minakata, S. Parke 1505.01826
PBD, H. Minakata, S. Parke 1604.08167
H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283

3. Get other two eigenvalues by picking two of A, B, C conditions

Requires one more $\sqrt{}$

The Approximation

- This is the only approximation used in the entire approach
- In vacuum the approximation returns to the correct value

Many approximations in the literature are not correct in vacuum limit
See G. Barenboim, PBD, S. Parke, C. Ternes 1902.00517

- In fact can iteratively improve λ_3 with rapid convergence via Newton's method:

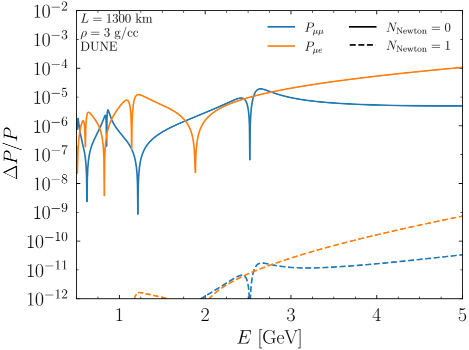
$$\lambda_3 \to \lambda_3 - \frac{X(\lambda_3)}{X'(\lambda_3)}$$

$$X(\lambda) = \lambda^3 - A\lambda^2 + B\lambda - C = 0$$

  - Precision improvement starts at 10^{-5} for the first step
  - The improvement is quadratic thereafter
- One line of code, just loop as many times as desired

NuFast-LBL Precision

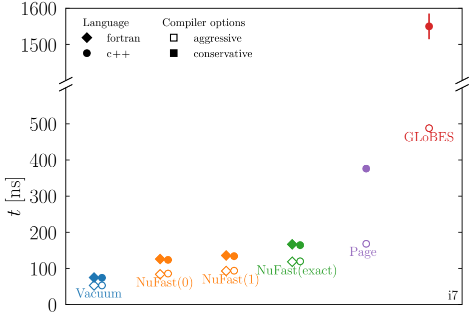
Is this approximation okay?
DUNE requires $\lesssim 1\%$ level precision



Slightly better for HK
$\sim 10^{-10}$ for JUNO
See backups

NuFast-LBL Speed

Is this algorithm fast?
How does it compare?



- “Conservative” = default, `-O0`
- “Aggressive” = `-Ofast` and `-ffast-math`
- Some variation expected due to architecture

See also J. Page 2309.06900
and P. Huber, et al. hep-ph/0701187

NuFast-LBL: The Code

- Code is on github: github.com/PeterDenton/NuFast-LBL
- Implementations in c++ and f90

```
// NuFast.cpp
// Calculate the probabilities:
Probability_Matter_LBL(s12sq, s13sq, s23sq, delta,
    Dmsq21, Dmsq31, L, E, rho, Ye, N_Newton,
    &probs_returned);
// Print out the probabilities to terminal
for (int alpha = 0; alpha < 3; alpha++)
{
    for (int beta = 0; beta < 3; beta++)
    {
        printf("%d %d %g\n", alpha, beta,
            probs_returned[alpha][beta]);
    } // beta, 3
} // alpha, 3
```

- $\bar{\nu}$: $E < 0$; IO: $\Delta m^2_{31} < 0$
- Folder called `Benchmarks` to make the plots and speed tests in the paper
- Used in `NuOscillator`, `MaCh3`, `GUNDAM`, and theory papers
- Speed up of $\gtrsim 5\times$ over state-of-the-art in realistic usage

Optimal Structure of the Probability for Atmospherics

1. Cannot apply the previous tricks through the Earth
2. Must compute the amplitude in Each layer

$$\prod_j \mathcal{A}_j$$

Optimal Structure of the Probability for Atmospherics

3. Work in the tilde basis with θ_{23} and δ pulled out
   - Can shift δ from θ_{13} to θ_{23}

$$\begin{pmatrix}1&&\\&c_{23}&s_{23}\\&-s_{23}&c_{23}\end{pmatrix}\begin{pmatrix}c_{13}&&s_{13}e^{-i\delta}\\&1&\\-s_{13}e^{i\delta}&&c_{13}\end{pmatrix}R_{12}\to\begin{pmatrix}1&&\\&c_{23}&s_{23}e^{i\delta}\\&-s_{23}e^{-i\delta}&c_{23}\end{pmatrix}\begin{pmatrix}c_{13}&&s_{13}\\&1&\\-s_{13}&&c_{13}\end{pmatrix}R_{12}$$

