

The best way to probe CP violation in the lepton sector is with long-baseline accelerator neutrino experiments in the appearance mode: the appearance of ν_e in predominantly ν_μ beams. Here we show that it is possible to discover CP violation with disappearance experiments only, by combining JUNO for electron neutrinos and DUNE or Hyper-Kamiokande for muon neutrinos. While the maximum sensitivity to discover CP is quite modest (1.6σ with 6 years of JUNO and 13 years of DUNE), some values of δ may be disfavored by $> 3\sigma$ depending on the true value of δ .

Neutrino oscillation experiments will be entering the precision era in the next decade with the advent of high statistics experiments like DUNE, HK, and JUNO. Correctly estimating the confidence intervals from data for the oscillation parameters requires very large Monte Carlo data sets involving calculating the oscillation probabilities in matter many, many times. In this paper, we leverage past work to present a new, fast, precise technique for calculating neutrino oscillation probabilities in matter optimized for long-baseline neutrino oscillations in the Earth's crust including both accelerator and reactor experiments. For ease of use by theorists and experimentalists, we provide fast c++ and fortran codes.

CP-Violation with Neutrino Disappearance and NuFast

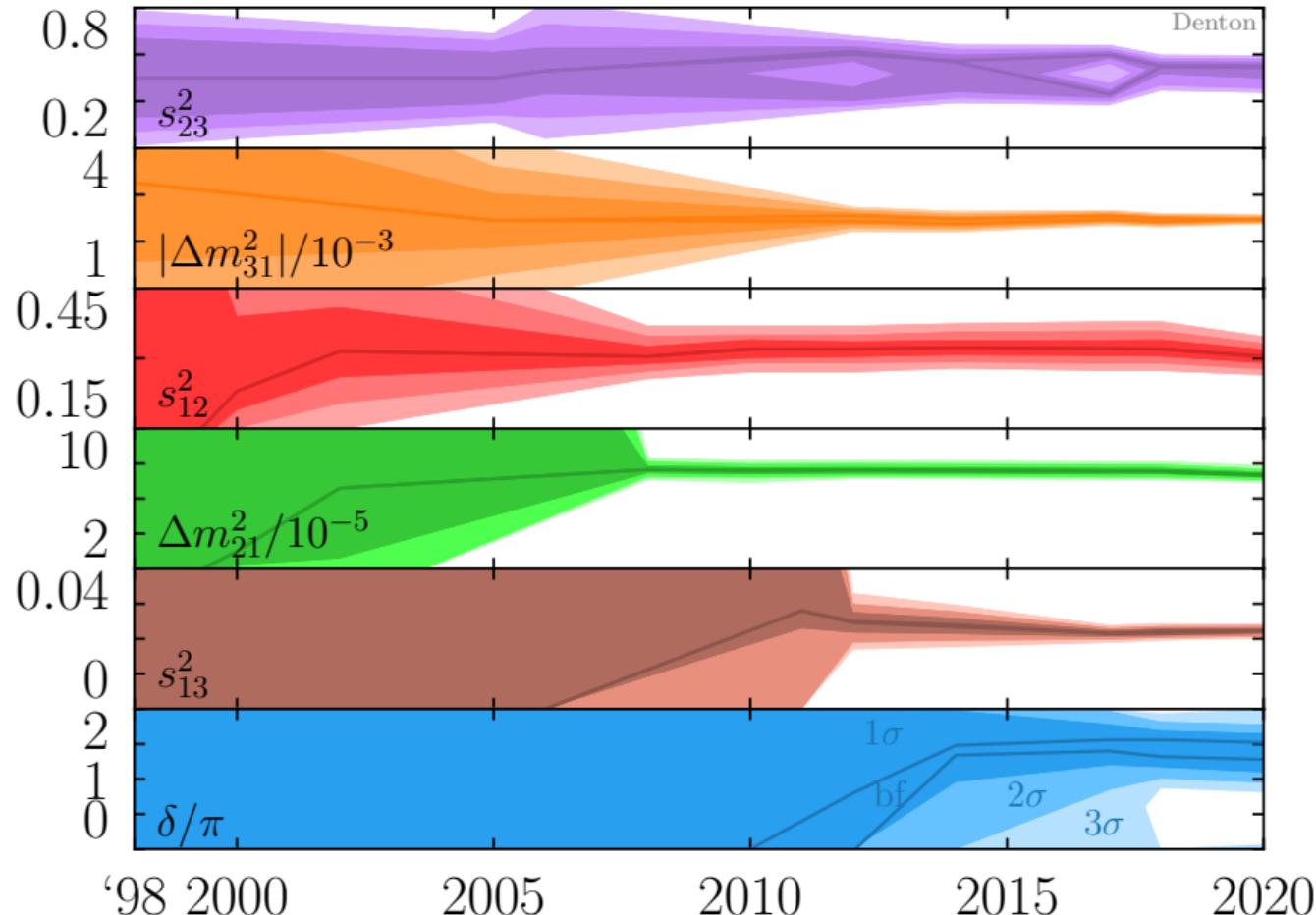
Peter B. Denton

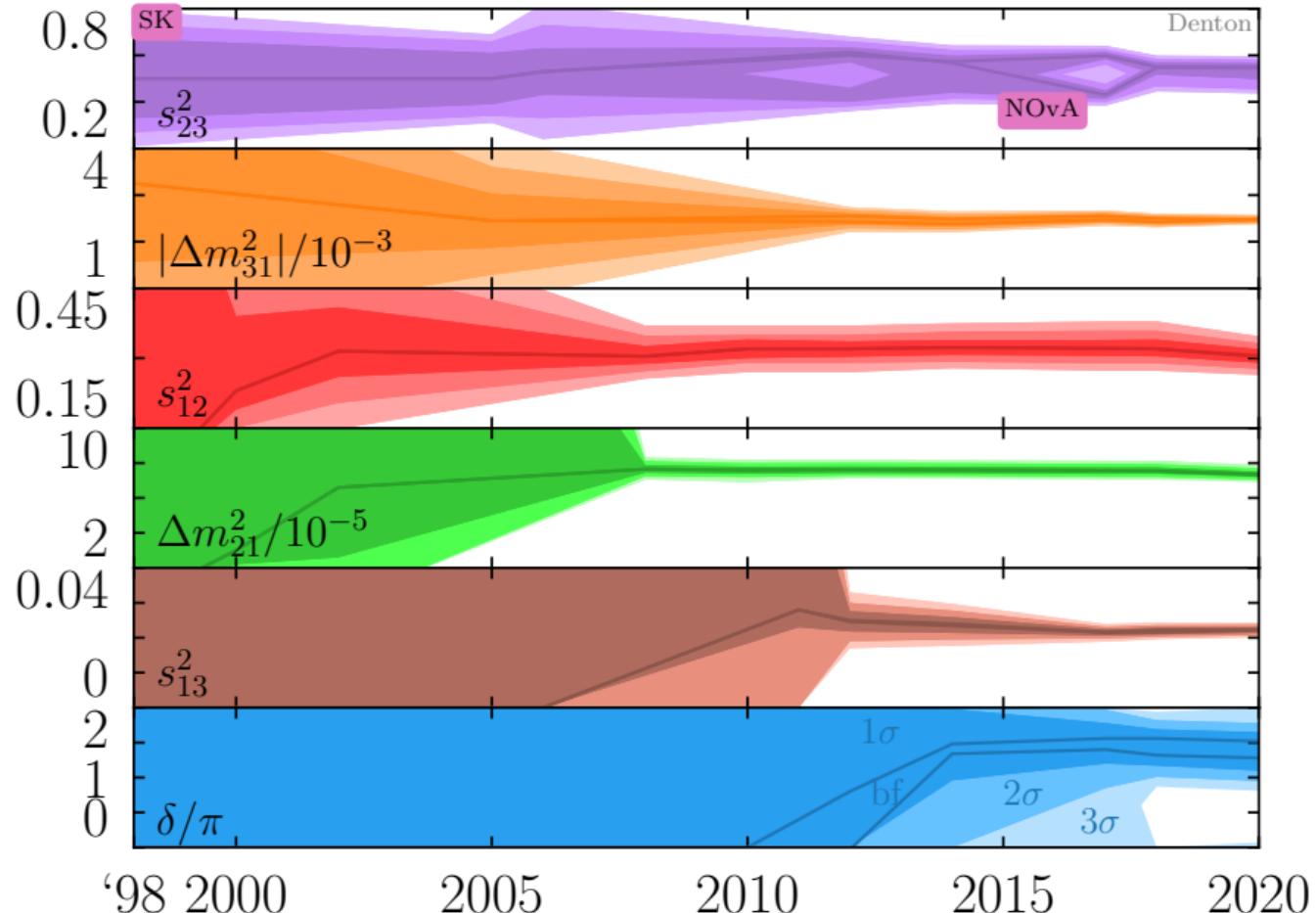
CETUP*

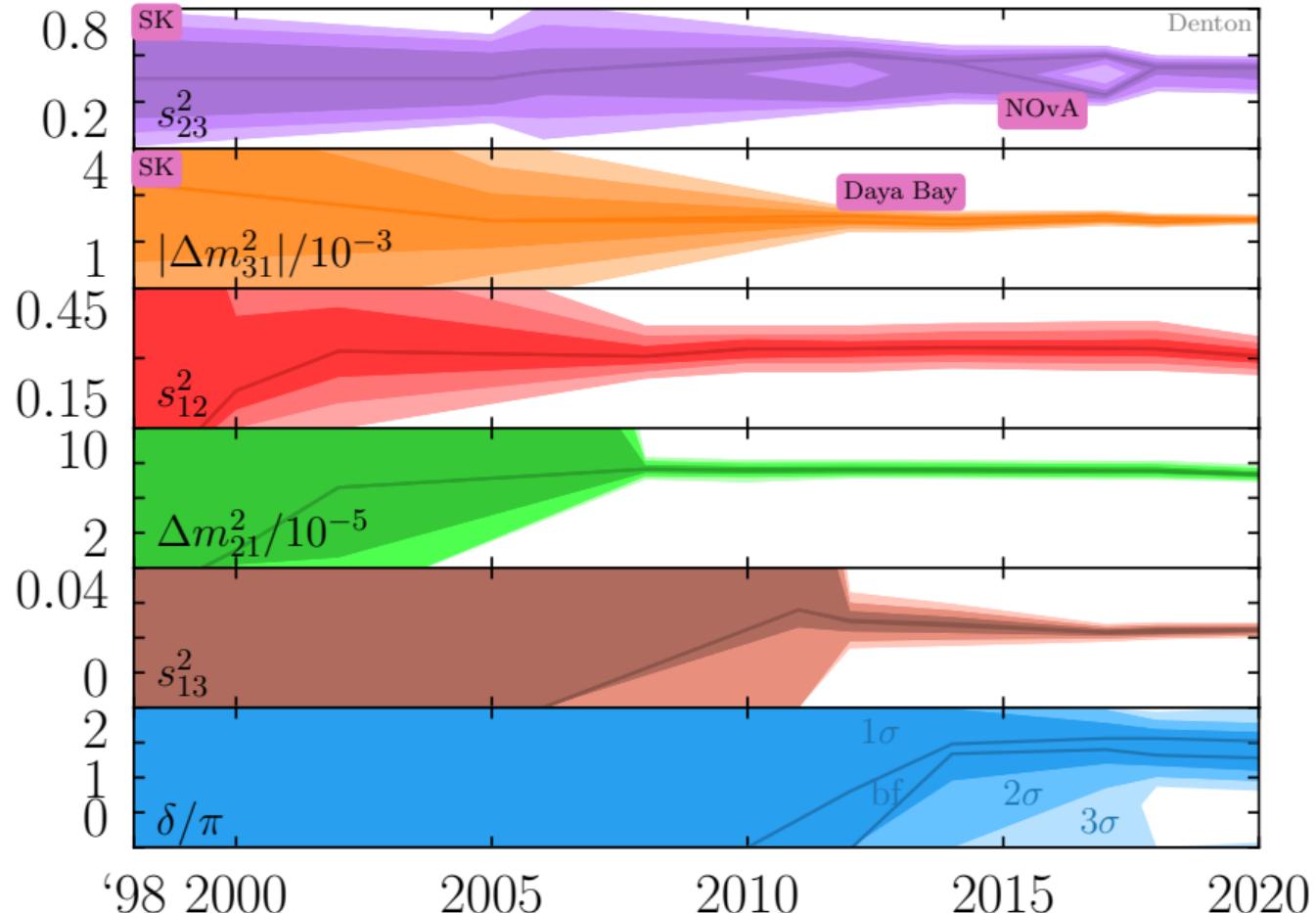
July 8, 2024

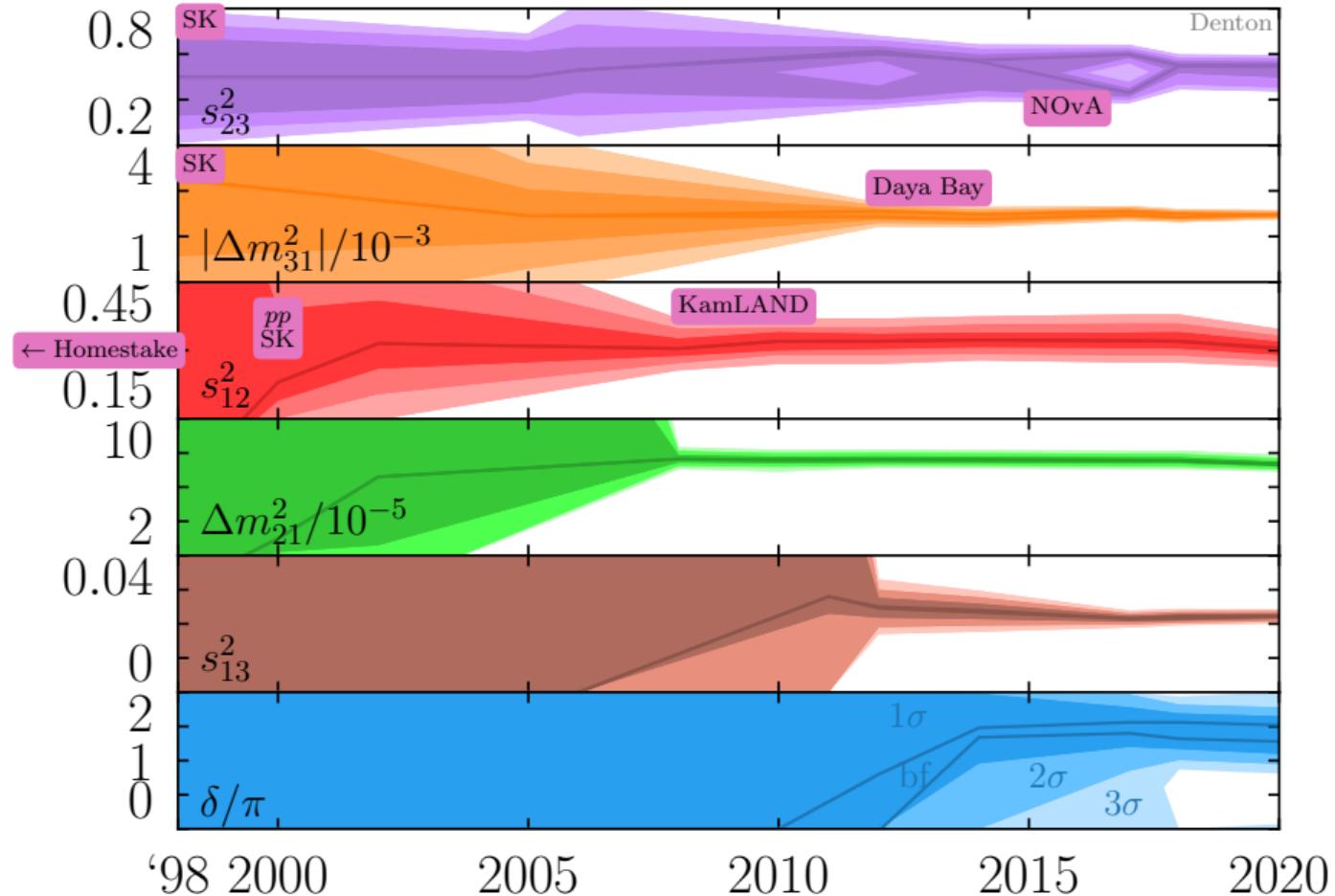
2309.03262 PRL (in press)
2405.02400 with S. Parke

