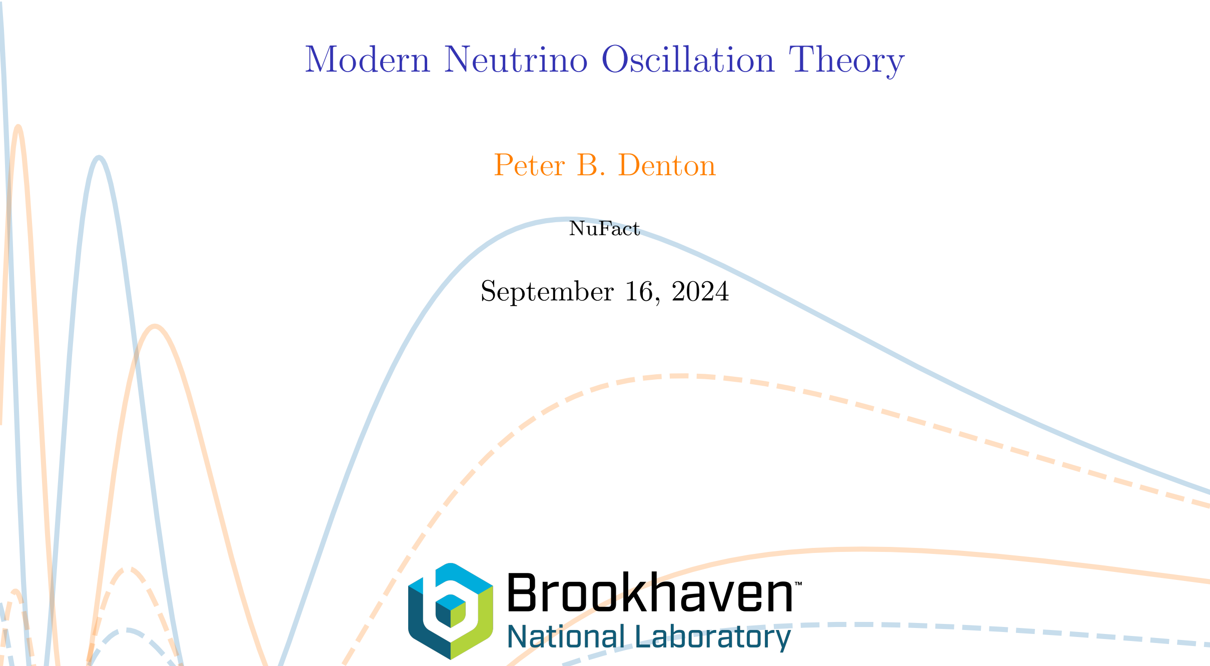
Modern Neutrino Oscillation Theory

Peter B. Denton

NuFact

September 16, 2024



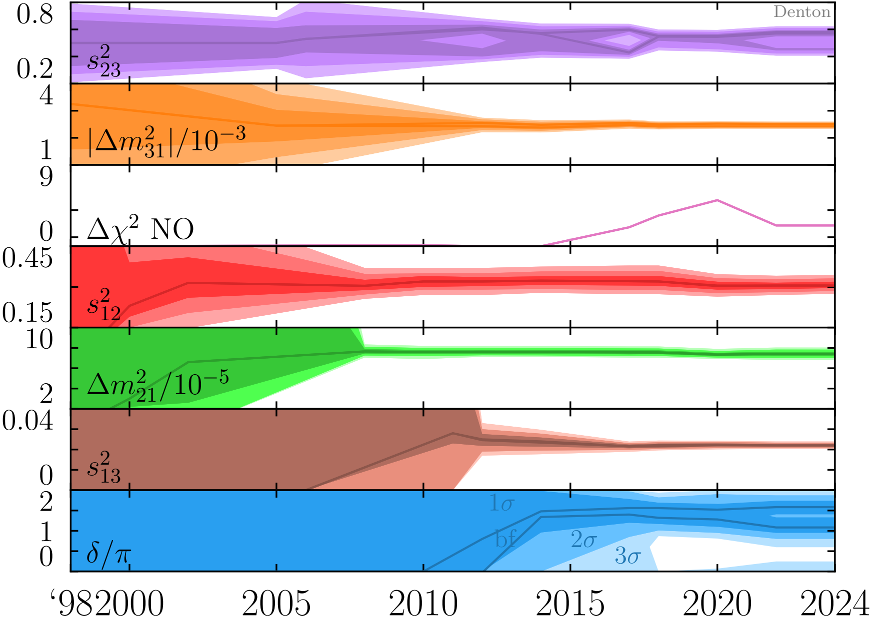
Brookhaven National Laboratory

Neutrino oscillations add ≥ 7 new parameters:

Measure them!

Theme of this talk:

Neutrinos don’t interact a lot,
But everything is connected:
You can’t do it alone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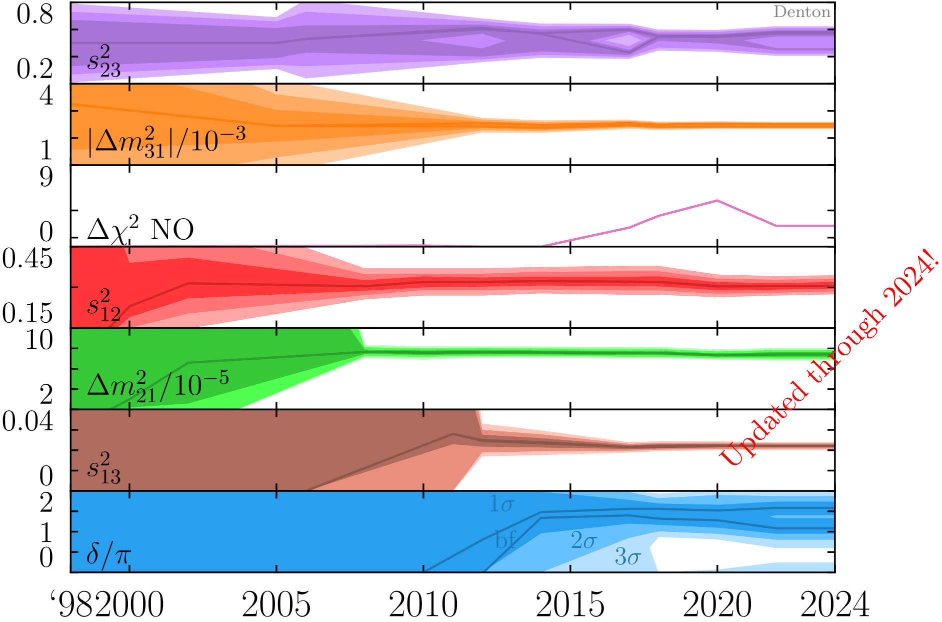
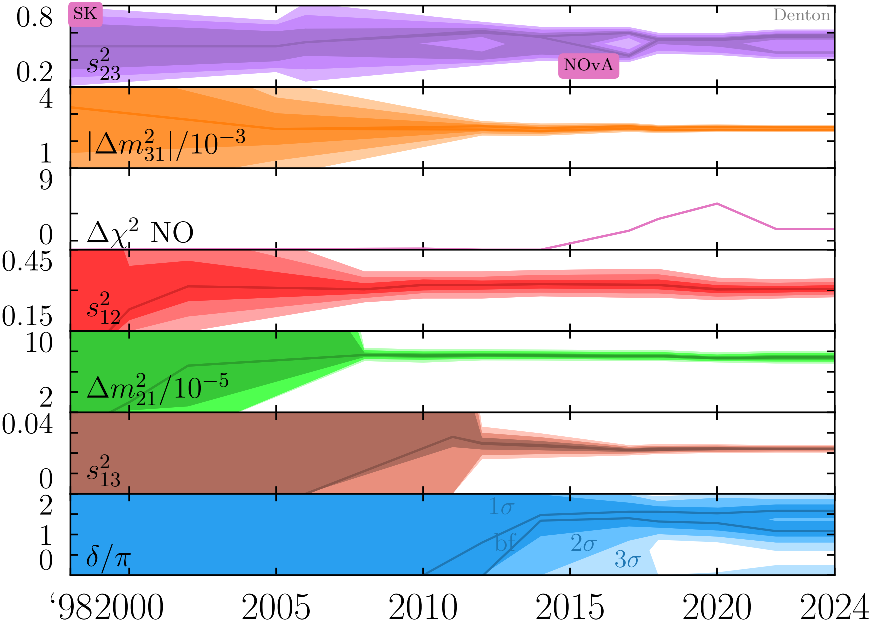


