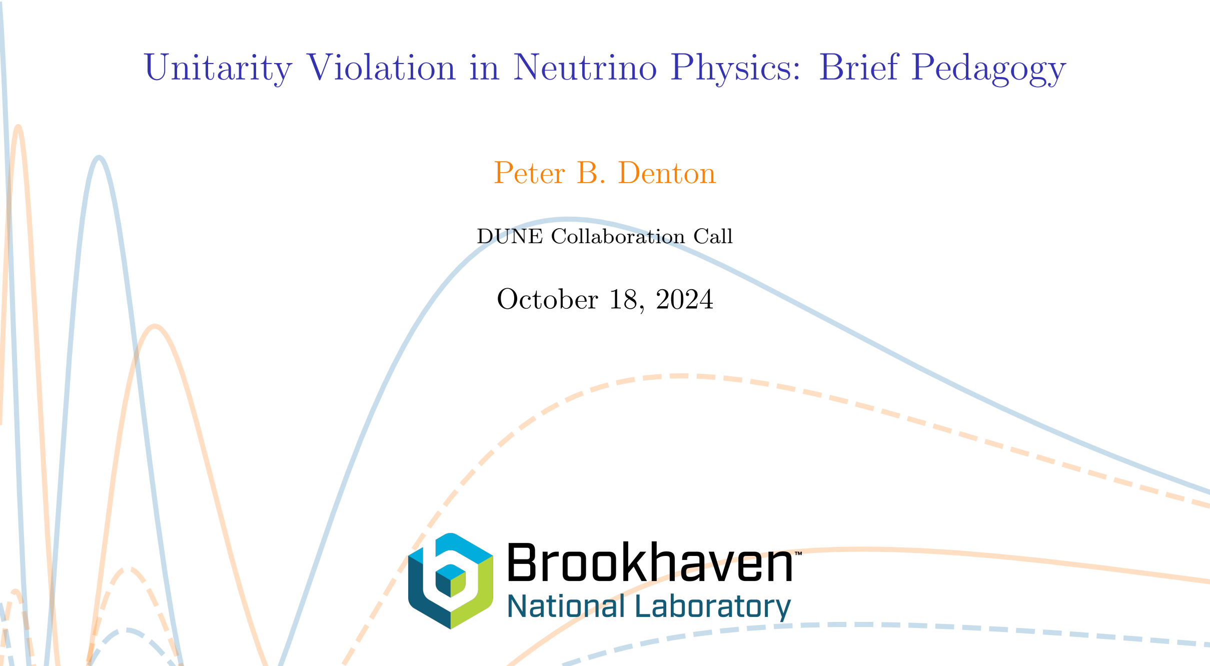
Abstract

Unitarity violation is one important framework for searching for new physics. I will discuss how neutrino oscillations are affected by unitarity violation and the importance of tau neutrinos. I will discuss exactly how they play the key role in constraining tau neutrino unitarity via a complex interplay of the matter effect, tau lepton production threshold, misreconstructed tau neutrino energy, and the matter effect. This allows one to identify tau neutrino with no event-by-event discrimination and without assuming unitarity and hopefully encourages experimentalists to perform these analyses in the future.



Unitarity Violation in Neutrino Physics: Brief Pedagogy

Peter B. Denton

DUNE Collaboration Call

October 18, 2024

Parameter counting

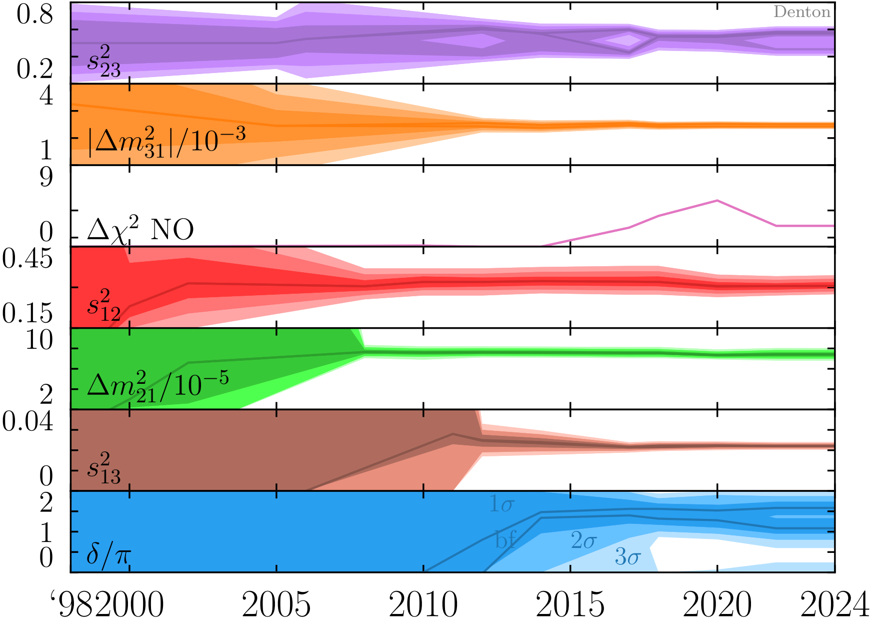
Neutrino oscillations implies 7+ new parameters:

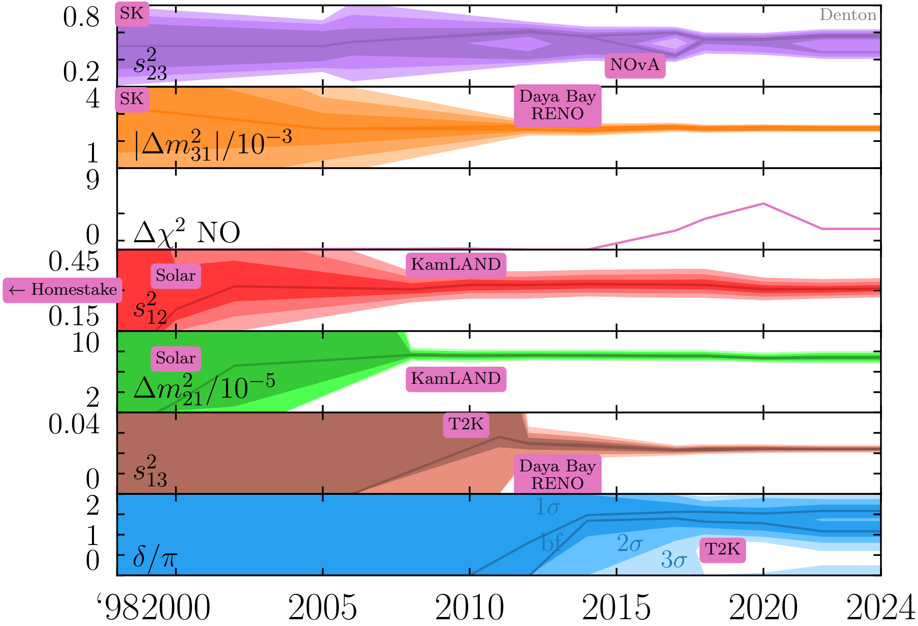
- Masses: 3 parameters
  - We've measured 2(ish)
  - Need DUNE/JUNO/atmospherics to complete these 2
  - Need cosmology for third
- Mixing matrix:
  - Start with a 3×3 complex matrix: 18 parameters
  - Unitarity (9 conditions): 9 parameters
  - Rephasing of charged leptons (3 conditions): 6 parameters
  - Rephasing of neutral leptons (3 conditions)*: 4 parameters