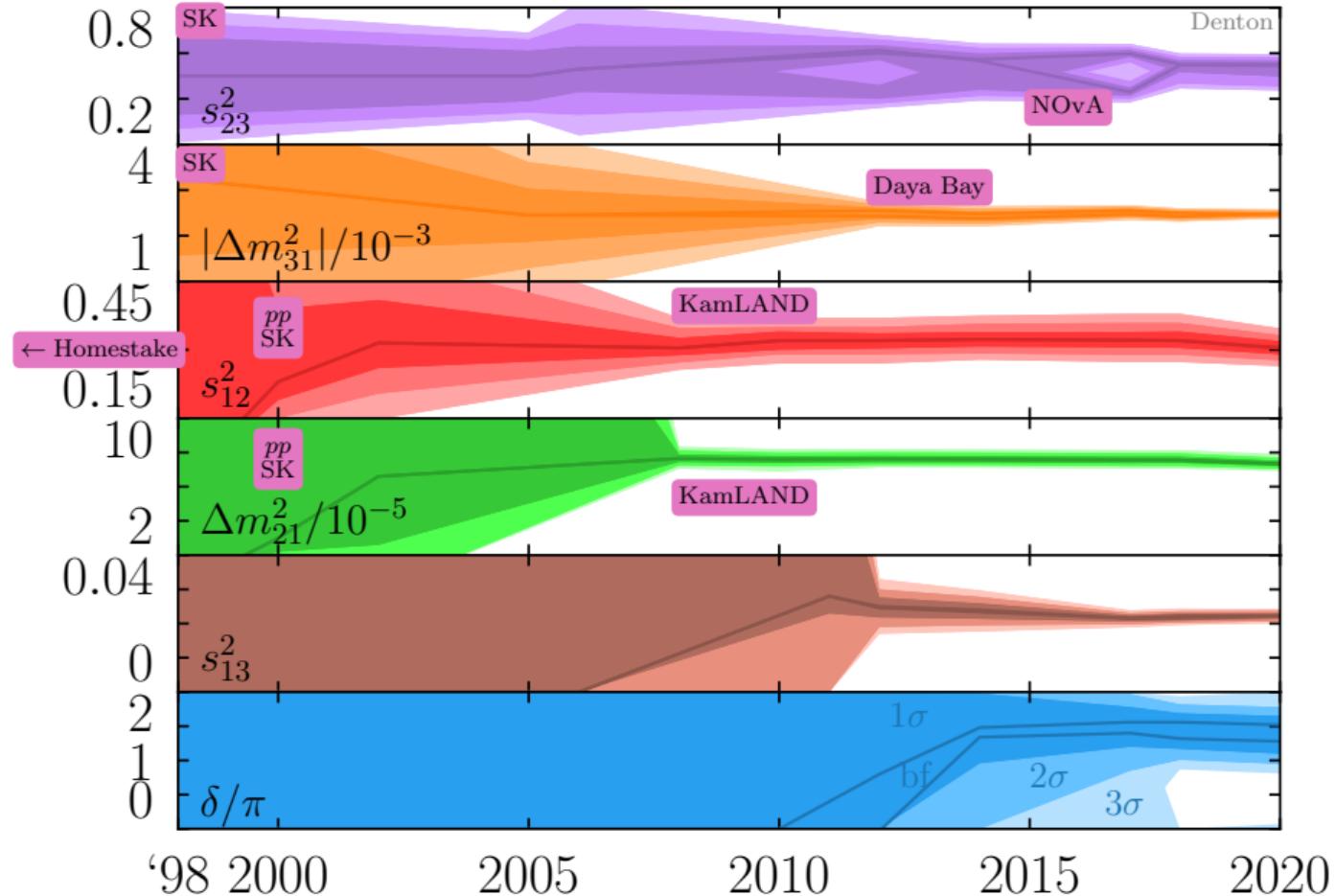


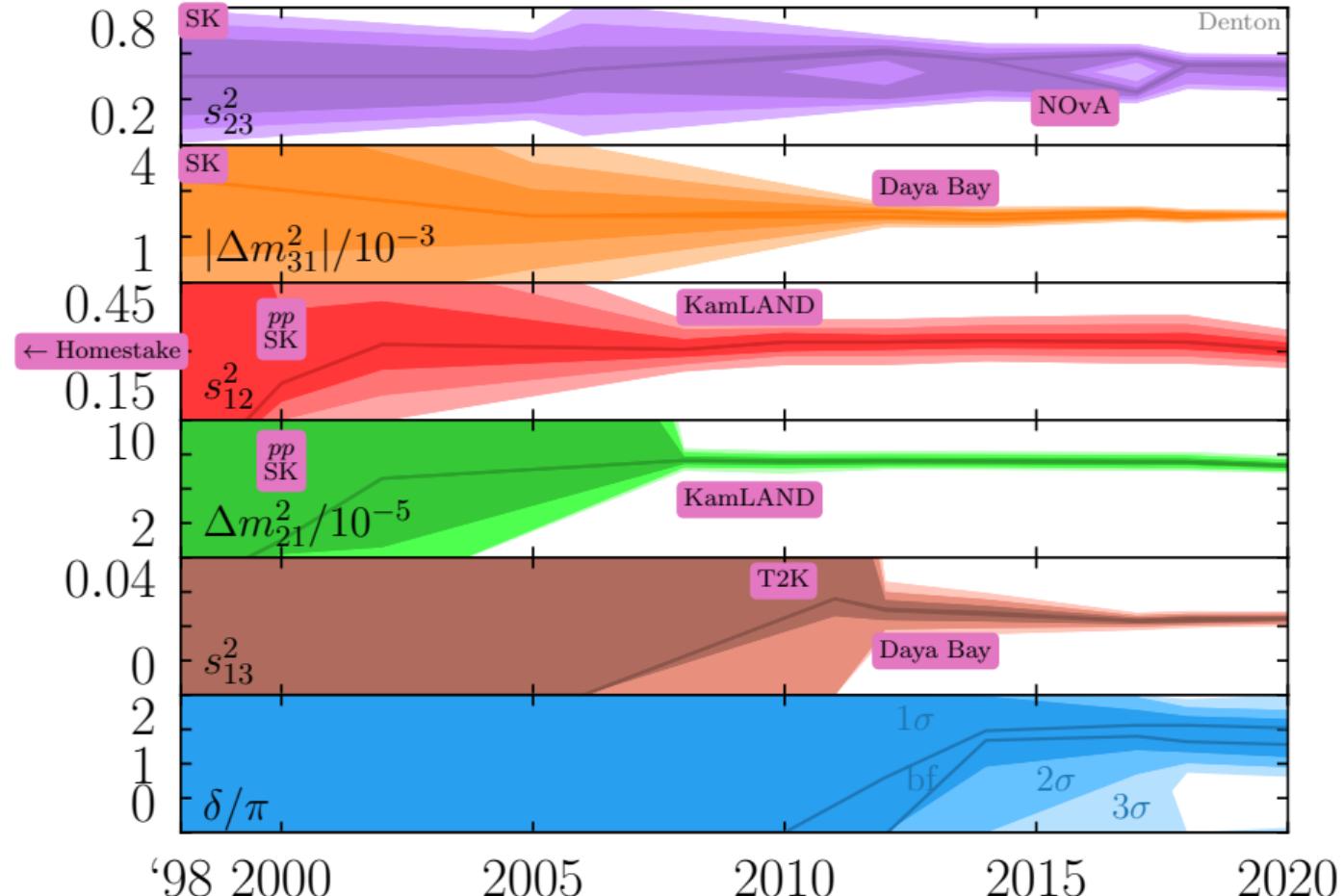


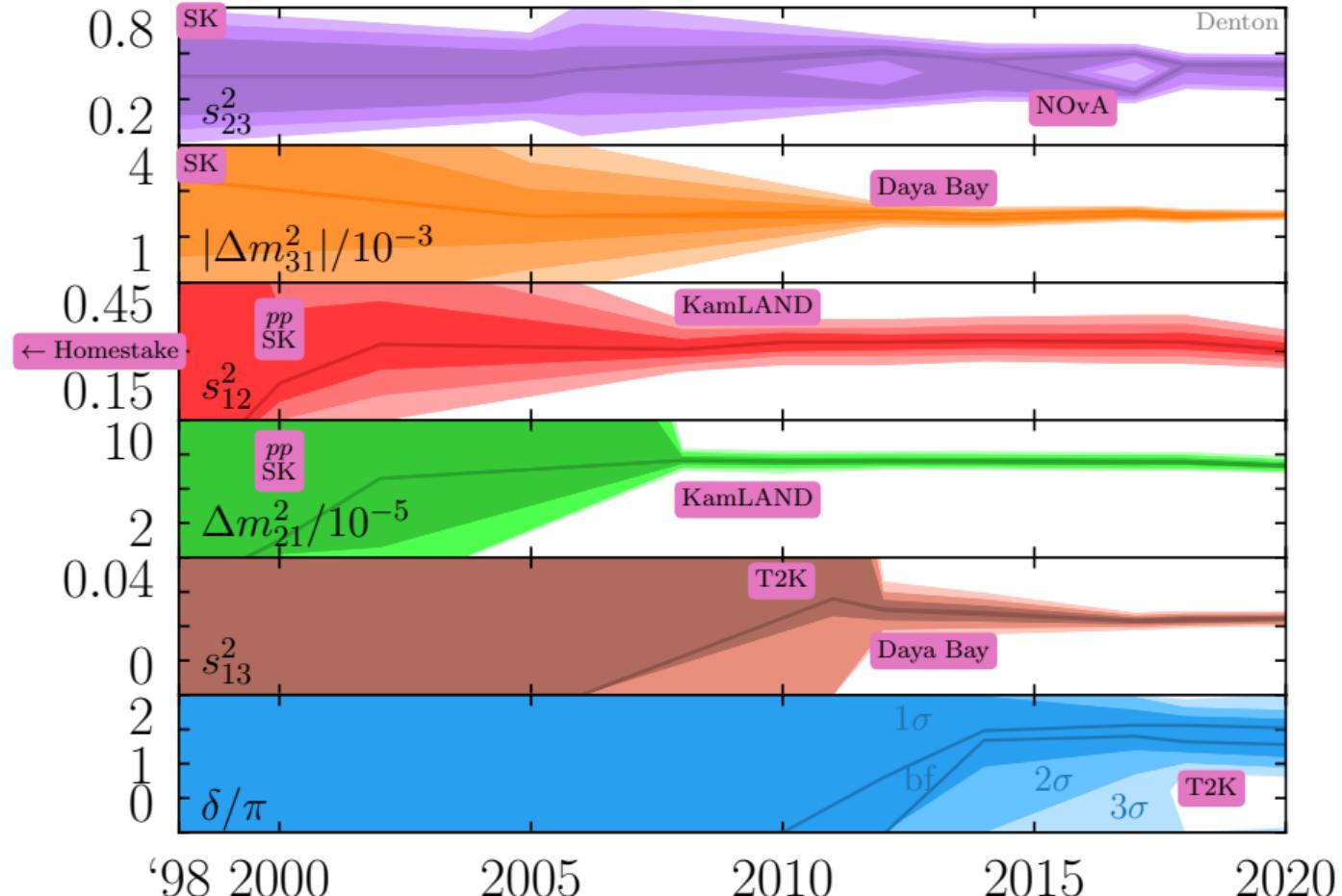












Four known unknown in particle physics: all neutrinos

Atmospheric mass ordering

θ_{23} octant

Complex phase

Absolute mass scale

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Outline

1. Why CPV is interesting
2. Other non-standard probes of CPV
3. Relationship between appearance, disappearance, CP, T, CPT
4. Three ways to see why there is CPV information in disappearance
 - 4.1 Parameter counting
 - 4.2 Direct analytic calculation
 - 4.3 Numerical test
5. Role of the matter effect
6. Sensitivities
7. NuFast

Why is CPV interesting?

δ and CP violation

$$J_{CP} = s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23} \sin \delta$$

C. Jarlskog [PRL 55, 1039 \(1985\)](#)



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1. Strong interaction: no observed EDM \Rightarrow CP (nearly) **conserved**

$$\frac{\bar{\theta}}{2\pi} < 10^{-11}$$

J. Pendlebury, et al. [1509.04411](#)

2. Quark mass matrix: non-zero but **small** CP violation

$$\frac{|J_{CKM}|}{J_{\max}} = 3 \times 10^{-4}$$

CKMfitter [1501.05013](#)