Updated through 2024!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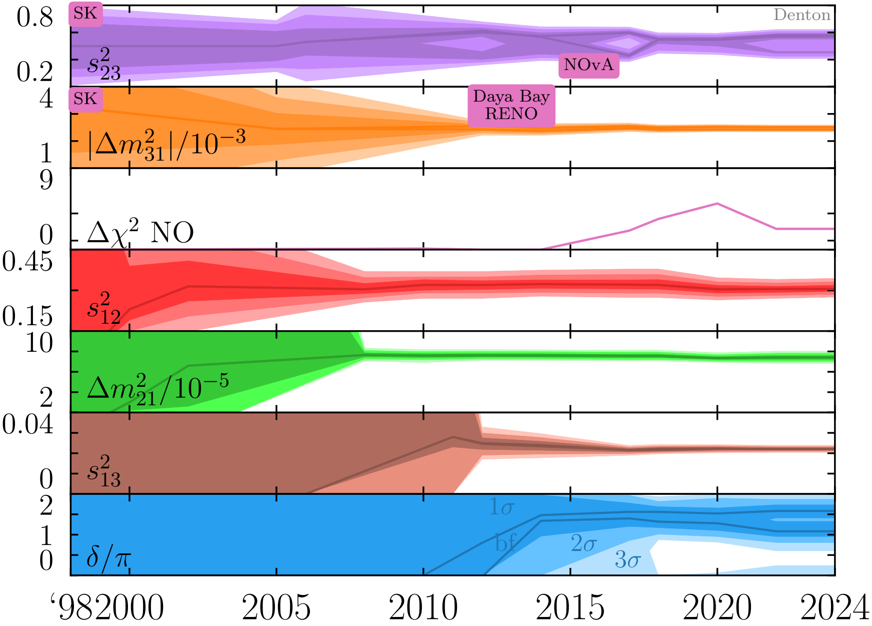
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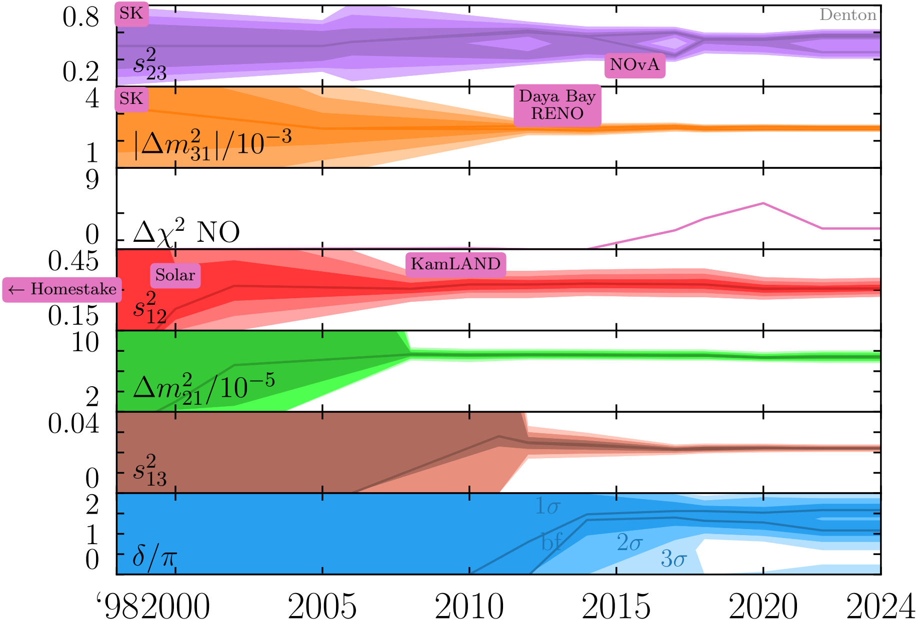
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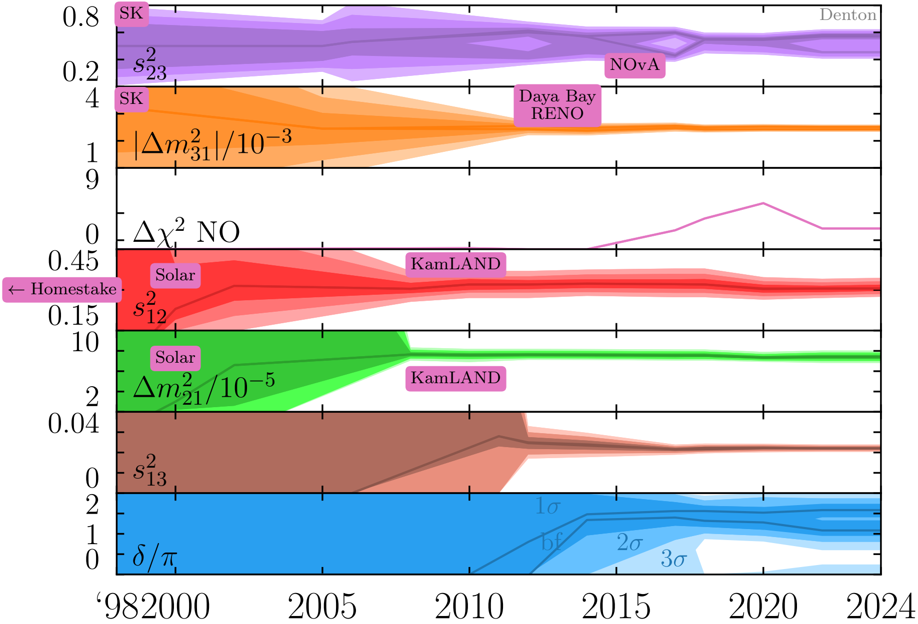
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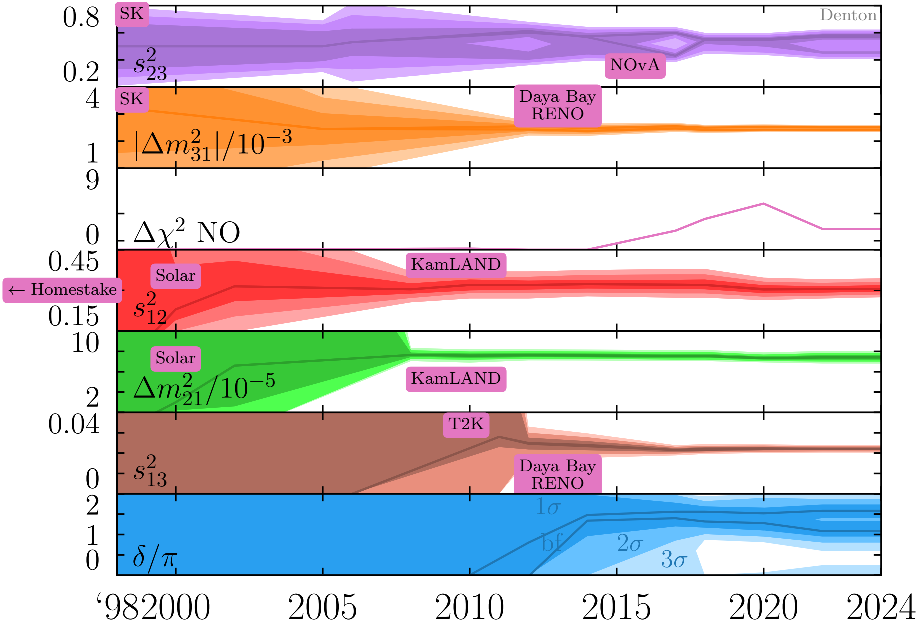
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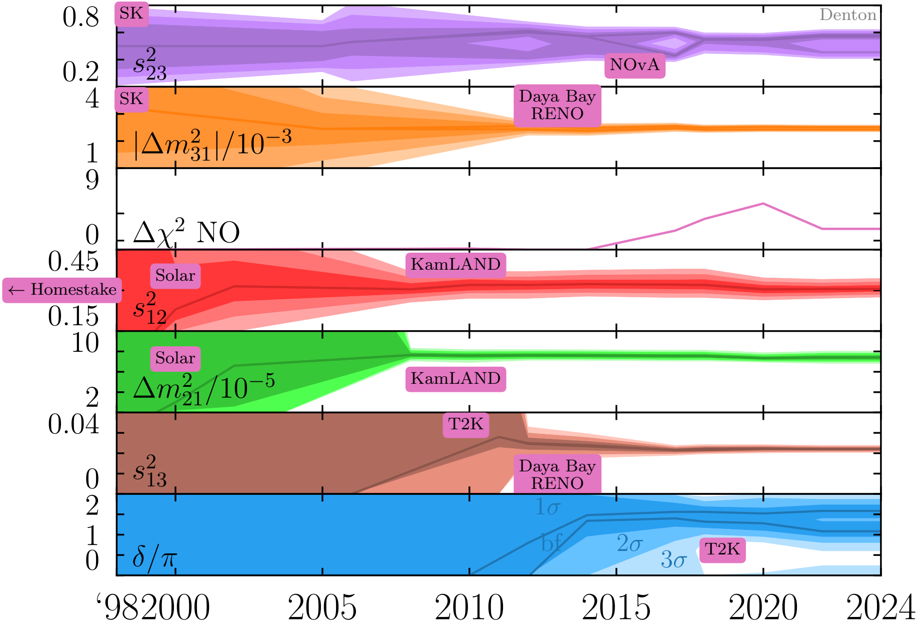
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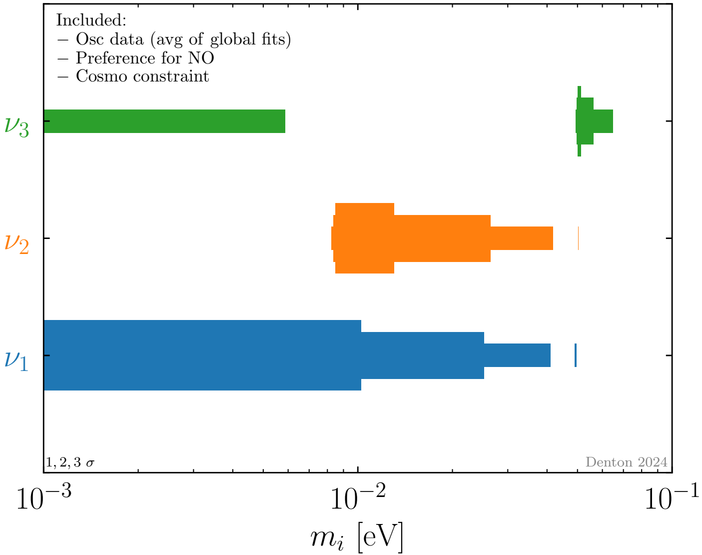
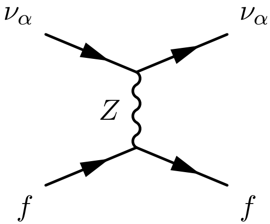
Absolute masses



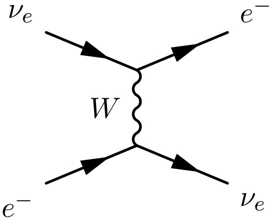
The Mass Ordering

There are 4+ ways of determining the mass ordering

All oscillation techniques require the matter effect



$$V_{\rm NC} = \mp\frac{1}{2}\sqrt{2}G_F n_n$$

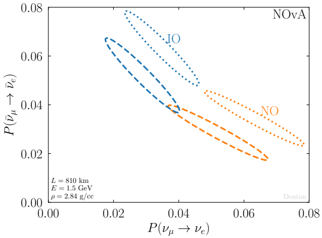


$$V_{\rm CC} = \pm\sqrt{2}G_F n_e$$

L. Wolfenstein, PRD 17 (1978)

The Mass Ordering: #1/4

Matter effect in appearance at NOvA/DUNE/Atmospherics



Only actually need either ν or $\bar\nu$;
Both help for systematics

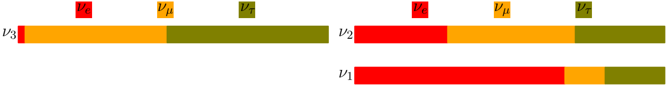
The appearance probability depends on terms like:

$$\simeq \left(\frac{\Delta m^2_{31}}{a-\Delta m^2_{31}}\right)^2 \sin^2\left(\frac{(a-\Delta m^2_{31})L}{4E}\right), \qquad \frac{a}{\Delta m^2_{31}} \simeq \begin{cases} 0.05 & \text{T2K/HK} \\ 0.15 & \text{NOvA} \\ 0.23 & \text{DUNE} \end{cases}$$

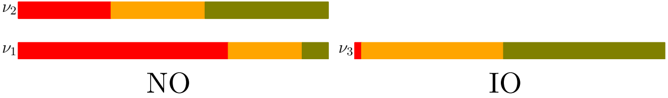
A. Cervera, et al. hep-ph/0002108
E. Akhmedov, et al. hep-ph/0402175
G. Barenboim, PBD, S. Parke, C. Ternes 1902.00517

The Mass Ordering: #2/4

Differentiate Δm^2_{31} and Δm^2_{32} at JUNO



Then if $\Delta m^2_{31} > \Delta m^2_{32} \Rightarrow$ Normal

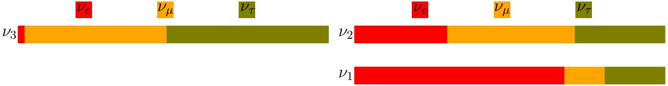


The Mass Ordering: #2/4

Differentiate Δm^2_{31} and Δm^2_{32} at JUNO

JUNO is essentially* in vacuum,
where is the matter effect?