*Valid for Dirac neutrinos,
or in environments where Dirac/Majorana
are indistinguishable, such as $p_\nu \gg m_\nu$

Many different ways to parameterize matrix:
Typical: θ_{23}, θ_{13}, θ_{12}, δ

Other parameterizations discussed: PBD, R. Pestes 2006.09384


Denton
0.8
0.2
s^2_{23}
4
1
$|\Delta m^2_{31}|/10^{-3}$
9
0
$\Delta\chi^2$ NO
0.45
0.15
s^2_{12}
10
2
$\Delta m^2_{21}/10^{-5}$
0.04
0
s^2_{13}
2
1
0
δ/π
1σ
bf
2σ
3σ
‘98
2000
2005
2010
2015
2020
2024


Denton
SK
NOvA
s_{23}^2
0.8
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SK
Daya Bay RENO
$|\Delta m_{31}^2|/10^{-3}$
4
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$\Delta\chi^2$ NO
9
0
Solar
KamLAND
← Homestake
s_{12}^2
0.45
0.15
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$\Delta m_{21}^2/10^{-5}$
10
2
T2K
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Unitarity violation meaning

Consistency of the three-flavor oscillation picture?

and/or

Searches for unitarity violation?

Unitarity violation meaning

Consistency of the three-flavor oscillation picture?

and/or

Searches for unitarity violation?

Not the same!

Lots of models to test standard three-flavor picture:
Sterile, unitarity violation, vector NSI, scalar NSI, neutrino decay, decoherence, CPTV/LIV, …

Unitarity violation: what is it?

Our 3 × 3 matrix isn't unitary:

$$U_3U_3^\dagger\neq\mathbb{1}$$

Addition of new flavor states $\nu_a,\nu_b,\nu_c,\dots$ and new mass states ν_4,ν_5,ν_6

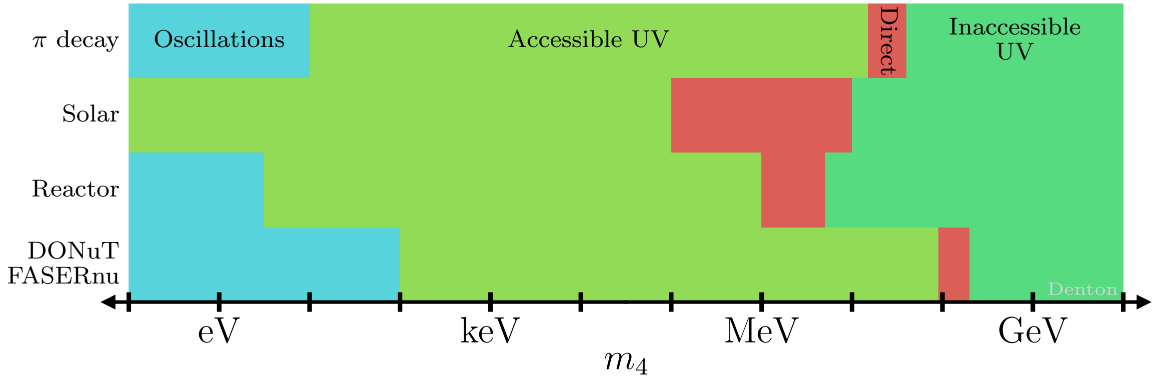
$$U\to\begin{pmatrix}U_{e1}&U_{e2}&U_{e3}&U_{e4}&\cdots\\U_{\mu1}&U_{\mu2}&U_{\mu3}&U_{\mu4}&\cdots\\U_{\tau1}&U_{\tau2}&U_{\tau3}&U_{\tau4}&\cdots\\U_{a1}&U_{a2}&U_{a3}&U_{a4}&\cdots\\\vdots&\vdots&\vdots&\vdots&\ddots\end{pmatrix}$$

Unitarity Violation ⇒

New mass states not directly accessible by oscillations or decay

Thus check if U_3 is what it should be

Unitarity violation: a tale of four regimes



*Details depends on the specific experiment/channel

Unitarity violation: mass ranges

experiment	(4,4) (m_4)	(5,3) (m_4)
atmospheric ν_μ disappearance	$\in [10\text{ eV}, 15\text{ MeV}]$	$\gtrsim 40$ MeV
atmospheric ν_τ appearance	$\in [10\text{ eV}, 15\text{ MeV}]$	$\gtrsim 40$ MeV
astrophysical ν_τ appearance	$\lesssim 15$ MeV	$\gtrsim 40$ MeV
solar ^{8}B	$\lesssim 5$ MeV	$\gtrsim 20$ MeV
DONuT/FASERnu	$\in [100\text{ eV}, 90\text{ MeV}]$	$\gtrsim 200$ MeV
LBL ν_τ appearance (OPERA)	$\in [1\text{ eV}, 15\text{ MeV}]$	$\gtrsim 40$ MeV
LBL ν_τ appearance (DUNE)	$\in [0.1\text{ eV}, 15\text{ MeV}]$	$\gtrsim 40$ MeV
LBL ν_μ disappearance (DUNE)	$\in [0.1\text{ eV}, 15\text{ MeV}]$	$\gtrsim 40$ MeV
CEvNS	$\in [10\text{ eV}, 15\text{ MeV}]$	$\gtrsim 40$ MeV