Optimal Structure of the Probability for Atmospherics

3. Work in the tilde basis with θ_{23} and δ pulled out
   - Can shift δ from θ_{13} to θ_{23}

$$\begin{pmatrix}1&&\\&c_{23}&s_{23}\\&-s_{23}&c_{23}\end{pmatrix}\begin{pmatrix}c_{13}&&s_{13}e^{-i\delta}\\&1&\\-s_{13}e^{i\delta}&&c_{13}\end{pmatrix}R_{12}\to\begin{pmatrix}1&&\\&c_{23}&s_{23}e^{i\delta}\\&-s_{23}e^{-i\delta}&c_{23}\end{pmatrix}\begin{pmatrix}c_{13}&&s_{13}\\&1&\\-s_{13}&&c_{13}\end{pmatrix}R_{12}$$

   - U_{23} commutes with the matter potential

$$(2E)H_f=U_{23}(\theta_{23},\delta)\left[R_{13}R_{12}\begin{pmatrix}0&&\\&\Delta m^2_{23}&\\&&\Delta m^2_{31}\end{pmatrix}R_{12}^TR_{13}^T+\begin{pmatrix}a&&\\&0&\\&&0\end{pmatrix}\right]U_{23}^\dagger(\theta_{23},\delta)$$

$$H_f=U_{23}(\theta_{23},\delta)\tilde{H}U^\dagger(\theta_{23},\delta)$$

Optimal Structure of the Probability for Atmospherics

3. Work in the tilde basis with θ_{23} and δ pulled out
   - Can shift δ from θ_{13} to θ_{23}

$$\begin{pmatrix}1&&\\&c_{23}&s_{23}\\&-s_{23}&c_{23}\end{pmatrix}\begin{pmatrix}c_{13}&&s_{13}e^{-i\delta}\\&1&\\-s_{13}e^{i\delta}&&c_{13}\end{pmatrix}R_{12}\to\begin{pmatrix}1&&\\&c_{23}&s_{23}e^{i\delta}\\&-s_{23}e^{-i\delta}&c_{23}\end{pmatrix}\begin{pmatrix}c_{13}&&s_{13}\\&1&\\-s_{13}&&c_{13}\end{pmatrix}R_{12}$$

   - U_{23} commutes with the matter potential

$$(2E)H_f=U_{23}(\theta_{23},\delta)\left[R_{13}R_{12}\begin{pmatrix}0&&\\&\Delta m^2_{23}&\\&&\Delta m^2_{31}\end{pmatrix}R^T_{12}R^T_{13}+\begin{pmatrix}a&&\\&0&\\&&0\end{pmatrix}\right]U^\dagger_{23}(\theta_{23},\delta)$$

$$H_f=U_{23}(\theta_{23},\delta)\tilde{H}U^\dagger(\theta_{23},\delta)$$

$$\mathcal{A}=\prod_j\mathcal{A}_j=\prod_j e^{-iH_{f,j}L_j}=U_{23}(\theta_{23},\delta)\left(\prod_j e^{-i\tilde{H}_jL_j}\right)U^\dagger_{23}(\theta_{23},\delta)$$

$$U_{23}U^\dagger_{23}=\mathbb{1}$$

Optimal Structure of the Probability for Atmospherics

3. Work in the tilde basis with θ_{23} and δ pulled out
   - Can shift δ from θ_{13} to θ_{23}

$$\begin{pmatrix}1&&\\&c_{23}&s_{23}\\&-s_{23}&c_{23}\end{pmatrix}\begin{pmatrix}c_{13}&&s_{13}e^{-i\delta}\\&1&\\-s_{13}e^{i\delta}&&c_{13}\end{pmatrix}R_{12}\to\begin{pmatrix}1&&\\&c_{23}&s_{23}e^{i\delta}\\&-s_{23}e^{-i\delta}&c_{23}\end{pmatrix}\begin{pmatrix}c_{13}&&s_{13}\\&1&\\-s_{13}&&c_{13}\end{pmatrix}R_{12}$$