*Y. Li, Y. Wang, Z. Xing 1605.00900
*A. Khan, H. Nunokawa, S. Parke 1910.12900



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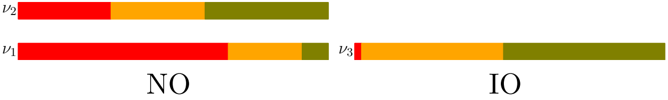

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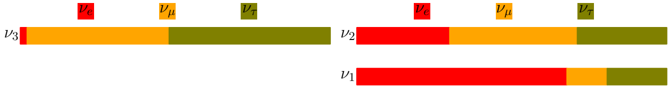

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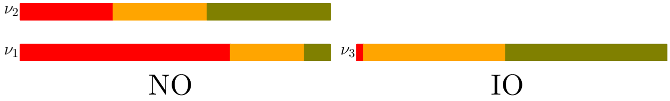
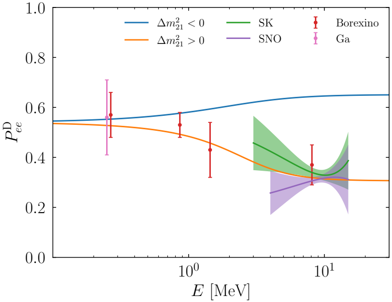


Solar neutrinos confirm that $\Delta m^2_{21} > 0$



The Mass Ordering: #3/4

Compare ν_e disappearance to ν_μ disappearance

The atmospheric Δm^2 measured with ν_e will be larger than that with ν_μ in the NO at the $\mathcal{O}$(few)% level

H. Nunokawa, S. Parke, R. Funchal `hep-ph/0503283`
S. Parke, R. Funchal `2404.08733`
See Stephen Parke's talk after lunch

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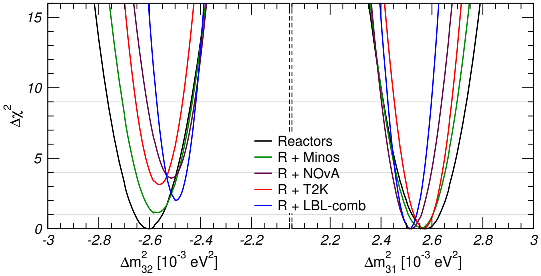
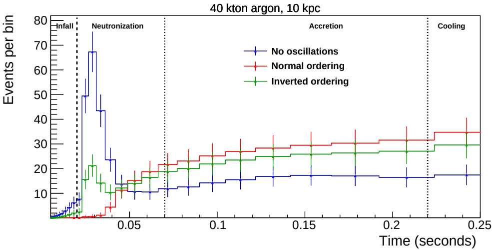


I. Esteban, et al. `2007.14792`

The Mass Ordering: #4/4

The neutronization burst in a supernova

$$F_{\nu_e}=\begin{cases}F^0_{\nu_x} & \text{NO}\\ s^2_{12}F^0_{\nu_e}+c^2_{12}F^0_{\nu_x} & \text{IO}\end{cases}$$



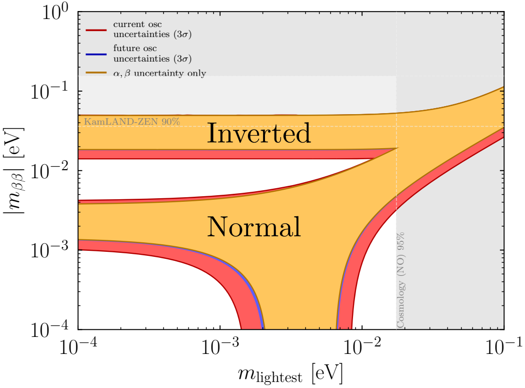
K. Scholberg 1707.06384
MSW L. Wolfenstein, PRD 17 (1978)
S. Mikheyev, A. Smirnov, Sov. J. Nucl. Phys. 42 (1985)

The DSNB has weak mass ordering sensitivity

A. Suliga et al. 1804.03157

The Mass Ordering: #5/4

- Affects cosmology: $\sum m_\nu$
- Affects $0\nu\beta\beta$
- Affects end point measurements
- Affects CνB



PBD, J. Gehrlein 2308.09737

The Mass Ordering: #5/4

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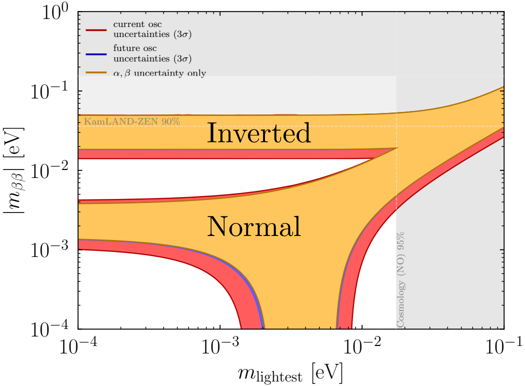
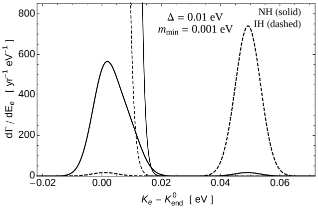
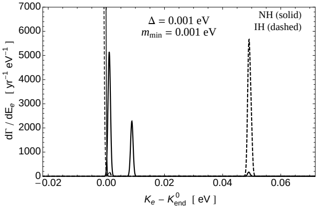


PBD, J. Gehrlein 2308.09737





CνB: A. Long, C. Lunardini, E. Sabancilar 1405.7654

Mass Ordering Status

- In general, prospects are good
- Care is required in interpreting significances (atmospherics)
- Many truly independent probes on the horizon
- High impact on non-oscillation neutrino experiments
- Can we test its robustness to new physics?

Mass ordering: new physics degeneracies

In the presence of new physics such as NSI we have:

$$[\mathrm{NO}] + [\epsilon = 0] \quad \equiv \quad [\mathrm{IO}] + [\epsilon_{ee} = -2]$$

$$[\mathrm{IO}] + [\epsilon = 0] \quad \equiv \quad [\mathrm{NO}] + [\epsilon_{ee} = -2]$$

Equivalences hold even if all oscillation probabilities are *perfectly* measured

P. Bakhti, Y. Farzan 1403.0744
P. Coloma, T. Schwetz 1604.05772
PBD, S. Parke 2106.12436
PBD, J. Gehrlein 2204.09060

This is known as the **LMA-Dark** solution

Is the mass ordering robust?

Need **scattering** to break

Can probe same NC $\epsilon = -2$ process in scattering, but. . .

1. CHARM and NuTeV for $M_{Z'} \gtrsim 10$ GeV

   PBD, et al. 1701.04828

2. COHERENT-CsI for $M_{Z'} \gtrsim 50$ MeV and cosmology for $M_{Z'} \lesssim 5$ MeV

   PBD, Y. Farzan, I. Shoemaker 1804.03660

3. Dresden-II for any mediator mass in ν_e sector

   PBD, J. Gehrlein 2204.09060

4. Can still evade with specific flavor structures

   $\epsilon_{\mu\mu} = \epsilon_{\tau\tau} = 2$ or certain u / d combinations

5. CCM or COHERENT can close all loopholes

   See Yuri Efremenko's COHERENT talk on Thursday

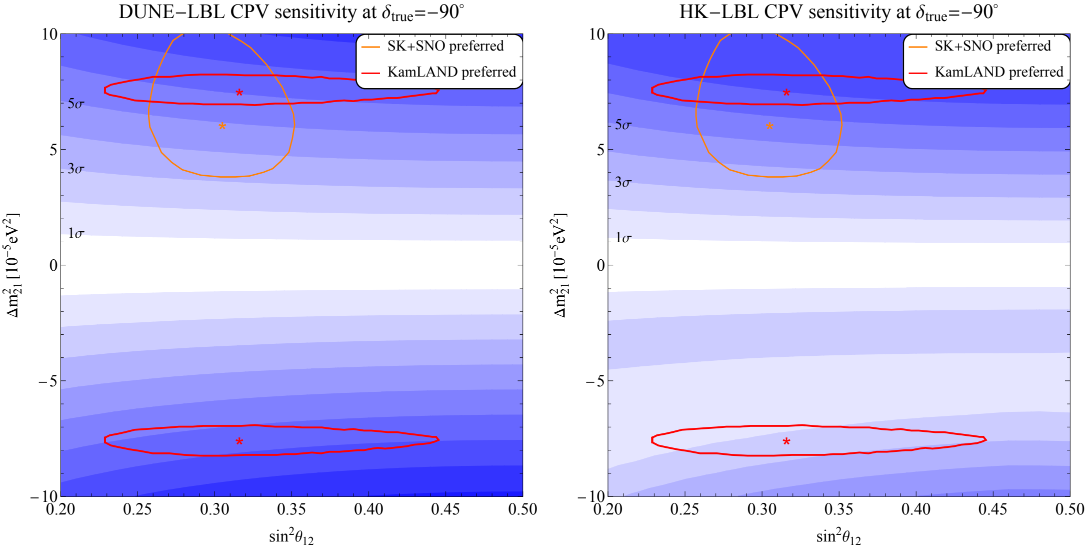
CP Violation

External inputs?
Novel probes of CPV?