3. Lepton mass matrix: ?

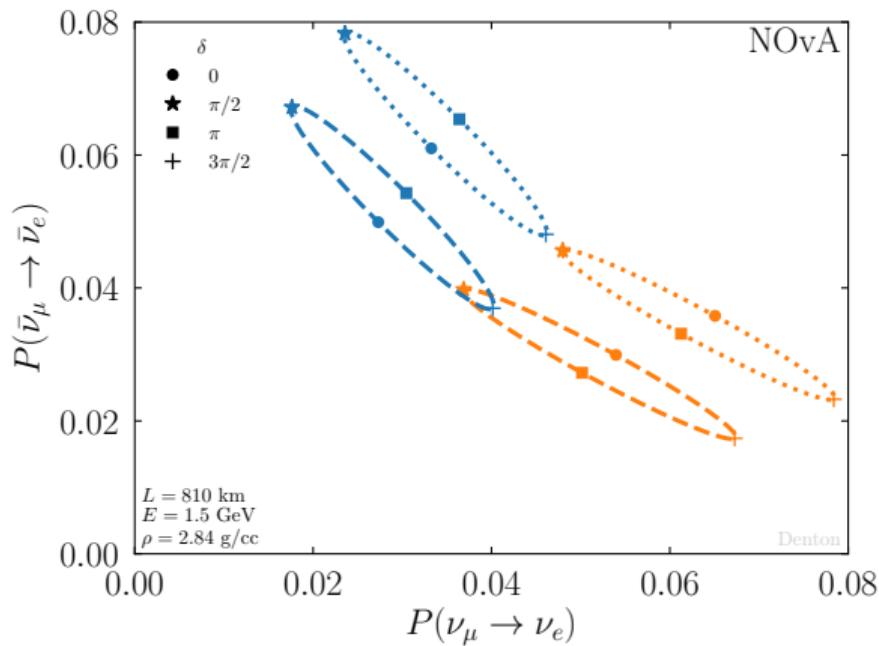
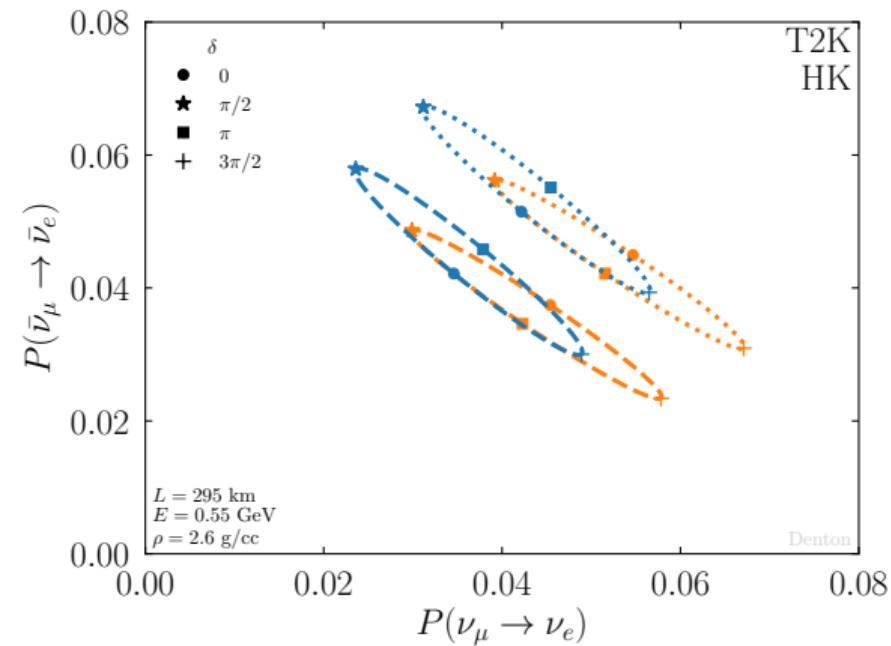
$$\frac{|J_{PMNS}|}{J_{\max}} < 0.34$$

PBD, J. Gehrlein, R. Peses [2008.01110](#)

$$J_{\max} = \frac{1}{6\sqrt{3}} \approx 0.096$$

CETUP*: July 8, 2024 6/50

δ : what is it really?



δ : what is it not?

$\delta \not\Rightarrow$ Baryogenesis/Leptogenesis

The amount of leptogenesis is a function of:

1. δ
2. the heavy mass scale
3. α, β (Majorana phases)
4. CP phases in the RH neutrinos
5. ...

C. Hagedorn, et al. [1711.02866](#)

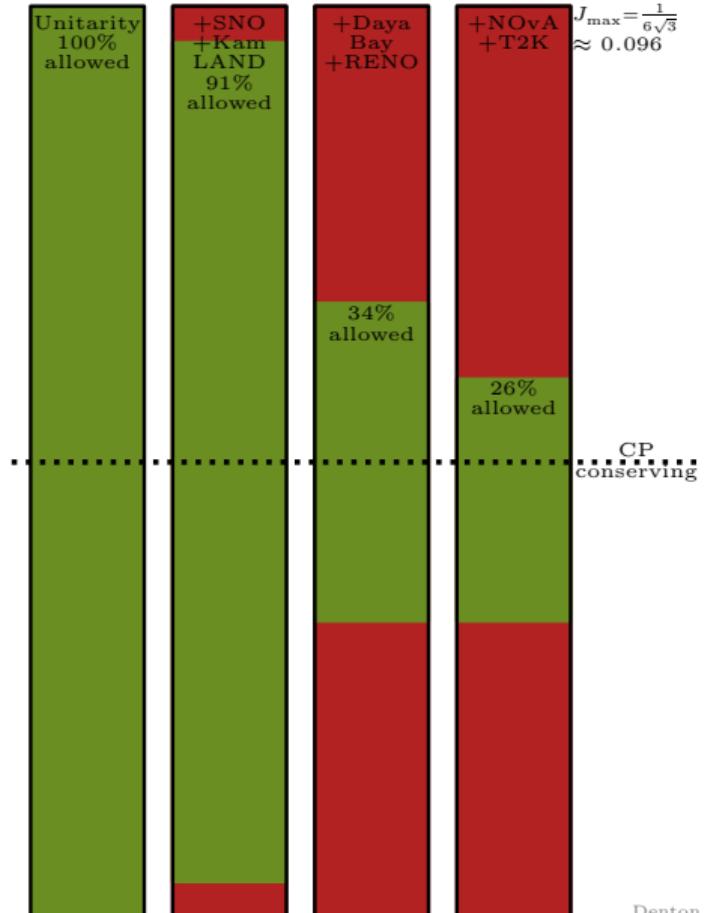
K. Moffat, et al. [1809.08251](#)