   - U_{23} commutes with the matter potential

$$(2E)H_f=U_{23}(\theta_{23},\delta)\left[R_{13}R_{12}\begin{pmatrix}0&&\\&\Delta m^2_{23}&\\&&\Delta m^2_{31}\end{pmatrix}R_{12}^TR_{13}^T+\begin{pmatrix}a&&\\&0&\\&&0\end{pmatrix}\right]U_{23}^\dagger(\theta_{23},\delta)$$

$$H_f=U_{23}(\theta_{23},\delta)\tilde{H}U^\dagger(\theta_{23},\delta)$$

$$\mathcal{A}=\prod_j\mathcal{A}_j=\prod_j e^{-iH_{f,j}L_j}=U_{23}(\theta_{23},\delta)\left(\prod_j e^{-i\tilde{H}_jL_j}\right)U_{23}^\dagger(\theta_{23},\delta)$$

$\tilde{H}$ is real! And doesn't depend on θ_{23} or δ

$U_{23}U_{23}^\dagger=\mathbb{1}$

Reduces many unnecessary computations

Account for Matter in Atmospherics

- Calculate eigenvalues λ_i as in NuFast-LBL: exactly or approximately with Newton-Raphson corrections
- Further unitarity reductions:

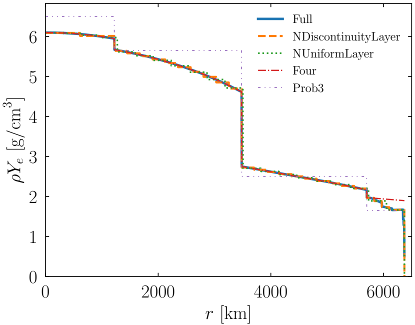
$$\mathcal{A}_{\alpha\beta} = \delta_{\alpha\beta} + V_{\alpha 2}V^*_{\beta 2}(e^{-i\Delta\lambda_{21}L/(2E)} - 1) + V_{\alpha 3}V^*_{\beta 3}(e^{-i\Delta\lambda_{31}L/(2E)} - 1)$$

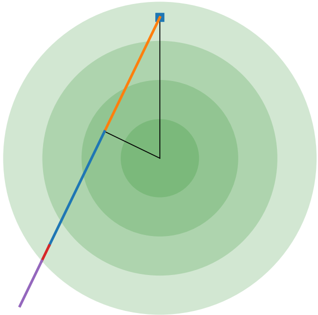
- In the tilde basis, the $\tilde{V}\tilde{V}^*$ terms are real
- Use adjugate matrices and advanced Eigenvector-Eigenvalue Identity

PBD, S. Parke, T. Tao, X. Zhang 1908.03795
A. Abdulahi, S. Parke 2212.12565

$$\tilde{V}_{\alpha i}\tilde{V}_{\beta i} = \frac{\mathrm{Adj}[\lambda_i \mathbb{1} - (2E)\tilde{H}]}{\prod_{k\neq i}(\lambda_i - \lambda_k)}$$