External Inputs to CP Violation Measurements

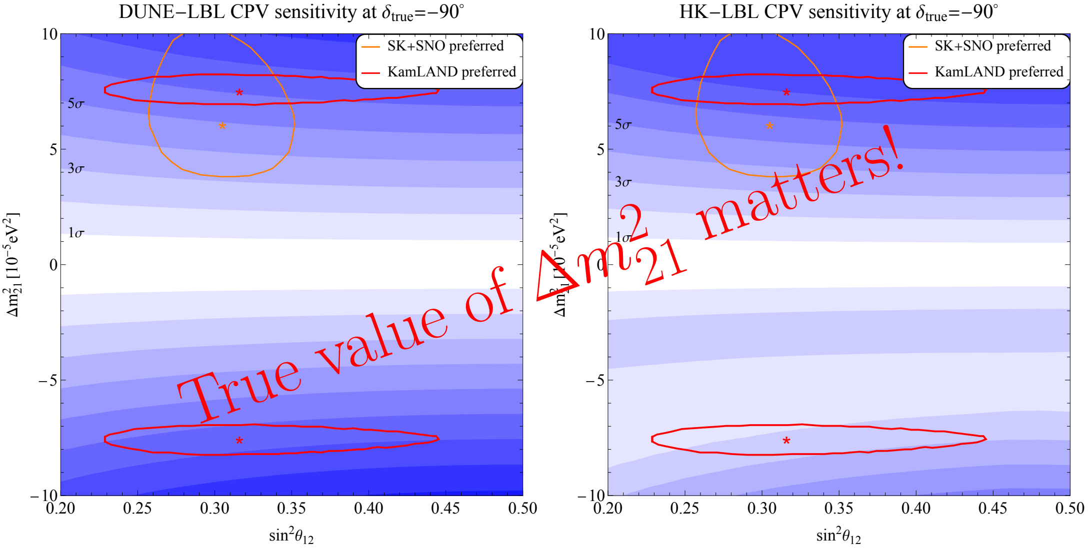
- DUNE and HK will have leading constraints on most parameters: $|\Delta m^2_{31}|$, MO, θ_{23}, δ, and even θ_{13}
- Solar parameters Δm^2_{21} and θ_{12} play a role in CPV searches
- CPV is a fundamentally three-flavor effect
- True values of solar parameter affect CPV sensitivities
- Precision will receive $\sim$ 10x improvement with JUNO

True values matter



PBD, J. Gehrlein 2302.08513

True values matter



True value of Δm^2_{21} matters!

PBD, J. Gehrlein 2302.08513

Novel CP Violation Probes

- Two independent measurements of this is *essential*
- Accelerator $\nu_\mu \to \nu_e$ (and anti-neutrino) appearance is best
- CPV can be discovered with only ν's (or only $\bar{\nu}$'s), but systematics
- Given the challenging systematics and the possibilities of new physics, are they enough?
- Are there other ways to probe CPV?

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Two other ways with appearance
and one with disappearance

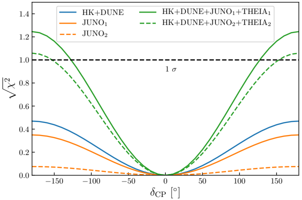
Other non-standard CPV probes with appearance

1. Some information in solar due to loops in elastic scattering

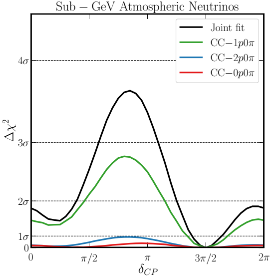
V. Brdar, X-J. Xu 2306.03160
K. Kelly, et al. 2407.03174 requires 3k Borexinos

2. Sub-GeV atmospherics

K. Kelly, et al. 1904.02751
Also JUNO, see also e.g. A. Suliga, J. Beacom 2306.11090



Solar (no systematics)



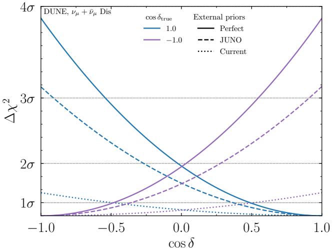
Atmospherics at DUNE

Other Non-standard CPV probes with disappearance

- Disappearance measurements are fundamentally CP conserving
- One good disappearance experiment can measure two $|U_{\alpha i}|^2$
- If four independent $|U_{\alpha i}|^2$ are measured, can extract CPV

$$J_{CP}^2 = |U_{e2}|^2|U_{\mu 2}|^2|U_{e3}|^2|U_{\mu 3}|^2 - \frac{1}{4}\left(1 - |U_{e2}|^2 - |U_{\mu 2}|^2 - |U_{e3}|^2 - |U_{\mu 3}|^2 + |U_{e2}|^2|U_{\mu 3}|^2 + |U_{e3}|^2|U_{\mu 2}|^2\right)^2$$

- Need JUNO and DUNE/HK



- Disappearance has different (better?) systematics than appearance
- Good cross check!

PBD 2309.03262

How to compute neutrino oscillation probabilities quickly?

`NuFast`

github.com/PeterDenton/NuFast

PBD, S. Parke `2405.02400`

Monte-Carlo Estimates of Statistical Significances

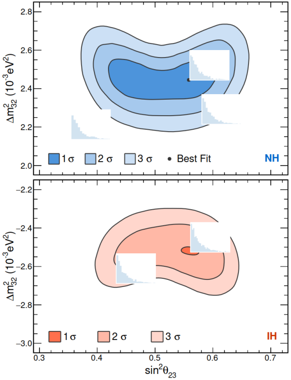
Wilks' theorem is often wrong

At each point in parameter space, simulate the experiment many times

"many" means $\gg 1/p$ for a desired p-value

This is sometimes called Feldman-Cousins

G. Feldman, R. Cousins physics/9711021

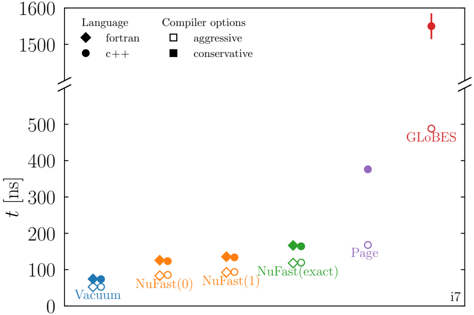


Study found most of
the time was spent
computing probabilities

NOvA/T2K are $\sim 3\sigma$ experiments,
but DUNE/HK will be $\gtrsim 5\sigma$ experiments!

The `NuFast` Approach

1. Inputs: 6 oscillation parameters, experimental details (L, E, ρ, Y_e)
2. Calculate λ_3 approximately
   - Iteratively improve with Newton's method, as needed
3. Calculate other two eigenvalues via sum rules
4. Calculate the $|V_{\alpha i}|^2$'s with the Eigenvector-Eigenvalue Identity
5. Calculate the sines of the kinematic terms
6. Get the T violating term with the NHS identity
7. Calculate key probabilities: P_{ee}, $P_{\mu\mu}$, & $P_{\mu e}$
8. Calculate remaining probabilities



PBD, S. Parke 2405.02400

New physics searches in oscillations

Unitarity Violation: How to Calculate

Kinematically **accessible** states

1. Unitary calculation of full $n \times n$ matrix
2. Oscillation averaged:

$$\sin^2\frac{\Delta m^2_{41}L}{4E} \to \frac{1}{2}$$

$$\sin\frac{\Delta m^2_{41}L}{4E} \to 0$$

3. No matter effect:

$$H^{\rm mat} = {\rm diag}(V_{\rm CC} + V_{\rm NC}, V_{\rm NC}, V_{\rm NC}, 0, \dots)$$

Unitarity Violation: How to Calculate

Kinematically **accessible** states

1. Unitary calculation of full $n \times n$ matrix
2. Oscillation averaged:

$$\sin^2 \frac{\Delta m_{41}^2 L}{4E} \to \frac{1}{2}$$

$$\sin \frac{\Delta m_{41}^2 L}{4E} \to 0$$

3. No matter effect:

$$H^{\text{mat}} = \text{diag}(V_{\text{CC}} + V_{\text{NC}}, V_{\text{NC}}, V_{\text{NC}}, 0, \dots)$$

Kinematically **inaccessible** states

1. Nonunitary calculation of $m \times m$ matrix

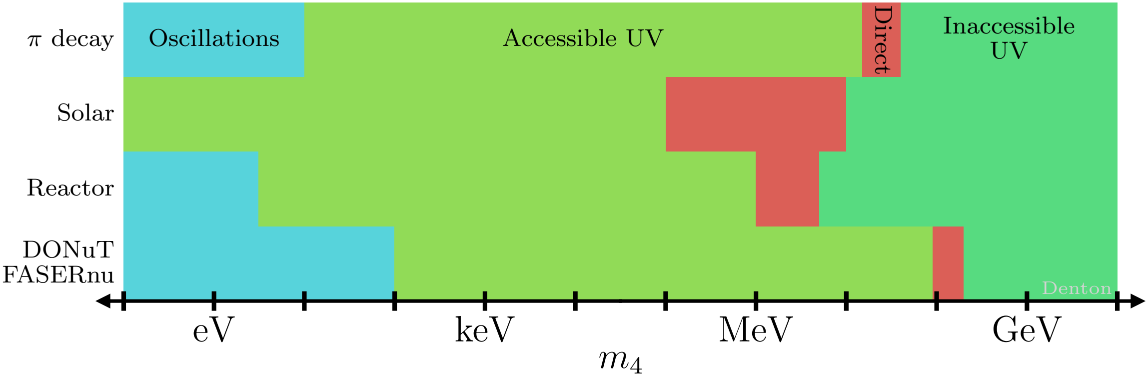
m = number of kinematically accessible states

2. Rescale probability:

$$P_{\alpha\beta} = \frac{|\sum_{i=1}^{\text{acc}} U_{\alpha i}^* e^{iP_iL} U_{\beta i}|}{(\sum_{i=1}^{\text{acc}} U_{\alpha i}^* U_{\alpha i})(\sum_{i=1}^{\text{acc}} U_{\beta i}^* U_{\beta i})}$$

3. Cannot subtract multiples of $\mathbb{1}$
4. Rescale cross section/flux as appropriate
5. Rescale G_F in matter effect

Unitarity Violation: a Tale of Two Regimes



*Details depends on the specific experiment/channel

Unitarity Violation: Tau Row

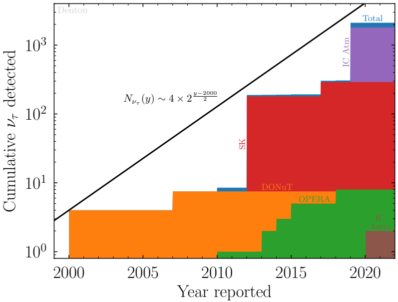
Leptons: tau row is the weakest

1. Existing global analyses use OPERA and SNO
2. More data from atmospheric ν_τ appearance!
   Tau neutrino data set doubles every two years!