(m, n): m total neutrinos, n accessible neutrinos

PBD, J. Gehrlein 2109.14575

Unitarity violation: how to calculate

Kinematically **accessible** states

1. Unitary calculation of full $n \times n$ matrix
2. Oscillation averaged:

$$\sin^2\frac{\Delta m^2_{41}L}{4E} \to \frac{1}{2}$$

$$\sin\frac{\Delta m^2_{41}L}{4E} \to 0$$

3. No matter effect:

$$H^{\rm mat} = {\rm diag}(V_{\rm CC}+V_{\rm NC}, V_{\rm NC}, V_{\rm NC}, 0, \ldots)$$

Unitarity violation: how to calculate

Kinematically **accessible** states

1. Unitary calculation of full $n \times n$ matrix
2. Oscillation averaged:

$$\sin^2 \frac{\Delta m_{41}^2 L}{4E} \to \frac{1}{2}$$

$$\sin \frac{\Delta m_{41}^2 L}{4E} \to 0$$

3. No matter effect:

$$H^{\text{mat}} = \text{diag}(V_{\text{CC}} + V_{\text{NC}}, V_{\text{NC}}, V_{\text{NC}}, 0, \dots)$$

Kinematically **inaccessible** states

1. Nonunitary calculation of $m \times m$ matrix

$m =$ number of kinematically accessible states

2. Rescale probability:

$$P_{\alpha\beta} = \frac{|\sum_{i=1}^{\text{acc}} U_{\alpha i}^* e^{iP_i L} U_{\beta i}|}{(\sum_{i=1}^{\text{acc}} U_{\alpha i}^* U_{\alpha i})(\sum_{i=1}^{\text{acc}} U_{\beta i}^* U_{\beta i})}$$

3. Cannot subtract multiples of $\mathbb{1}$
4. Rescale cross section/flux as appropriate
5. Rescale G_F in matter effect

Unitarity violation

- Oscillations could conceivably differentiate: 2 new states from 1, but not 3+ from 2
- Zero distance effect ⇒ near detector **with flux prediction**

E.g. RAA, Gallium

- Numerous parameterizations: α matrix, η matrix, submatrix & Cauchy-Schwartz

All apply to the inaccessible cases only

- There is an approximate correspondence to sterile and NSI

$$\alpha_{ee} \approx \frac{1}{2}(s_{14}^2 + s_{15}^2 + s_{16}^2) \approx -\epsilon_{ee}\,, \quad \ldots$$

M. Blennow, et al. 1609.08637

Caveats apply! Applies to one experiment at a time

- Additional EW precision information: W, Z, π, μ, τ decays

Care is required

S. Antush, et al. hep-ph/0607020
S. Antusch, O. Fischer 1407.6607

Unitarity violation status from oscillations

3σ maximal deviations from unitarity

Leptons

	Parke+ (2015)	Hu+ (2020)	Ellis+ (2020)
ν_e row	0.073	0.003	0.05
ν_μ row	0.064	0.02	0.04
ν_τ row	0.43	0.2	0.82
ν_1 col	0.17	0.06	0.22
ν_2 col	0.23	0.09	0.27
ν_3 col	0.31	0.12	0.40

Quarks

u row	0.0015	$\sim 3\sigma$ tension
c row	0.06	
t row	-	
d col	0.005	
s col	0.06	
b col	-	

Lepton constraints don't include anomalies
Care is required

S. Ellis, K. Kelly, S. Li 2008.01088
Z. Hu, et al. 2008.09730
S. Parke, M. Ross-Lonergan 1508.05095
PDG

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Vastly different mixing angle hierarchy

$\Rightarrow$

Like comparing apples and hairstyles

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S. Ellis, K. Kelly, S. Li 2008.01088
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Global tau neutrino data set

The global tau neutrino data set:

Experiment	Source	∼Events detected
DONuT	Production	7.5
OPERA	Long-baseline	8
SK	Atmospheric	291[1]
IceCube	Atmospheric	1804[2]
IceCube	Astrophysical	2

[1] will increase to ∼ 430, see H. Tanaka and M. P. Zezula's talks

[2] with ∼ 10k en route "soon," see J. Koskinen IceCube NuTau2021 talk

Dominant unitarity constraint comes from atmospheric ν_τ appearance

PBD, J. Gehrlein `2109.14575`

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A word on solar neutrinos:

1. SK 1998: showed that ν_μ-ν_τ mixing is large (no ν_e appearance detected)
2. SNO 2001,2002: ES and NC measured a statistically significant non-ν_e flux
3. $\Rightarrow \nu_e \to \nu_\tau$ at SNO with input from SK

Unitarity violation framework

- Suppose there are m total neutrinos and n kinematically accessible: (m,n)

Accessible: [10 eV, 15 MeV]; inaccessible: $\gtrsim$ 40 MeV

ν_τ is an exception to this that requires care

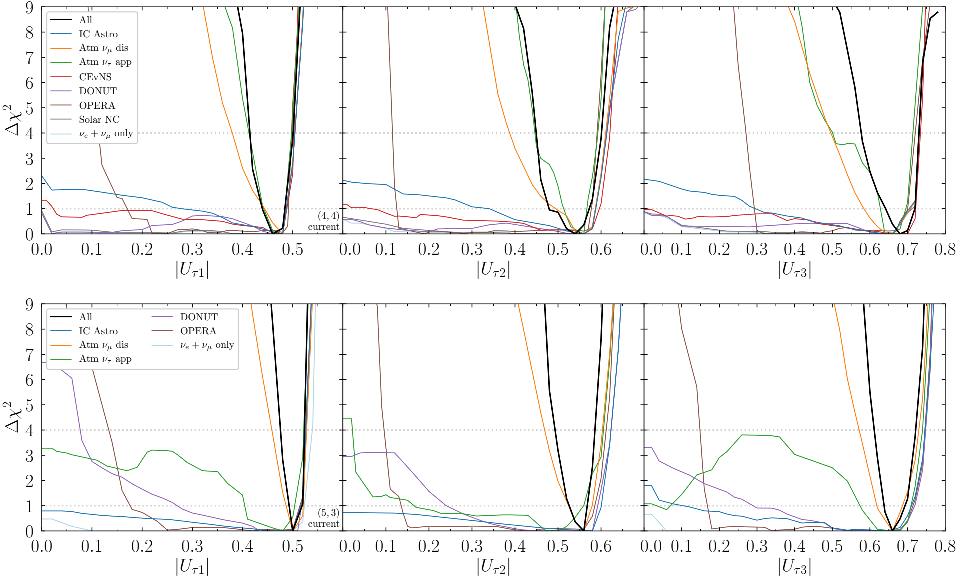
  - Standard: (3,3)
  - One accessible sterile: (4,4)
  - Two heavy steriles: (5,3)
- Include matter effect
  - Steriles don't experience it - relevant for $m=n$
  - It modifies the probability - relevant for $m>n$
- For $m=n$ oscillation probabilities can be calculated in the usual fashion
- For $m>n$ care is required:
  - Flux, cross sections, and weak interaction need to be rescaled
  - Oscillation probability needs to be rescaled and carefully calculated:

$$P^r_{\alpha\beta}=\left|\left[N^*We^{-i\Lambda L}W^\dagger N^T\right]_{\alpha\beta}\right|^2$$