Earth Trajectories



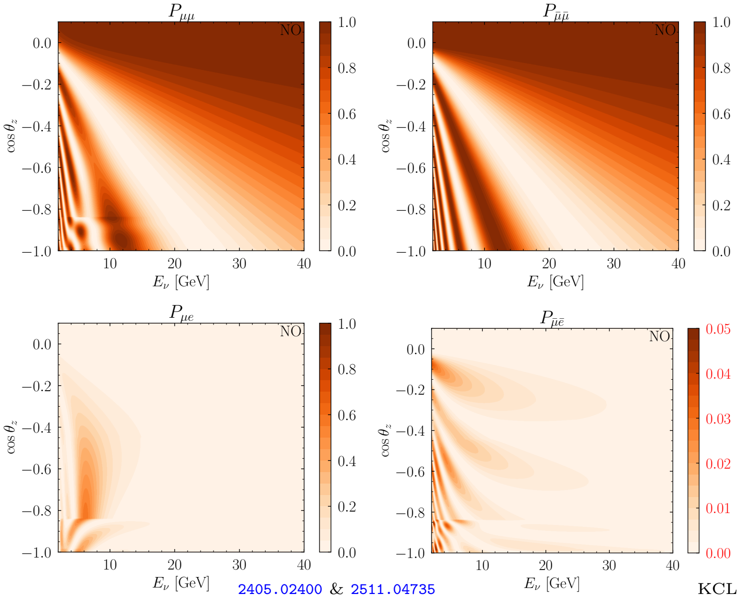


- Various Earth density profiles
- Varying or constant density in each shell

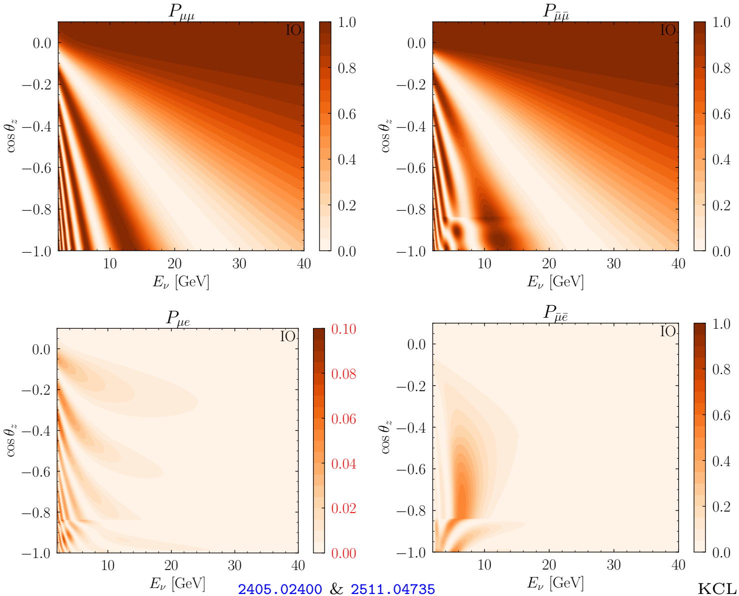
Trajectory goes from:

1. Production height to surface
2. Surface to detector depth
3. Detector depth to deepest point
4. Deepest point to detector depth: this is the transpose of #3

Atmospheric Results: NO



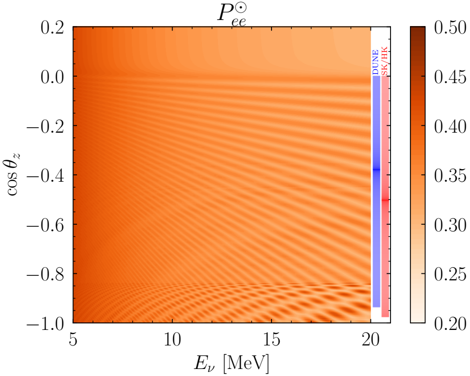
Atmospheric Results: IO

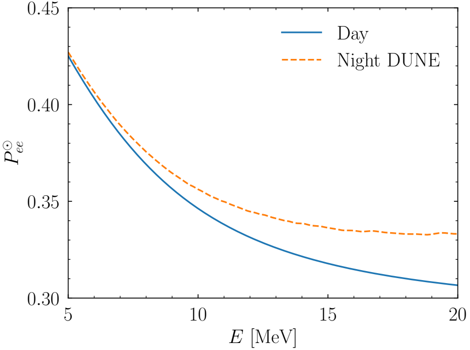


Nighttime Solar Neutrinos

At night solar neutrinos experience partial regeneration:
There are more ν_e's from the Sun at night than during the day!

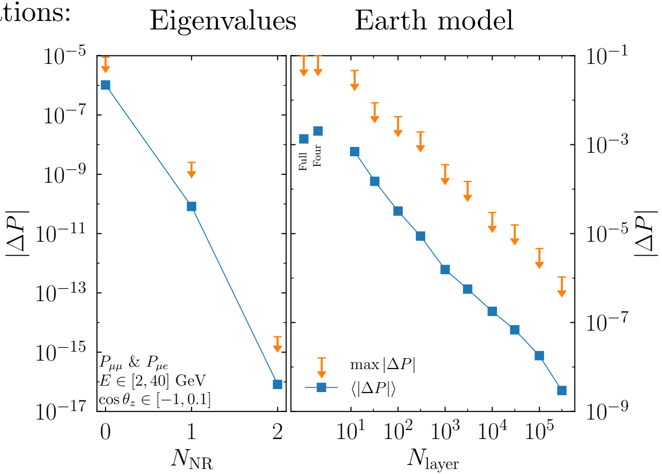
SuperK has $\sim 2\sigma$ evidence for this effect; DUNE and HK aim to measure it well