PBD 2109.14576

Also astrophysical ν_τ appearance; weak but distinct!

PBD, J. Gehrlein 2109.14575



Atmospheric works because τ is in direct region

PBD, et al. 2203.05591

Unitarity Violation: Tau Row

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PBD 2109.14576

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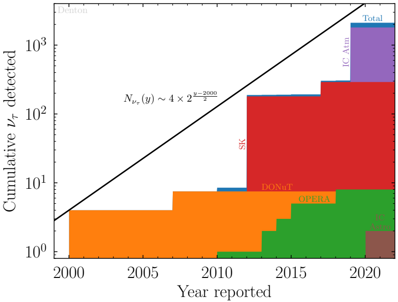
PBD, J. Gehrlein 2109.14575



PBD, et al. 2203.05591

Atmospheric works because τ is in direct region

Atmospheric ν_μ disappearance is most constraining for UV in ν_τ row

Estimate Size of NSI Effect

Suppose two LBL experiments measure different values of δ due to NSI:

$$|\epsilon_{e\beta}| \approx \frac{s_{12}c_{12}c_{23}\pi\Delta m^2_{21}}{2s_{23}w_\beta}\left|\frac{\sin\delta_{\rm T2K}-\sin\delta_{\rm NOvA}}{a_{\rm NOvA}-a_{\rm T2K}}\right| \approx \begin{cases} 0.22 & \text{for } \beta=\mu \\ 0.24 & \text{for } \beta=\tau \end{cases}$$

$a \propto \rho E$

$w_\beta = s_{23},\ c_{23}$ for $\beta = \mu, \tau$

Assumed upper octant $\theta_{23} > 45^\circ$ for numbers

Consistency checks:

- $\sin\delta_{\rm NOvA} = \sin\delta_{\rm T2K} \Rightarrow |\epsilon| = 0$
- $\sin\delta_{\rm NOvA} \neq \sin\delta_{\rm T2K}$ and $a_{\rm NOvA} = a_{\rm T2K} \Rightarrow |\epsilon| \to \infty$
- Octant:
  1. LBL is governed by ν_3
  2. Upper octant $\Rightarrow$ ν_3 is more ν_μ
  3. More ν_μ $\Rightarrow$ need less new physics coupling to ν_μ to produce a given effect

PBD, J. Gehrlein, R. Pestes 2008.01110

Beyond Expected Sensitivities with a Neutrino Factory

Latest P5 mentions several interesting possibilities:

"*Such a [10 TeV muon collider] demonstrator might produce intense muon and neutrino beams*"

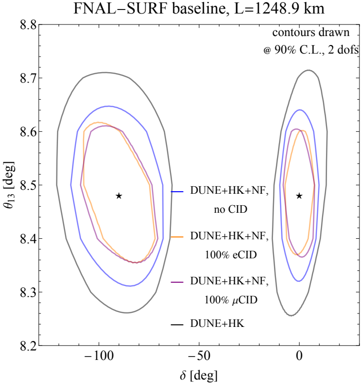
"*The upgraded facility would also generate bright, well-characterized neutrino beams bringing natural synergies with studies of neutrinos beyond DUNE.*"

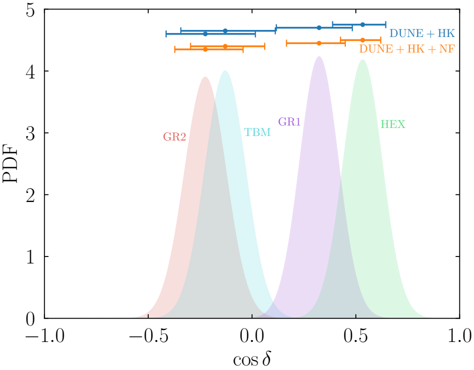
"*20-Year Vision: . . . could entail the deployment of a low-energy muon storage ring, as exemplified by the Neutrinos from Stored Muons (nuSTORM) experiment*"

⋮

See P5 Panel Discussion on Friday

Neutrino Factory and Oscillations





Enhanced sensitivity to δ and flavor model discrimination

Rich near detector physics

Can probe new physics and resolve anomalies

See Julia Gehrlein's talk Tuesday after lunch

PBD, J. Gehrlein 2407.02572

A. Bogacz, et al. 2203.08094

R. Kitano, J. Sato, S. Sugama 2407.05807

Flavor Models

Upcoming oscillation measurements have the capability to confirm/reject predictions of flavor models

How will the theory develop?

More models?

Different way of looking at predictions

Flavor Models

Upcoming oscillation measurements have the capability to confirm/reject predictions of flavor models

How will the theory develop?

More models?

Different way of looking at predictions

Many model classes:

- Generalized CP
- Charged lepton corrections
- Modular symmetries
- Sum rules

$$c_1 e^{i\chi_1}(m_1 e^{i\alpha})^d + c_2 e^{i\chi_2}(m_2 e^{i\beta})^d + m_3^d = 0$$

- Texture zeros

  Found that 13/15 two texture zeros are ruled out

Asked: do models predict the funnel?

Sometimes!

Flavor Models

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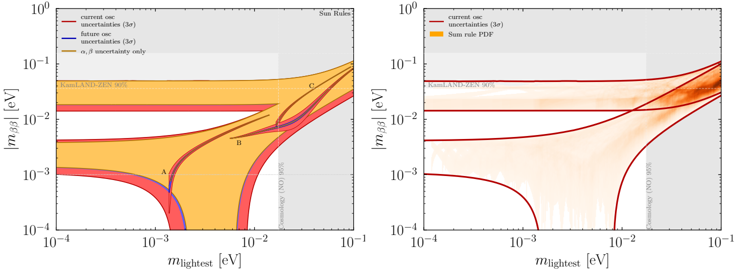


PBD, J. Gehrlein 2308.09737

Asked: do models predict the funnel?

$c_1 = 1,\ c_2 = 2,\ d = -\frac{1}{2},\ |\chi_1| = \frac{\pi}{2},\ \chi_2 = 0 \Rightarrow P_{\rm funnel} = 0.74$

Sometimes!

Too Much Physics

Neutrino theory not covered:

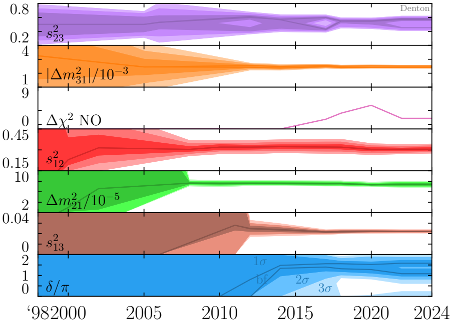
- $0\nu\beta\beta$ ab inition matrix element calculations
- Neutrino nucleus cross sections $\sim$GeV
- Neutrinos at the LHC
- High energy astrophysical neutrino searches
- Many new physics scenarios including self interactions, neutrino decay, decoherence, LIV, CPT violation, . . .
- Neutrino model building
- $\vdots$

Neutrino Theory Summary

- Many ways to combine data for the mass ordering
- Some new ideas for CPV discovery
- JUNO has several key roles in CPV
- Cutting edge techniques for fast computation of probabilities
- A neutrino factory would push oscillation physics beyond the DUNE/HK/JUNO generation
- Many new physics scenarios to investigate;
  useful approximations are important!
- Can expect new information about flavor models

Backups

References



SK hep-ex/9807003

M. Gonzalez-Garcia, et al. hep-ph/0009350

M. Maltoni, et al. hep-ph/0207227

SK hep-ex/0501064

SK hep-ex/0604011

T. Schwetz, M. Tortola, J. Valle 0808.2016

M. Gonzalez-Garcia, M. Maltoni, J. Salvado 1001.4524

T2K 1106.2822

D. Forero, M. Tortola, J. Valle 1205.4018

D. Forero, M. Tortola, J. Valle 1405.7540

P. de Salas, et al. 1708.01186

F. Capozzi et al. 2003.08511

I. Esteban et al. 2007.14792

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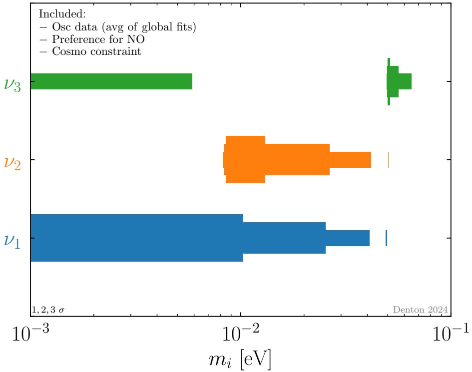


DESI 2404.0300

I. Esteban et al. 2007.14792

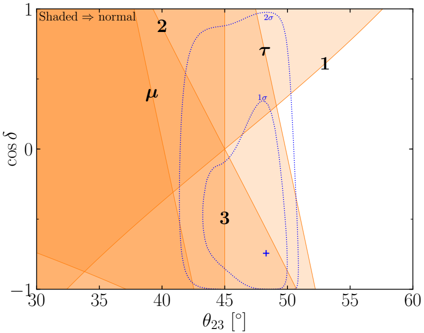
P. de Salas, et al. 2006.11237

F. Capozzi, et al. 2107.00532

θ_{23}: Broader Implications

Normalcy

Is the heaviest neutrino mostly ν_τ?
Is the lightest neutrino least ν_τ?