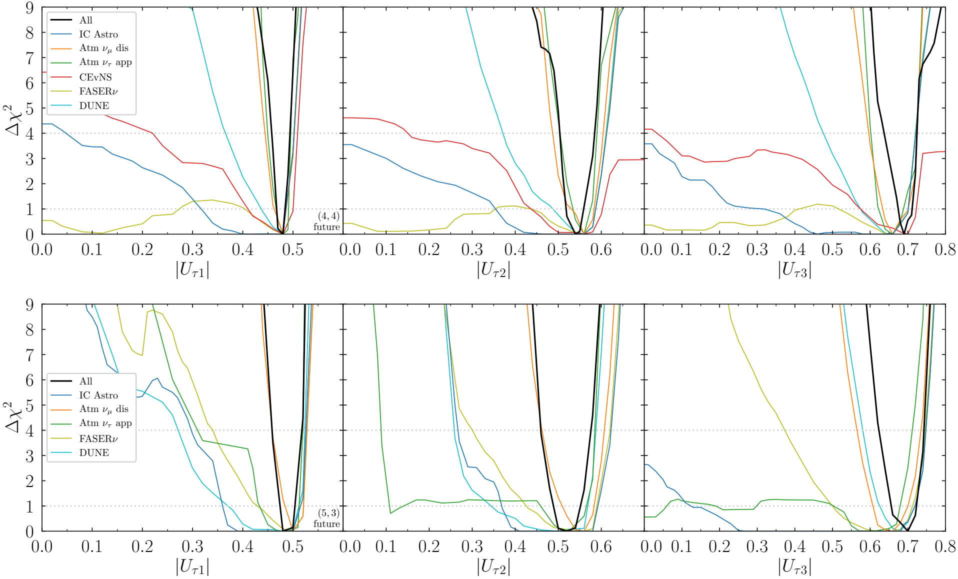
N: $m\times m$ submatrix

W,Λ eigenvectors/eigenvalues of Hamiltonian in mass basis with matter effect

Modern tau row picture



Future tau row picture



Strong CLFV constraints

Prediction in MUV	Prediction in the SM	Experiment
$[R_\ell]_{\rm SM}\,(1-0.15(\epsilon_{ee}+\epsilon_{\mu\mu}))$	20.744(11)	20.767(25)
$[R_b]_{\rm SM}\,(1+0.03(\epsilon_{ee}+\epsilon_{\mu\mu}))$	0.21577(4)	0.21629(66)
$[R_c]_{\rm SM}\,(1-0.06(\epsilon_{ee}+\epsilon_{\mu\mu}))$	0.17226(6)	0.1721(30)
$\left[\sigma^0_{had}\right]_{\rm SM}\,(1-0.25(\epsilon_{ee}+\epsilon_{\mu\mu})-0.27\epsilon_\tau)$	41.470(15) nb	41.541(37) nb
$[R_{inv}]_{\rm SM}\,(1+0.75(\epsilon_{ee}+\epsilon_{\mu\mu})+0.67\epsilon_\tau)$	5.9723(10)	5.942(16)
$[M_W]_{\rm SM}(1-0.11(\epsilon_{ee}+\epsilon_{\mu\mu}))$	80.359(11) GeV	80.385(15) GeV
$[\Gamma_{\rm lept}]_{\rm SM}(1-0.59(\epsilon_{ee}+\epsilon_{\mu\mu}))$	83.966(12) MeV	83.984(86) MeV
$[(s^{\ell,{\rm lep}}_{W,{\rm eff}})^2]_{\rm SM}(1+0.71(\epsilon_{ee}+\epsilon_{\mu\mu}))$	0.23150(1)	0.23113(21)
$[(s^{\ell,{\rm had}}_{W,{\rm eff}})^2]_{\rm SM}(1+0.71(\epsilon_{ee}+\epsilon_{\mu\mu}))$	0.23150(1)	0.23222(27)

S. Antush, O. Fischer 1407.6607

Additional constraints

- Lepton flavor universality: $\tau \to \mu\nu\nu$ vs. $\tau \to e\nu\nu$, etc.
- CKM unitarity constraints often include leptons
- Scattering NC, CC at NuTeV
- θ_W measurements

Low energy (θ_W and others) experiments dominate fits,
EWPO are comparably important

S. Antush, O. Fischer 1407.6607

CLFV results

$$\left|NN^\dagger\right| = \begin{pmatrix} 0.9979 - 0.9998 & < 10^{-5} & < 0.0021 \\ < 10^{-5} & 0.9996 - 1.0 & < 0.0008 \\ < 0.0021 & < 0.0008 & 0.9947 - 1.0 \end{pmatrix}$$

S. Antush, O. Fischer 1407.6607

Precision at the 10^{-3} level

Further improvements possible on multiple fronts

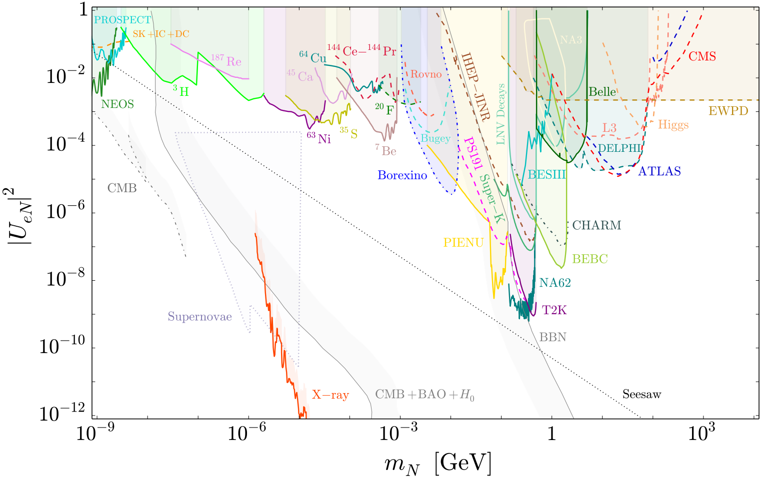
Oscillations at the $10^{-1} - 10^{-2}$ level