NuFast-Earth Precision

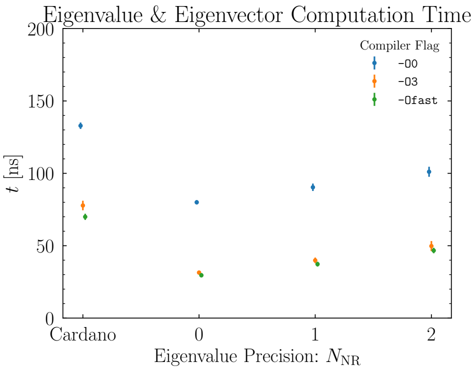
Two approximations:

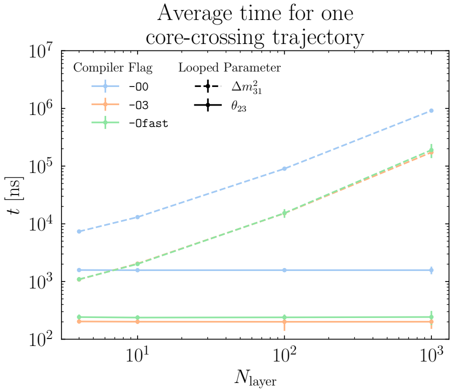


"True" has exact eigenvalues and 1M layers

Calculate $|\Delta P|$ across 100×100 grid in energy and angle

NuFast-Earth Speed





`nuSQuIDS` takes >1M ns per trajectory

C. Argüelles, J. Salvado, C. Weaver 2112.13804

`OscProb` takes 40 000 ns per trajectory at 44 layers (compare to 6000 ns or 200 ns)

J. Coelho, R. Pestes, et al. github.com/joaoabcoelho/OscProb

Other codes are designed with different goals in mind

NuFast-Earth Flowchart

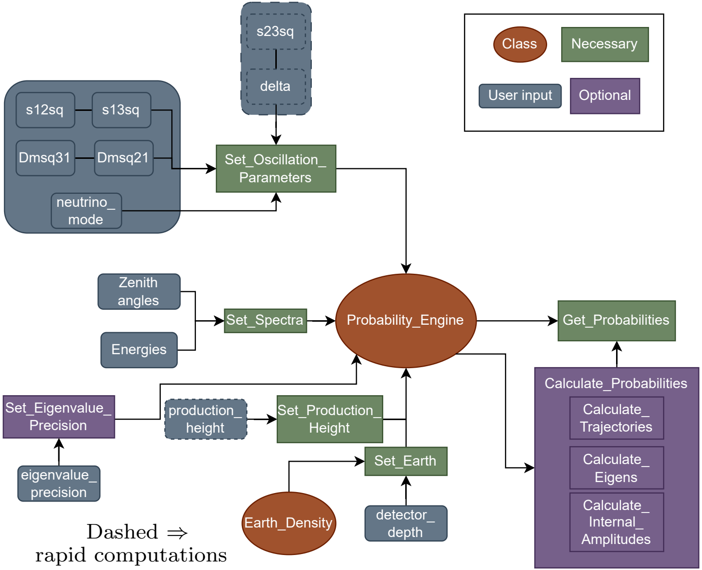
```
// Initialize the engine
Probability_Engine probability_engine;

// Set the oscillation parameters to nu-fit 6 best fit
    values:
probability_engine.Set_Oscillation_Parameters(0.307,
    0.02195, 0.561, 177 * M_PI / 180, 7.49e-5, 2.534e-3,
    true); //(s12sq, s13sq, s23sq, delta, Dmsq21, Dmsq31,
     neutrino_mode)

// Set energy and zenith angle arrays
std::vector<double> energies = {1, 2, 3, 4, 5}; // GeV
std::vector<double> coszs = {-1, -0.5, 0, 1}; // core-
    crossing to horizontal to down-going
probability_engine.Set_Spectra(energies, coszs);

// Create Earth model instance
PREM_NDiscontinuityLayer earth_density(2, 10, 10, 5); //
    2 layers in the inner core, 10 layers in the outer
    core, 10 layers in the inner mantle, and 5 layers in
    the outer mantle
// Set Earth details
probability_engine.Set_Earth(2, &earth_density); //
    detector depth in km
// Set production height (optional)
probability_enginge.Set_Production_Height(10); // km,
    recalculations after changing this are fast

// Do the calculations
std::vector<std::vector<Matrix3r>> probabilities =
    probabilitie_engine.Get_Probabilities();
```