Quarks easily satisfy normalcy PBD 2003.04319

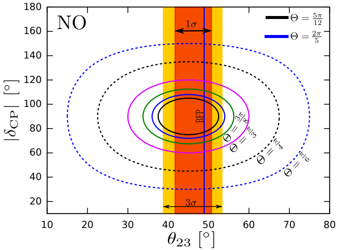
μ-τ interchange/reflection symmetry

$$\nu_\mu \leftrightarrow \nu_\tau$$

$$M_\nu^* = X M_\nu X^T \qquad X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$M_\nu \equiv U D_\nu U^\dagger$$

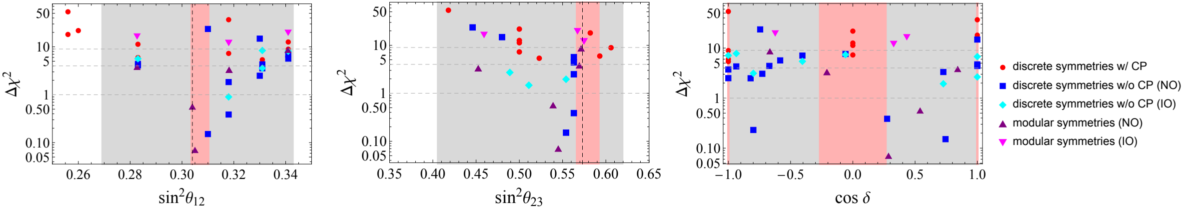
Predicts: $\theta_{23} = 45^\circ$, often $\theta_{13} = 0$



P. Chen, et al. 1512.01551

Parameter Interplay

Models predict specific correlations among the parameters



J. Gehrlein, et al. 2203.06219

CP Violation in Oscillations

In vacuum at first maximum:

$$P_{\mu e}-\bar{P}_{\mu e}\approx 8\pi J\frac{\Delta m^2_{21}}{\Delta m^2_{32}}$$

$$J\equiv s_{12}c_{12}s_{13}c^2_{13}s_{23}c_{23}\sin\delta$$

C. Jarlskog PRL 55, 1039 (1985)

- Extracting δ from data requires every other oscillation parameter
- J requires only Δm^2_{21} (up to matter effects)

Matter effects are easily accounted for

$$\hat{J}\simeq\frac{J}{\sqrt{(c_{212}-c^2_{13}a/\Delta m^2_{21})^2+s^2_{212}}\sqrt{(c_{213}-a/\Delta m^2_{ee})^2+s^2_{213}}}$$

PBD, S. Parke `1902.07185`

PBD, H. Minakata, S. Parke `1604.08167`

Many Interesting New Physics Scenarios in Oscillations

1. Sterile neutrinos

   PBD, Y. Farzan, I. Shoemaker 1811.01310
   PBD 2111.05793

2. Non-standard neutrino interactions (NSI)

   with any Lorentz structure: SPVAT

   PBD, J. Gehrlein, R. Pestes 2008.01110
   P. Coloma, PBD, M. Gonzalez-Garcia, M. Maltoni, T. Schwetz 1701.04828
   PBD, J. Gehrlein 2008.06062, 2204.09060
   PBD, A. Giarnetti, D. Meloni 2210.00109

3. Non-standard neutrino SELF interactions G. Barenboim, PBD, I. Oldengott 1903.02036

4. Neutrino decay

   with visible or invisible final states

   PBD, I. Tamborra 1805.05950
   PBD, A. Abdullahi 2005.07200

5. Unitarity violation

   PBD 2109.14576
   PBD, J. Gehrlein 2109.14575

6. Neutrino – dark matter interactions

   A. Dev, et al. 2205.06821
   C. Boehm, P. Fayet, R. Schaeffer astro-ph/0012504

7. Decoherence

   T. Stuttard, M. Jensen 2007.00068
   A. Gouvêa, V. Romeri, C. Ternes 2104.05806

8. Lorentz invariance or CPT violation

   S. Ge, H. Murayama 1904.02518

Differentiating New Physics

If there is new physics in oscillations, can DUNE tell what it is?

Used the best fit points to NOvA+T2K 2020 data as benchmarks

$\Delta\chi^2$	SM	$\eta_{e\mu}$	$\eta_{e\tau}$	$\eta_{\mu\tau}$	$\varepsilon_{e\mu}$	$\varepsilon_{e\tau}$	$\varepsilon_{\mu\tau}$	3+1
$\varepsilon_{e\mu}$ NO	200	140	140	170	/	180	160	80
$\varepsilon_{e\tau}$ NO	60	48	50	45	50	/	50	40
$\varepsilon_{\mu\tau}$ NO	200	180	170	180	160	180	/	80
$\varepsilon_{e\mu}$ IO	170	80	75	90	/	10	13	3
$\varepsilon_{e\tau}$ IO	70	50	50	45	45	/	60	20
$\varepsilon_{\mu\tau}$ IO	500	400	400	400	300	350	/	160

In general, DUNE will have excellent model discrimination capability!

PBD, A. Giarnetti, D. Meloni 2210.00109

NSI Review

$$\mathcal{L}_{\rm NSI} = -2\sqrt{2}G_F \sum_{\alpha,\beta,f,P} \epsilon_{\alpha\beta}^{f,P} (\bar\nu_\alpha\gamma^\mu\nu_\beta)(\bar f\gamma_\mu f)$$

Models with large NSIs consistent with CLFV:

Y. Farzan, I. Shoemaker 1512.09147 Y. Farzan, J. Heeck 1607.07616 D. Forero and W. Huang 1608.04719
K. Babu, A. Friedland, P. Machado, I. Mocioiu 1705.01822 PBD, Y. Farzan, I. Shoemaker 1804.03660
U. Dey, N. Nath, S. Sadhukhan 1804.05808 Y. Farzan 1912.09408

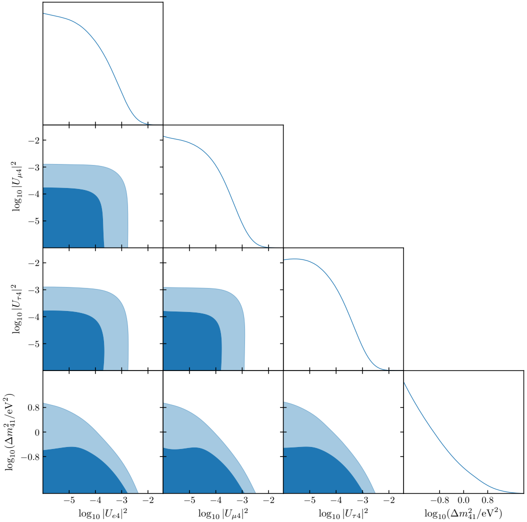
Affects oscillations via new matter effect

$$H = \frac{1}{2E}\left[UM^2U^\dagger + a\begin{pmatrix}1+\epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau}\\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau}\\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau}\end{pmatrix}\right]$$

Matter potential $a \propto G_F\rho E$

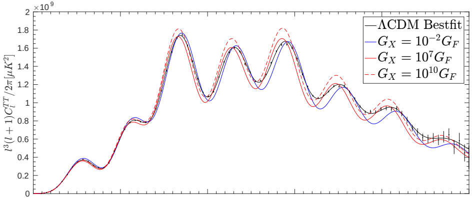
B. Dev, K. Babu, PBD, P. Machado, et al. 1907.00991

Cosmological Bounds



1σ, 2σ S. Hagstotz, et al. 2003.02289

- Includes CMB temperature, polarization, and lensing, and BAO
- No local H_0 constraint
- Bounds independent of flavor
- To be consistent with data must have small mixing **and** small mass
- Much more than just N_{eff} and $\sum m_\nu$
- Just adding a new interaction is not straightforward



N. Song, M. Gonzalez-Garcia, J. Salvado 1805.08218

Shape-Shifting Sterile Neutrinos

How to evade constraints?

Suppose:

1. Sterile neutrinos talk to dark matter

DM is ultralight boson

2. Dark matter talks to baryons

Then:

1. Sterile neutrinos aren’t abundantly produced in the early universe
2. Mixing angle in the Sun is suppressed
3. Reactor constraints still exist

H. Davoudiasl, PBD 2301.09651

PBD 2301.11106

Mass Ordering Current Status: All

Cosmology: $m_1 + m_2 + m_3 < 90$ meV at 95% CL

E. Valentino, S. Gariazzo, O. Mena 2106.15267

→ 20 meV precision with DESI, EUCLID, …

From oscillations:

$$\text{Normal}: \quad m_1 + m_2 + m_3 > 60 \text{ meV} \qquad \text{Inverted}: \quad m_1 + m_2 + m_3 > 100 \text{ meV}$$

See also KATRIN 2105.08533

PRIORS?

Some claim "decisive" Bayesian evidence for normal

R. Jimenez, et al. 2203.14247

More general prior assumptions ⇒ no significant information from cosmology

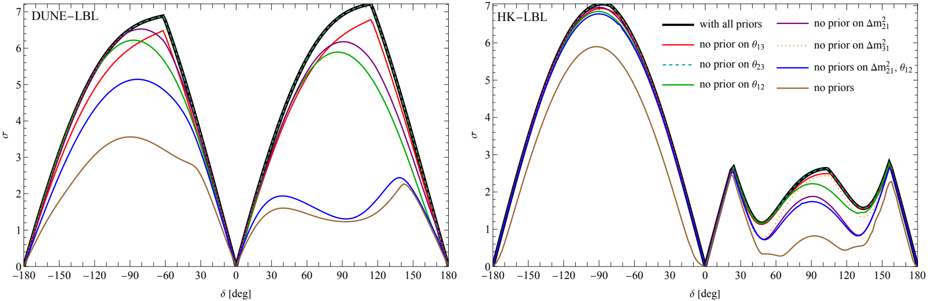
S. Gariazzo, et al. 1801.04946

S. Gariazzo, et al. 2205.02195

δ: Future Sensitivities

DUNE and HK will make great measurements via appearance $\nu_\mu \to \nu_e$

$\nu+\bar{\nu}$ helps systematics but isn't strictly necessary



PBD, J. Gehrlein 2302.08513

Need to know solar parameters to measure δ!