Non-oscillation constraints apply for heavy* m_4

*Depends on the exact probe

Mass dependent constraints

Kinematically accessible constraints:



Mass dependent constraints

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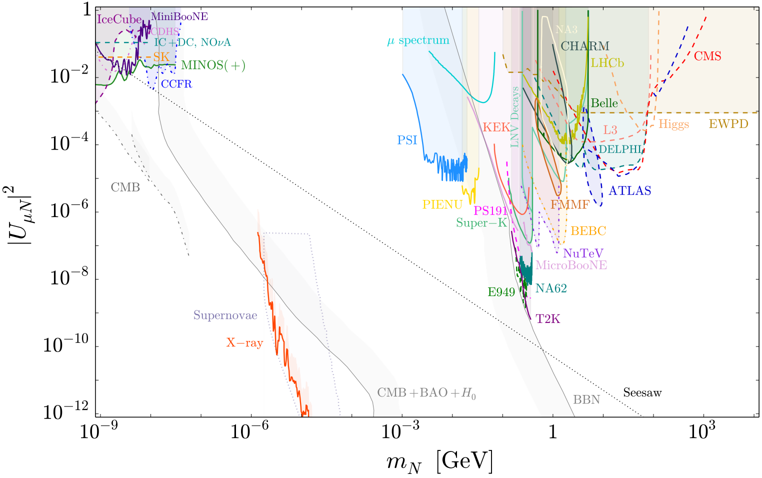

Mass dependent constraints

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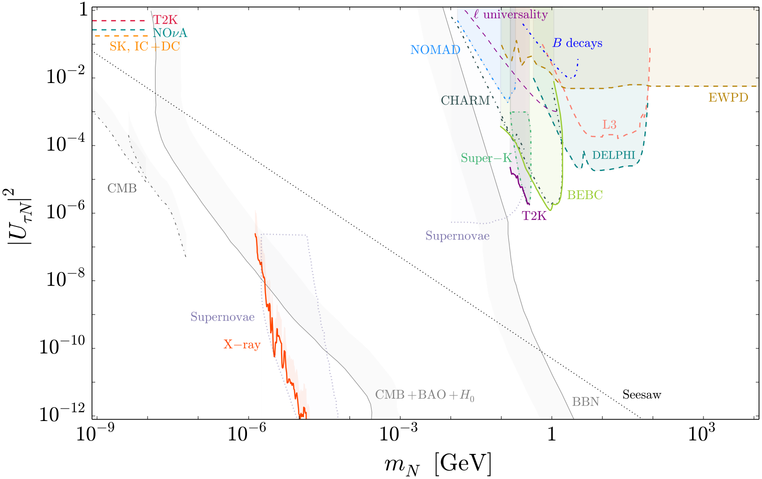


Neutrino oscillation summary

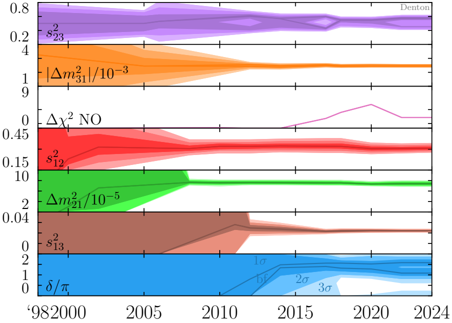
- Unitarity violation is phenomenologically very rich
- Atmospheric works for ν_τ because τ is in direct region
- CLFV and EW tests are stronger than oscillations, apply for m_4 large
- Oscillations dominate for 10 eV $\lesssim m_4 \lesssim$ 10 MeV

Precision is coming to neutrinos!

Thanks!
Questions?

Backups

References



SK hep-ex/9807003

M. Gonzalez-Garcia, et al. hep-ph/0009350

M. Maltoni, et al. hep-ph/0207227

SK hep-ex/0501064

SK hep-ex/0604011

T. Schwetz, M. Tortola, J. Valle 0808.2016

M. Gonzalez-Garcia, M. Maltoni, J. Salvado 1001.4524

T2K 1106.2822

D. Forero, M. Tortola, J. Valle 1205.4018

D. Forero, M. Tortola, J. Valle 1405.7540

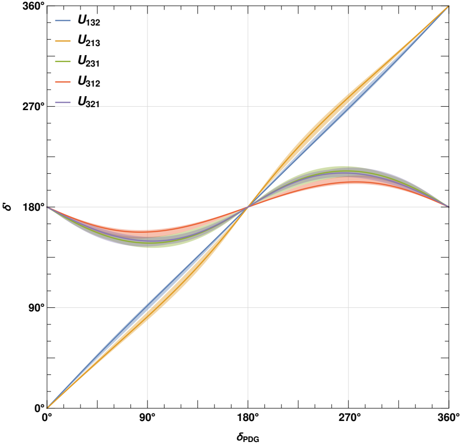
P. de Salas et al. 1708.01186

F. Capozzi et al. 2003.08511

I. Esteban et al. 2007.14792

Complex phase in different parameterizations

- Can relate the complex phase in one parameterization to that in another
- U_{132} and U_{213} similar to U_{123}
- δ constrained to $\sim [150°, 210°]$ in U_{231}, U_{312}, U_{321}
- Bands indicate 3σ uncertainty on θ_{12}, θ_{13}, θ_{23}
- "50% of possible values of δ"
  $\Rightarrow$ parameterization dependent



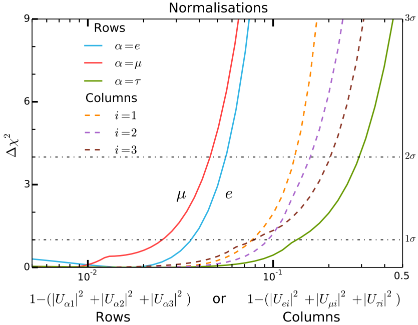
DUNE TDR II 2002.03005

Unitarity constraints on tau neutrinos

Past studies used:

1. $\nu_\mu \to \nu_\tau$ at OPERA
2. SNO NC and CC data

S. Ellis, K. Kelly, S. Li 2008.01088
Z. Hu, J. Ling, J. Tang, T. Wang 2008.09730



S. Parke M. Ross-Lonergan 1508.05095

Unitarity violation: tau row

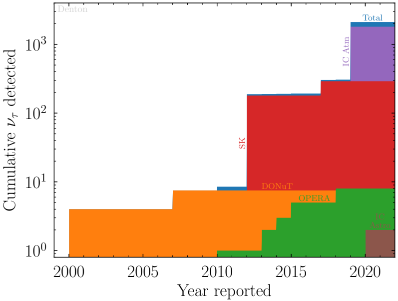
Leptons: tau row is the weakest

1. Existing global analyses use OPERA and SNO
2. More data from atmospheric ν_τ appearance!

PBD 2109.14576

Also astrophysical ν_τ appearance; weak but distinct!