NuFast-Earth Recommendations

Many choices in how to use `NuFast-Earth`
What is the best way for most cases?

1. Eigenvalue precision: set the number of Newton-Raphson corrections to 1

```
probability_engine.Set_Eigenvalue_Precision(1);
```

2. Earth model: 2, 10, 10, 5 layers in each of the four main zones

```
PREM_NDiscontinuityLayer earth_density(2, 10, 10, 5);
```

3. Looping order: put θ_{23}, δ, and production height on the innermost loops

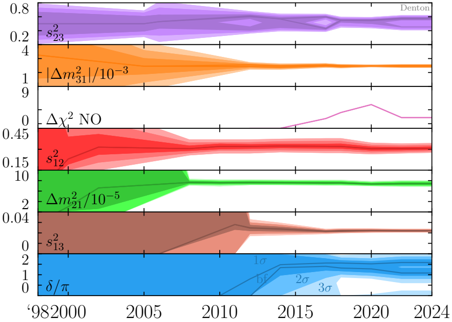
Discussion and Conclusions

- **Disappearance can discover CPV**
- Requires two good measurements: JUNO and DUNE/HK
- Can rule out some values of δ at $> 3\sigma$
- **LBL Experiments should break down δ analyses into app vs. dis**
- Since systematics are different, provides a good cross check

- `NuFast-LBL` and `NuFast-Earth` provide **fast, precise, usable code**
- Leverages modern linear algebra and neutrino theory
- Orders of magnitude impact in computational cost
- LBL code implemented in pipelines for NOvA, T2K, DUNE, HK, & JUNO
- Atmospheric code being implemented now

Backups

References



SK hep-ex/9807003

M. Gonzalez-Garcia, et al. hep-ph/0009350

M. Maltoni, et al. hep-ph/0207227

SK hep-ex/0501064

SK hep-ex/0604011

T. Schwetz, M. Tortola, J. Valle 0808.2016

M. Gonzalez-Garcia, M. Maltoni, J. Salvado 1001.4524

T2K 1106.2822

D. Forero, M. Tortola, J. Valle 1205.4018

D. Forero, M. Tortola, J. Valle 1405.7540

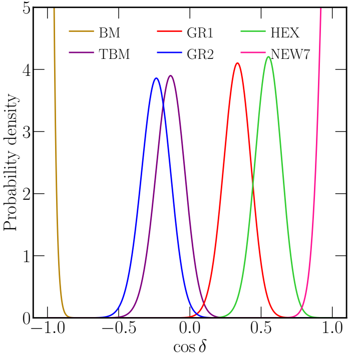
P. de Salas, et al. 1708.01186

F. Capozzi et al. 2003.08511

The Importance of $\cos\delta$

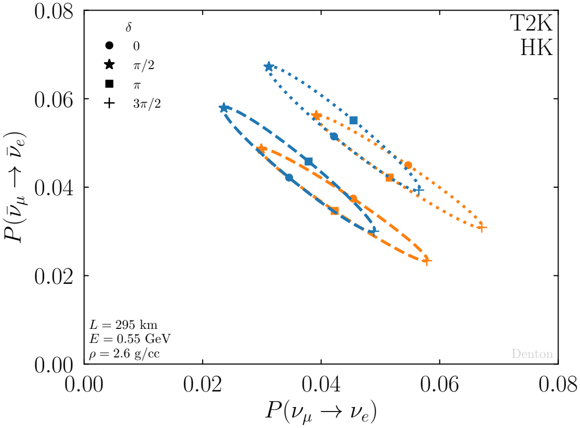
- If only $\sin\delta$ is measured $\Rightarrow$ sign degeneracy: $\cos\delta = \pm\sqrt{1-\sin^2\delta}$
- Most flavor models predict $\cos\delta$