Current solar knowledge: okay
Future (JUNO): excellent

Solar Parameter Status

Data	Δm^2_{21} $[10^{-5}$ eV$^2]$	$\sin^2\theta_{12}$	Ref.
SK+SNO	+6.10	0.305	SK Neutrino 2022
KamLAND	±7.54	0.316	1303.4667 SK Neutrino 2022
SK+SNO+KamLAND	**7.49**	**0.305**	SK Neutrino 2022
Global fit	7.42	0.304	Esteban+ 2007.14792
	7.5	0.318	de Salas+ 2006.11237
	7.36	0.303	Capozzi+ 2107.00532

		$\delta x/x$		
Generation	Data	Δm^2_{21}	$\sin^2\theta_{12}$	Ref.
Current	SK+SNO	15%	4.6%	SK Neutrino 2022
	KamLAND	2.5%	9.5%	1303.4667 SK Neutrino 2022
	SK+SNO+KamLAND	**2.4%**	**4.3%**	SK Neutrino 2022
	Global fit	2.8%	4.3%	Esteban+ 2007.14792
		2.9%	5.0%	de Salas+ 2006.11237
		2.2%	4.3%	Capozzi+ 2107.00532
Future	DUNE-solar	5.9%	3.0%	Capozzi+ 1808.08232
	JUNO	0.3%	0.5%	JUNO 2204.13249

Other CP Violating NSI Constraints

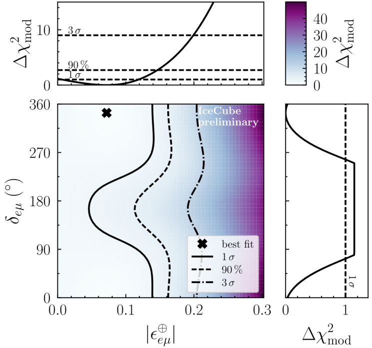
NSI effects grow with energy, density, and distance

Best probes:

- $\epsilon_{\mu\tau}$: atmospheric
- $\epsilon_{e\mu}$, $\epsilon_{e\tau}$: LBL appearance, atmospheric

- IceCube
  - Constraint is at LBL best fit with 3 yrs

    10 yrs of data in the bank
  - Prefers non-zero $|\epsilon_{e\mu}|$ at $\sim 1\sigma$



T. Ehrhardt, IceCube PPNT (2019)

- Super-K
  - Only consider real NSI
  - Comparable sensitivity as IceCube

Super-K `1109.1889`

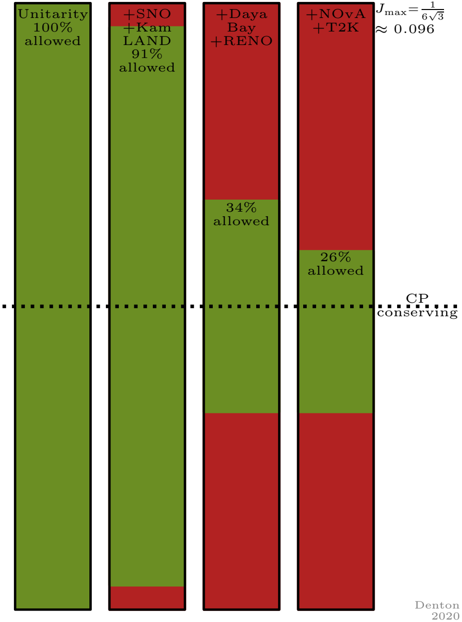
- COHERENT
  - Only applies to NSI models with $M_{Z'} \gtrsim 10$ MeV
  - NSI u, d, e configuration matters
  - Comparable constraints

COHERENT `1708.01294`
PBD, Y. Farzan, I. Shoemaker `1804.03660`
PBD, J. Gehrlein `2008.06062`

δ, J: Current Status

Maximal CP violation is already ruled out:

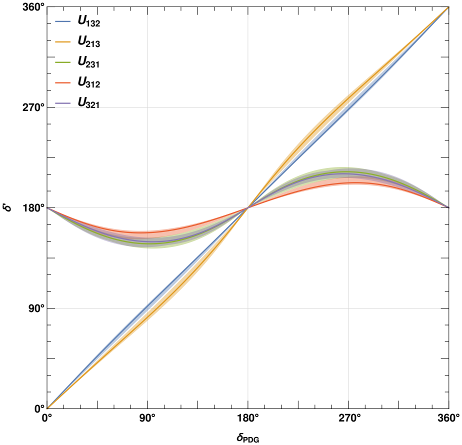
1. $\theta_{12} \neq 45^\circ$ at $\sim 15\sigma$
2. $\theta_{13} \neq \tan^{-1}\frac{1}{\sqrt{2}} \approx 35^\circ$ at many (100) σ
3. $\theta_{23} = 45^\circ$ allowed at $\sim 1\sigma$
4. $|\sin\delta| = 1$ allowed



Complex Phase in Different Parameterizations

- Can relate the complex phase in one parameterization to that in another
- U_{132} and U_{213} similar to U_{123}
- δ constrained to $\sim [150°, 210°]$ in U_{231}, U_{312}, U_{321}
- Bands indicate 3σ uncertainty on θ_{12}, θ_{13}, θ_{23}
- "50% of possible values of δ"
  $\Rightarrow$ parameterization dependent

DUNE TDR II 2002.03005



PBD, R. Pestes 2006.09384

Quark Mixing

From the PDG, V_{CKM} in the V_{123} parameterization is

$$\theta_{12} = 13.09^\circ \quad \theta_{13} = 0.2068^\circ \quad \theta_{23} = 2.323^\circ \quad \delta_{\mathrm{PDG}} = 68.53^\circ$$

Looks like "large" CPV:

$$\sin\delta_{\mathrm{PDG}} = 0.93 \sim 1$$

yet $J_{\mathrm{CKM}}/J_{\max} = 3 \times 10^{-4}$.

Switch to V_{212} parameterization, $\Rightarrow \delta' = 1^\circ$ and $\sin\delta' = 0.02$.

When δ and When J?

If the goal is **CP violation** the Jarlskog invariant should be used

however

If the goal is **measuring the parameters** one must use δ

Given θ_{12}, θ_{13}, θ_{23}, and J, I can't determine the sign of $\cos\delta$ which is physical

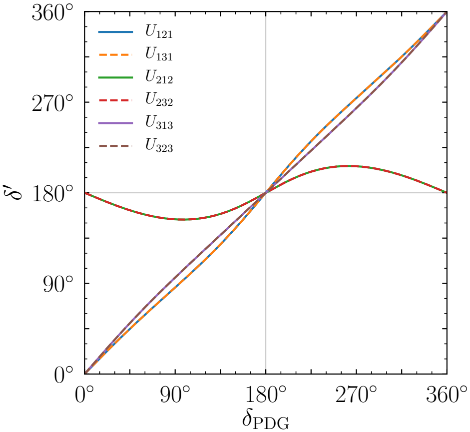
e.g. $P(\nu_\mu \to \nu_\mu)$ depends on $\cos\delta$ a tiny bit
PBD 2309.03262

- T2K/HK are mostly sensitivity to $\sin\delta$; they should focus on J

T2K does this now!

- NOvA/DUNE has modest $\cos\delta$ sensitivity; both J and δ should be reported

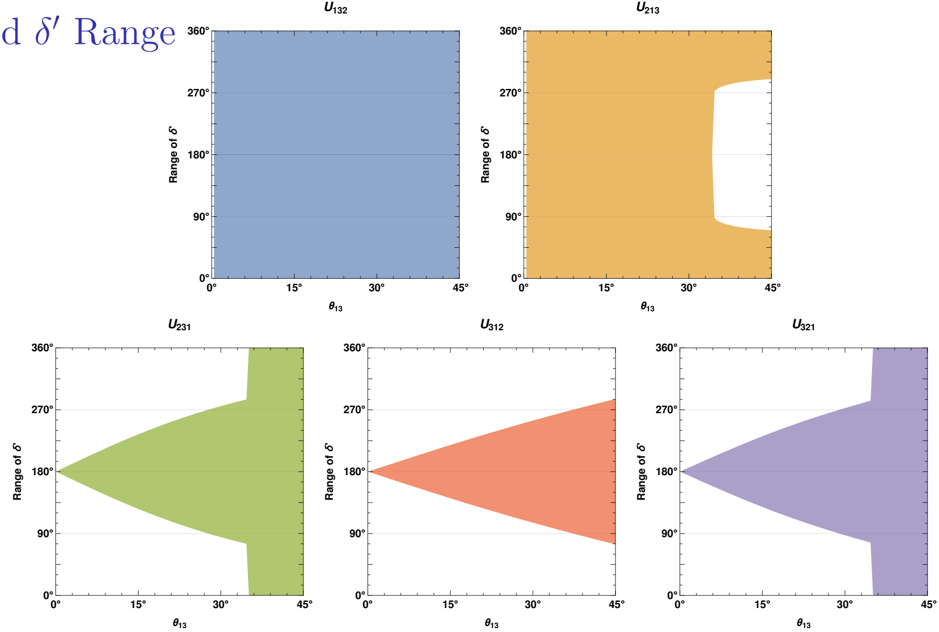
Repeated Rotations



	U_{121}	U_{131}	U_{212}	U_{232}	U_{313}	U_{323}
$\lvert U_{e2}\rvert$	✓	✓	✓	✓	✗	✗
$\lvert U_{e3}\rvert$	✓	✓	✗	✗	✓	✓
$\lvert U_{\mu3}\rvert$	✗	✗	✓	✓	✓	✓

Note that $e^{i\delta}$ must be on first or third rotation

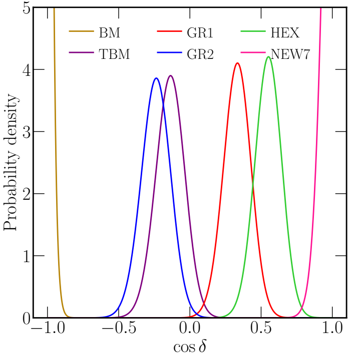
Allowed δ' Range



The Importance of $\cos\delta$

- If only $\sin\delta$ is measured $\Rightarrow$ sign degeneracy: $\cos\delta = \pm\sqrt{1-\sin^2\delta}$
- Most flavor models predict $\cos\delta$