PBD, J. Gehrlein 2109.14575



PBD, et al. 2203.05591 (whitepaper)

Atmospheric works because τ is in direct region
Strong kinematic dependence due to τ mass in energy range of interest

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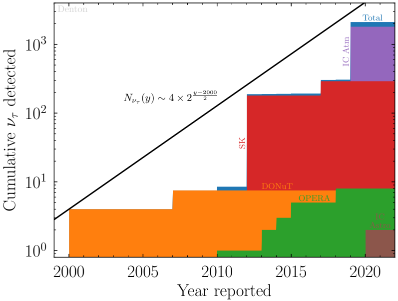

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PBD, J. Gehrlein 2109.14575

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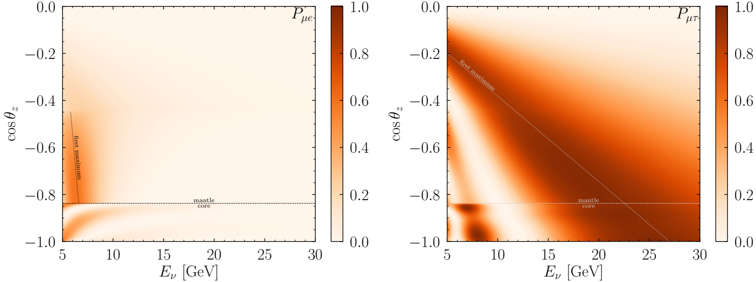
Strong kinematic dependence due to τ mass in energy range of interest

Tau neutrino data set doubles every two years!

PBD, et al. 2203.05591 (whitepaper)

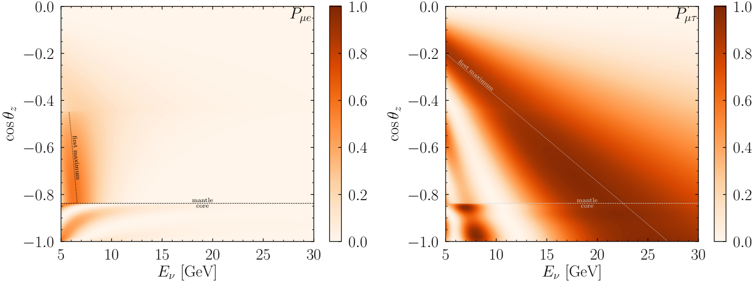
Atmospheric tau neutrino appearance

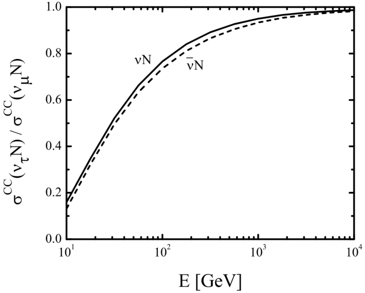
- Atmospheric neutrinos begin as ν_μ and mostly oscillate away to ν_τ



Atmospheric tau neutrino appearance

- Atmospheric neutrinos begin as ν_μ and mostly oscillate away to ν_τ
- High tau lepton production threshold diminishes events

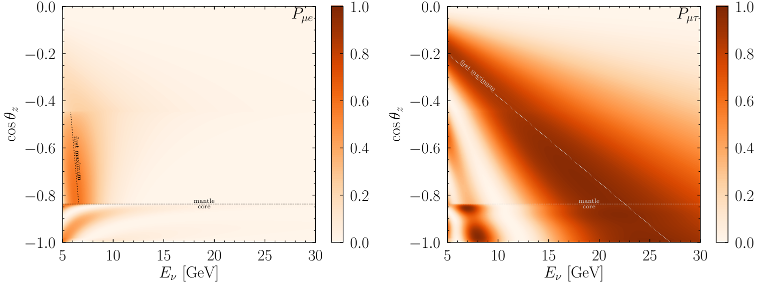


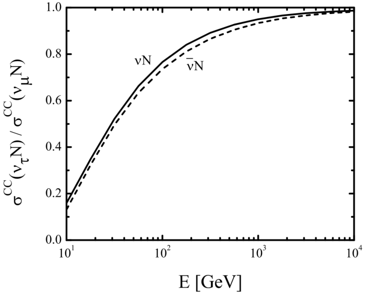


Y. Jeong, M. Reno 1007.1966

Atmospheric tau neutrino appearance

- Atmospheric neutrinos begin as ν_μ and mostly oscillate away to ν_τ
- High tau lepton production threshold diminishes events
- Identifying tau lepton in large coarse detectors is hard



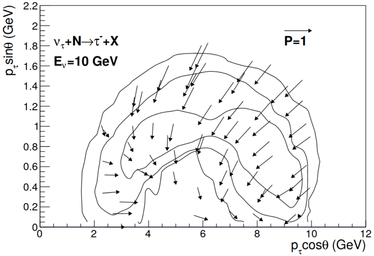


Y. Jeong, M. Reno 1007.1966

Tau neutrino appearance at SuperK

SuperK used:

1. Hadronic tau decay information
2. Tau polarization information
3. Neural net
4. *and standard oscillations*



2006.09384

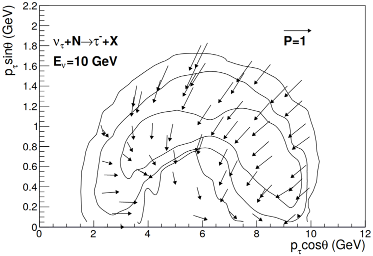
Tau neutrino appearance at SuperK

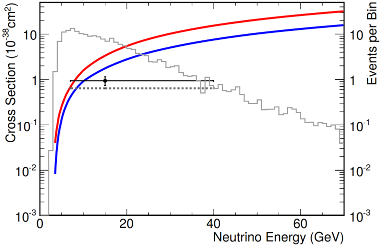
SuperK used:

1. Hadronic tau decay information
2. Tau polarization information
3. Neural net
4. *and standard oscillations*

Detected few hundred tau neutrino events,
constrained the ν_τ "normalization"
e.g. weighted cross section: (1.47 ± 0.32) x SM