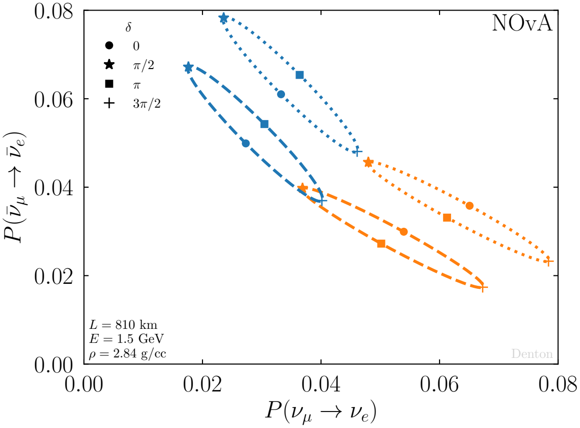
J. Gehrlein, et al. 2203.06219



L. Everett, et al. 1912.10139

δ: What is it Really?





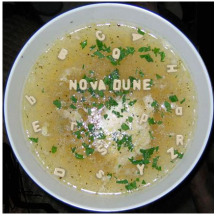
Appearance vs. Disappearance

Oscillation experiments can do
appearance or disappearance experiments:

Disappearance
K2K, MINOS, T2K, NOνA
KamLAND, Daya Bay, RENO, Double CHOOZ
(Sort of) SNO, Borexino, SK-solar
JUNO, DUNE, HK

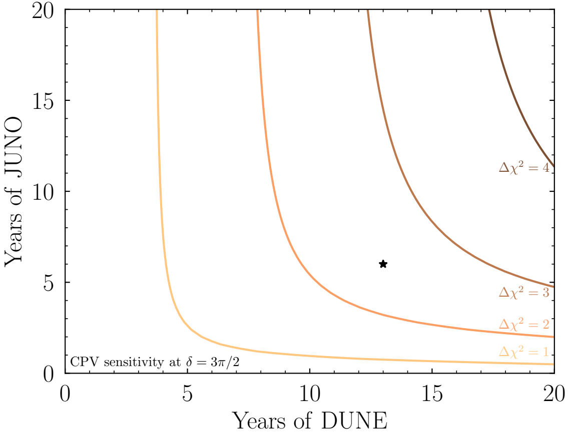
Appearance
T2K, NOνA
OPERA
Atm ν_τ hints @ SK & IceCube
DUNE, HK

Neither appearance nor disappearance
SK-atm, IceCube



Varying Runtime/Power

Significance to disfavor $|\cos\delta| = 1$ at $\cos\delta = 0$



Improvement requires **both** experiments!

Start at the End

What is needed for LBL experiments?

1. All 9 channels ($\nu_\alpha \to \nu_\beta$)
   - DUNE will certainly do ν_τ appearance

     See e.g. P. Machado, H. Schulz, J. Turner 2007.00015
   - $\nu_\tau \to \nu_\beta$ channels are not needed, but come from free from unitarity
   - JUNO needs only $\bar{\nu}_e \to \bar{\nu}_e$
2. Different energies, baselines, and densities
3. ν and $\bar{\nu}$
4. NO and IO
5. Oscillation parameters are mostly known
   - Don't need to consider e.g. $\Delta m^2_{21} > |\Delta m^2_{31}|$ or $\theta_{23} \sim 10^\circ$

All 9 Channels

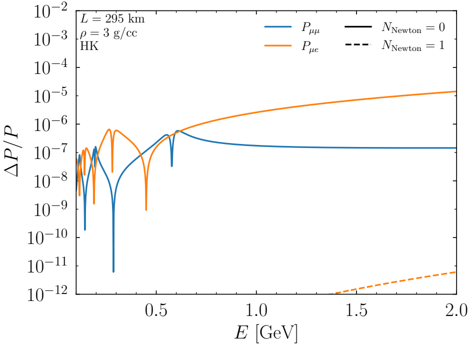
Given P_{ee}, $P_{\mu\mu}$, $P_{\mu e}^{TC}$, and $P_{\mu e}^{TV}$:

	$P_{\alpha e}$	$P_{\alpha\mu}$	$P_{\alpha\tau}$
$P_{e\beta}$	P_{ee}	$P_{\mu e}^{TC} - P_{\mu e}^{TV}$	$1 - P_{ee} - P_{\mu e}^{TC} + P_{\mu e}^{TV}$
$P_{\mu\beta}$	$P_{\mu e}^{TC} + P_{\mu e}^{TV}$	$P_{\mu\mu}$	$1 - P_{\mu\mu} - P_{\mu e}^{TC} - P_{\mu e}^{TV}$
$P_{\tau\beta}$	$1 - P_{ee} - P_{\mu e}^{TC} - P_{\mu e}^{TV}$	$1 - P_{\mu\mu} - P_{\mu e}^{TC} + P_{\mu e}^{TV}$	$-1 + P_{ee} + P_{\mu\mu} + 2P_{\mu e}^{TC}$

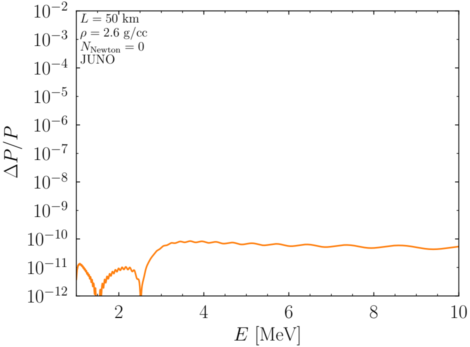
Total LBL Approach

1. Inputs: 6 oscillation parameters, experimental details (L, E, ρ, Y_e)
2. Calculate λ_3 approximately
   - Iteratively improve with Newton's method, if desired
3. Calculate the $|V_{\alpha i}|^2$'s with the Eigenvector-Eigenvalue Identity
4. Calculate the sines of the kinematic terms
5. Calculate the CP odd term with the NHS identity
6. Calculate key probabilities: P_{ee}, $P_{\mu\mu}$, and $P_{\mu e}$
7. Calculate remaining probabilities

Precision

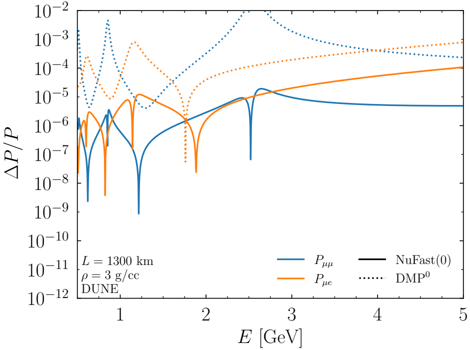


$L = 295$ km
$\rho = 3$ g/cc
HK
$P_{\mu\mu}$
$P_{\mu e}$
$N_{\rm Newton} = 0$
$N_{\rm Newton} = 1$
$\Delta P/P$
E [GeV]



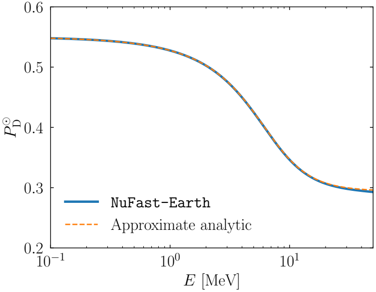
$L = 50$ km
$\rho = 2.6$ g/cc
$N_{\rm Newton} = 0$
JUNO
$\Delta P/P$
E [MeV]

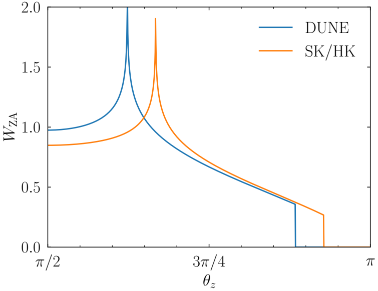
Comparison to DMP



DMP: PBD, H. Minakata, S. Parke 1604.08167

NuFast-Earth Solar Neutrino Validation





NuFast-Earth LBL Validation