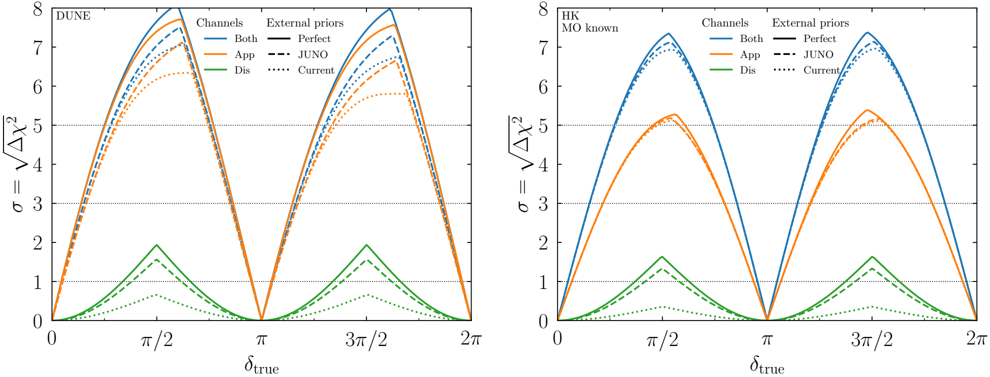
J. Gehrlein, et al. 2203.06219



L. Everett, et al. 1912.10139

CP Violation Discovery with Disappearance

Need JUNO and DUNE or HK



PBD 2309.03262

Gallium Experiments

- Low energy solar neutrino experiments measure the *pp* flux
  - Consistent with KamLAND

SAGE 0901.2200

GALLEX 1001.2731

- Callibrate detectors with intense radioactive sources
- See fewer ν_e than expected:

3.0σ: C. Giunti, M. Laveder 1006.3244

2.3σ: J. Kostensalo, et al. 1906.10980

$> 4\sigma$: BEST 2109.11482

- Cannot be easily explained with SM physics

C. Giunti, et al. 2212.09722

V. Brdar, J. Gehrlein, J. Kopp 2303.05528

W. Haxton, et al. 2303.13623

- Prefers:
  - $\Delta m^2_{41} \gtrsim 0.5$ eV^2
  - $\sin^2 2\theta_{ee} = 4|U_{e4}|^2(1-|U_{e4}|^2) \sim 0.4$

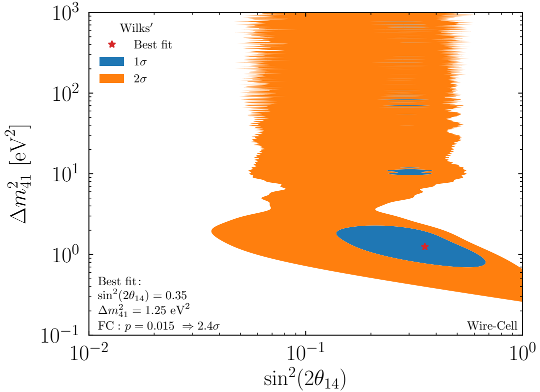
MicroBooNE Disappearance

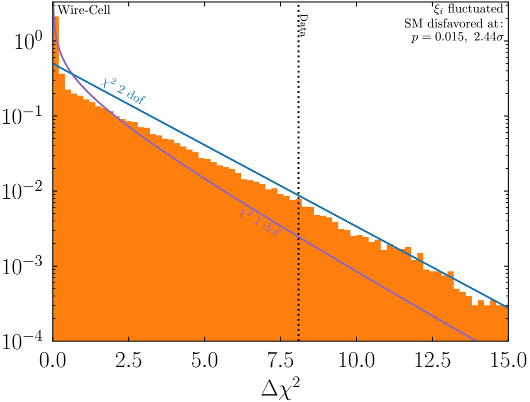
MicroBooNE is focused on ν_e appearance
Can do ν_μ and ν_e disappearance too!

See also D. Cianci, et al. 1702.01758

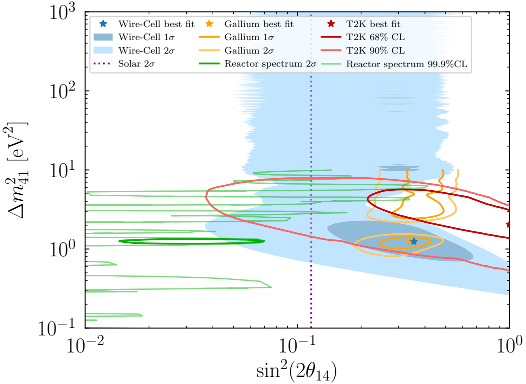
MiniBooNE backgrounds too big, plus anomaly

Results and Monte Carlo Significance





Global ν_e Disappearance Picture



- Gallium and my MicroBooNE regions agree
- Constraints from solar and reactor
- Cosmology disfavors entire plane

Other MicroBooNE Analysis Channels

Analysis	$\sin^2(2\theta_{14})$	Δm^2_{41} (eV^2)	$N\sigma$ (FC)
Wire-Cell	$0.35^{+0.19}_{-0.16}$	$1.25^{+0.74}_{-0.39}$	2.4
Deep-Learning	$0.88^{+0.12}_{-0.41}$	$3.91^{+0.40}_{-0.40}$	1.8
Pandora-Np	$0.81^{+0.19}_{-0.47}$	$[1.28,2.44]$ $6.73^{+1.75}_{-0.90}$ $\vdots$	2.4
Pandora-0p	$1_{-0.29}$	$2.21^{+0.82}_{-0.60}$ $\vdots$	1.8

2111.05793

Unitarity Violation: What is It?

Suppose the measurable 3×3 matrix is not unitary:

$$U_3 U_3^\dagger \neq \mathbb{1}$$

Addition of new flavor states $\nu_a, \nu_b, \nu_c, \ldots$ and new mass states ν_4, ν_5, ν_6

$$U \to \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & \cdots \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} & \cdots \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} & \cdots \\ U_{a1} & U_{a2} & U_{a3} & U_{a4} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Unitarity Violation ⇒

New mass states not directly accessible by oscillations or decay

Thus check if U_3 is what it should be

Unitarity Violation Status from Oscillations

3σ maximal deviations from unitarity

Leptons

	Hu+	Ellis+
ν_e row	0.003	0.05
ν_μ row	0.02	0.04
ν_τ row	0.2	0.82
ν_1 col	0.06	0.22
ν_2 col	0.09	0.27
ν_3 col	0.12	0.40

Quarks

u row	0.0015	$\sim 2.2\sigma$ tension
c row	0.06	
t row	-	
d col	0.005	
s col	0.06	
b col	-	

Lepton constraints don't include anomalies
Care is required

S. Ellis, K. Kelly, S. Li 2008.01088
Z. Hu, et al. 2008.09730
S. Parke, M. Ross-Lonergan 1508.05095
PDG

Vastly different mixing angle hierarchy

$\Rightarrow$

Like comparing strawberries and squid

Unitarity Violation

Consistency of the three-flavor oscillation picture?

and/or

Searches for unitarity violation?

Not the same!

Lots of models to test standard three-flavor picture:
Sterile, unitarity violation, NSI, neutrino decay, decoherence, …

Unitarity Constraints

Unitary violation: the study of how $U_{3\times3}$ is not unitary independent of m_4, m_5, …
Constraints vary considerably in the literature:

$$1-|U_{e1}|^2-|U_{e2}|^2-|U_{e3}|^2<\begin{cases}0.05\\0.001\end{cases}\quad\text{at }2\sigma$$

All analyses *assume* unitarity
Throw out LSND, MiniBooNE, RAA, gallium, etc.

S. Parke, M. Ross-Lonergan 1508.05095

Z. Hu, et al. 2008.09730

Unitarity Violation

- Could conceivably differentiate: 2 new states from 1, but not 3+ from 2
- Zero distance effect ⇒ near detector **with flux prediction**

E.g. RAA, Gallium

- Numerous parameterizations: α matrix, η matrix, submatrix & Cauchy-Schwartz

All apply to the inaccessible cases only

- There is an approximate correspondence to sterile and NSI

$$\alpha_{ee} \approx \frac{1}{2}(s_{14}^2 + s_{15}^2 + s_{16}^2) \approx -\epsilon_{ee}\,, \quad \ldots$$

M. Blennow, et al. 1609.08637

Applies one experiment at a time

- Additional EW precision information: W, Z, π, μ, τ decays

Care is required

S. Antush, et al. hep-ph/0607020

S. Antusch, O. Fischer 1407.6607

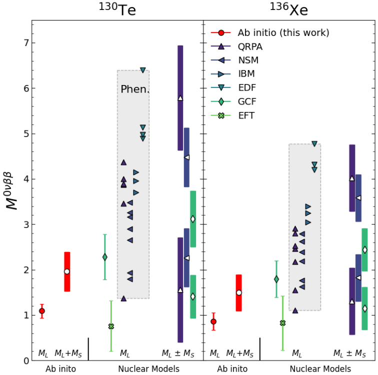
Unitarity Violation: Mass Ranges for Tau Neutrinos

experiment	(4,4) (m_4)	(5,3) (m_4)
atmospheric ν_μ disappearance	$\in [10\text{ eV}, 15\text{ MeV}]$	$\gtrsim 40$ MeV
atmospheric ν_τ appearance	$\in [10\text{ eV}, 15\text{ MeV}]$	$\gtrsim 40$ MeV
astrophysical ν_τ appearance	$\lesssim 15$ MeV	$\gtrsim 40$ MeV
solar ^{8}B	$\lesssim 5$ MeV	$\gtrsim 20$ MeV
DONuT/FASERnu	$\in$ [100 eV, 90 MeV]	$\gtrsim 200$ MeV
LBL ν_τ appearance (OPERA)	$\in$ [1 eV, 15 MeV]	$\gtrsim 40$ MeV
LBL ν_τ appearance (DUNE)	$\in$ [0.1 eV, 15 MeV]	$\gtrsim 40$ MeV
LBL ν_μ disappearance (DUNE)	$\in$ [0.1 eV, 15 MeV]	$\gtrsim 40$ MeV
CEvNS	$\in$ [10 eV, 15 MeV]	$\gtrsim 40$ MeV

PBD, J. Gehrlein `2109.14575`

Matrix Element Computation Progress

Recent years have seen real uncertainty estimates
And now some ab initio results!



Unfortunately the matrix elements are somewhat smaller than previously estimated

A. Belley, et al. 2307.15156