Super-KamiokaNDE 1711.09436
see H. Tanaka and M. P. Zezula's talks





Tau neutrino appearance at IceCube

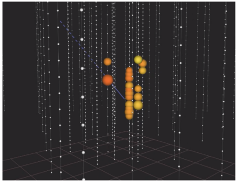
IceCube/DeepCore:

1. Much bigger than SuperK
2. 3D compared to SuperK's 2D
3. Much worse detector than SuperK
4. No ability to differentiate:
    - ν_τ CC that goes to a muon
    - ν_μ CC

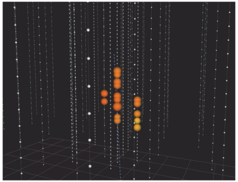
    or

    - ν_τ CC (that go to an electron or hadrons)
    - ν_e CC
    - ν NC

Track with energy of 26 GeV



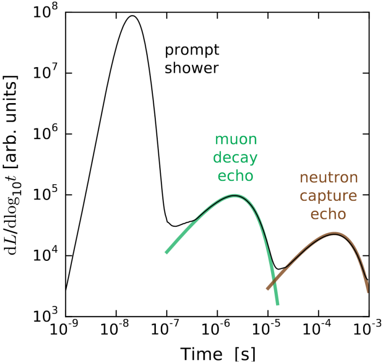
Cascade with energy of 30 GeV



M. Rodriguez IceCube slides

Possible means of identifying tau neutrinos event-by-event

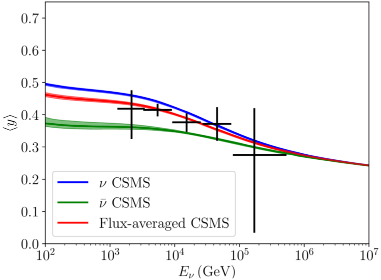
Hadronic showers contain far more muons and neutrons than electromagnetic showers



In practice, not possible

S. Li, M. Bustamante, J. Beacom 1606.06290

Inelasticity correlates with E_ν not E_{dep} and could be used

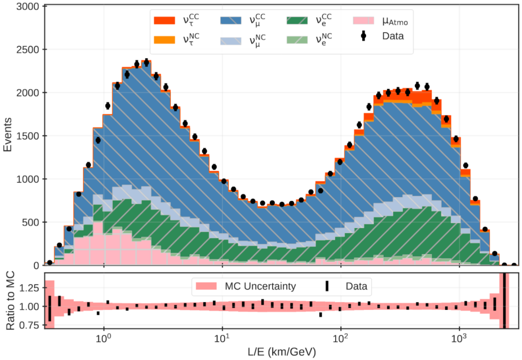


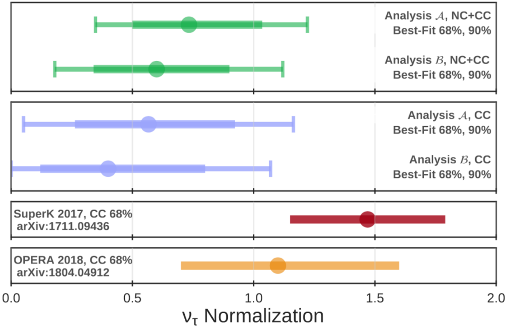
IceCube 1808.07629

Too hard to measure y at low atm. energies

IceCube results

Using oscillation parameters IceCube finds:





IceCube 1901.05366

Past work

Tau neutrino appearance in a large coarse detector is possible with:

1. Tau neutrino threshold
2. NC

T. Stanev astro-ph/9907018

Seeing extra low energy tau neutrinos could indicate astrophysical sources

H. Athar, F. Lee, G. Lin hep-ph/0407183

Both papers largely overlooked

My motivation

- Tau neutrino identification is relevant for unitarity

yet neither SuperK nor IceCube constrained unitarity with their data

- IceCube has the biggest data sets
- IceCube has extremely limited particle identification

cascades vs. tracks

- It would seem like $\nu_\mu \to \nu_e$ could mimic $\nu_\mu \to \nu_\tau$

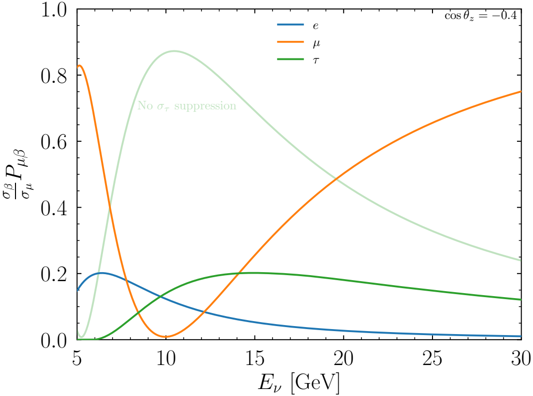
For different oscillation parameters or with unitarity violation

What, if any, physical effects allows for the identification of tau neutrinos without particle identification and without assuming unitarity?

Mimicry isn't always flattery

How to mimic $\nu_\mu \to \nu_\tau$ with $\nu_\mu \to \nu_e$ in the Earth:

Through the mantle:



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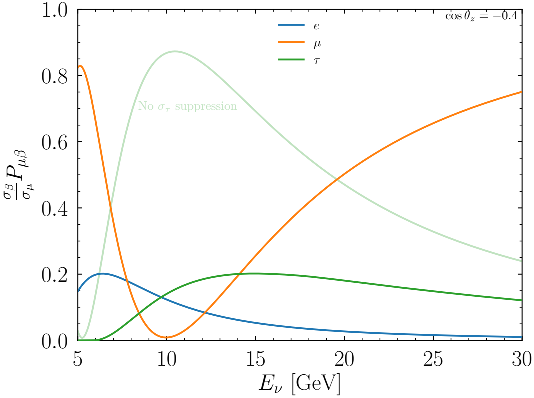
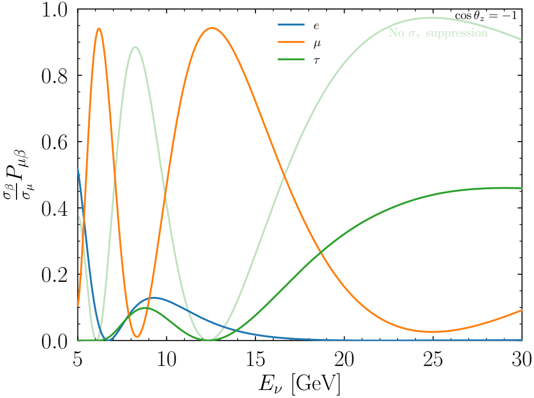


Through the core:



Back to IceCube observables

Define this cascade ratio:

$$\mathcal{R}_c(E_{\rm reco},\cos\theta_z)\equiv\frac{\frac{d^2N_c}{dE_{\rm reco}\,d\cos\theta_z}}{\Phi_i(E_{\rm reco})\sigma_{\rm tot}(E_{\rm reco})}$$

$$=f_{\rm CC}\left[P^r_{\mu e}(E_{\rm reco},\cos\theta_z)+\eta_{\nu_\tau}^{\gamma-1}R_{\tau\mu}(E_{\rm reco}/\eta_{\nu_\tau})(1-f_{\tau\mu})P^r_{\mu\tau}(E_{\rm reco}/\eta_{\nu_\tau},\cos\theta_z)\right]$$

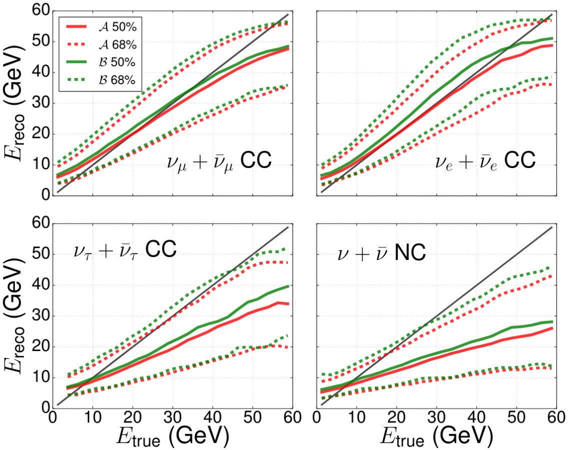
$$+(1-f_{\rm CC})\eta_{\rm NC}^{\gamma-1}\sum_{\beta\in\{e,\mu,\tau\}}P^r_{\mu\beta}(E_{\rm reco}/\eta_{\rm NC},\cos\theta_z)$$

- ν_e CC appearance
- ν_τ CC appearance with $\tau\to\nu_\tau+(e,X)$
- τ production threshold
- Reconstructed energy shift from spectrum and cross section

Different for $\tau\to\nu_\tau$ and NC

- NC

Reconstructed vs. true energy



τ's always decay to invisible energy ν_τ

$$\eta_{\nu_\tau} = 0.625$$

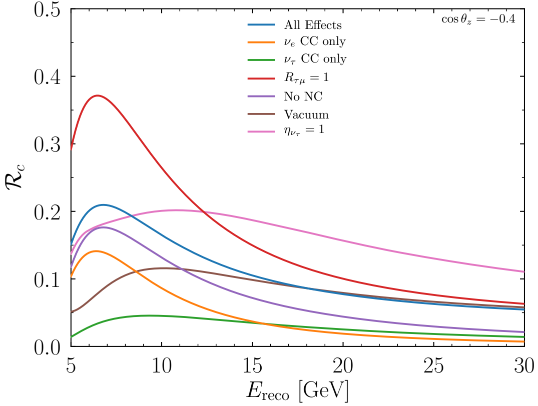
NC always loses some energy

$$\eta_{\rm NC} \simeq \frac{1}{3}$$

2006.09384

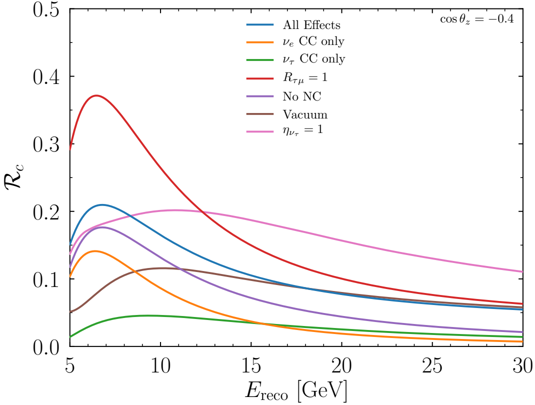
Impact of effects

Through the mantle:

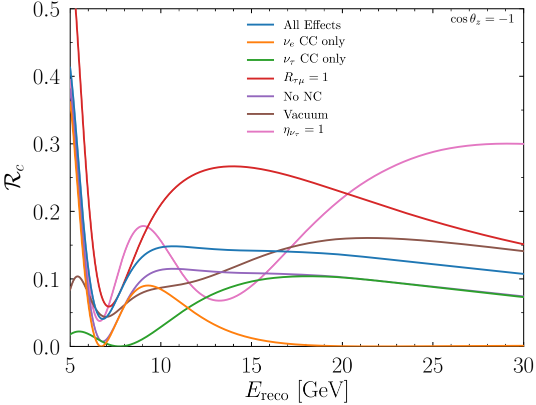


Impact of effects

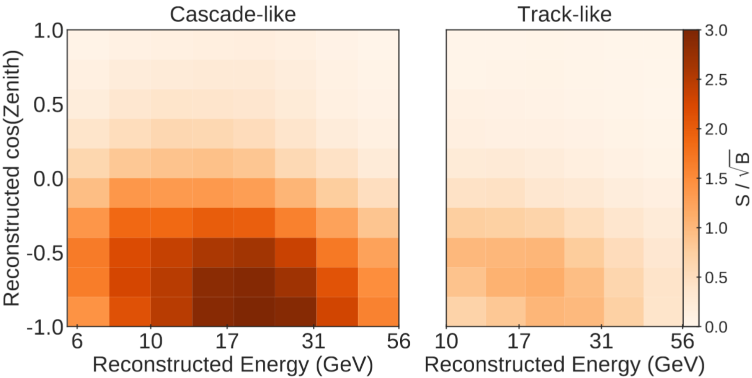
Through the mantle:



Through the core:



IceCube detector sensitivities



Contains all information on detector efficiencies, flux, and track/cascade misidentification

Tau identification in atmospherics

Effects considered:

1. NC
2. Matter effect
3. η_{ν_τ}: Tau neutrino reconstruction
4. $R_{\tau\mu}$: Tau lepton production threshold
5. External Δm^2_{31} constraint
6. External ν_e row constraint

Conclusions:

1. **With all known effects tau neutrinos can be identified even without assuming unitarity**
2. With all effects off and no unitarity: ν_τ's cannot be identified. Dial up ν_e to match
3. Including NC doesn't matter much
4. Turning on $R_{\tau\mu}$, η_{ν_τ}, or the matter significantly enhances sensitivity
5. Certain combinations approximately cancel: Just $R_{\tau\mu}$ and η_{ν_τ} has almost no sensitivity