Measuring $\delta = 0, \pi$	$\not\Rightarrow$	no leptogenesis
Measuring $\delta \neq 0, \pi$	$\not\Rightarrow$	leptogenesis

δ, J : current status

Maximal CP violation is already ruled out:

1. $\theta_{12} \neq 45^\circ$ at $\sim 15\sigma$
2. $\theta_{13} \neq \tan^{-1} \frac{1}{\sqrt{2}} \approx 35^\circ$ at many (100) σ
3. $\theta_{23} = 45^\circ$ allowed at $\sim 1\sigma$
4. $|\sin \delta| = 1$ allowed



When δ and when J ?

If the goal is **CP violation** the Jarlskog invariant should be used

however

If the goal is **measuring the parameters** one must use δ

Given θ_{12} , θ_{13} , θ_{23} , and J , I can't determine the sign of $\cos \delta$ which is physical

e.g. $P(\nu_\mu \rightarrow \nu_\mu)$ depends on $\cos \delta$

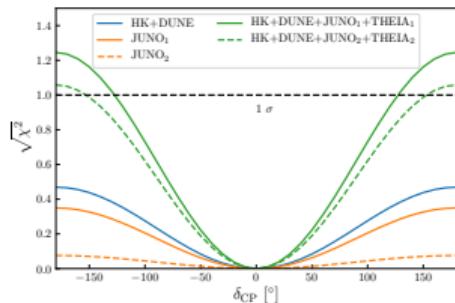
Other non-standard CPV probes

1. Some information in solar due to loops in elastic scattering

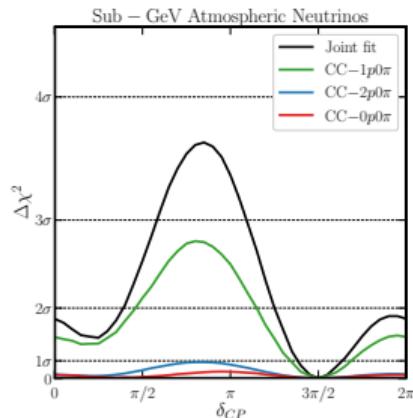
V. Brdar, X-J. Xu [2306.03160](#)
K. Kelly, et al. [2407.03174](#) requires 3k Borexinos

2. Sub-GeV atmospherics

K. Kelly, et al. [1904.02751](#)
See also e.g. A. Suliga, J. Beacom [2306.11090](#)



Solar (no systematics)



Atmospherics at DUNE

Appearance, disappearance, and CP

Appearance vs. Disappearance

Oscillation experiments can do
appearance or disappearance experiments:

Disappearance

K2K, MINOS, T2K, NO ν A
KamLAND, Daya Bay, RENO, Double CHOOZ
(Sort of) SNO, Borexino, SK-solar
JUNO, DUNE, HK

Appearance

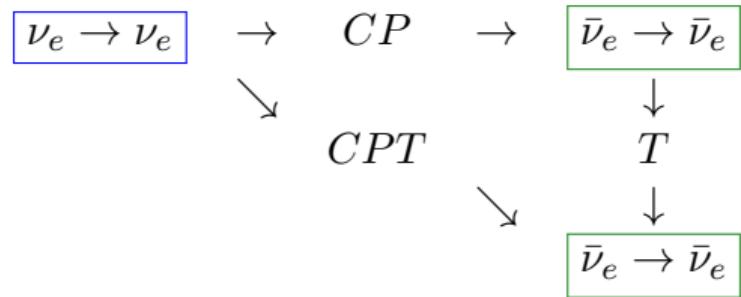
T2K, NO ν A
OPERA
Atm ν_τ hints @ SK & IceCube
DUNE, HK

Neither appearance nor disappearance

SK-atm, IceCube



CP, T: Disappearance

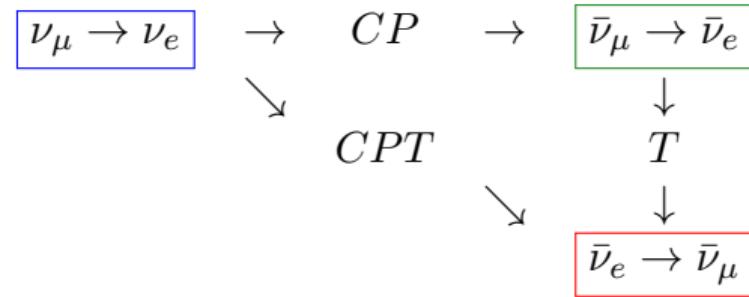


Disappearance measurements are even eigenstates of CP

$$CP[P(\nu_e \rightarrow \nu_e)] = P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \stackrel{CPT}{=} P(\nu_e \rightarrow \nu_e)$$

Assume that CPT is a good symmetry

CP, T: Appearance



Appearance measurements are not eigenstates of CP

Appearance and Disappearance, CP even and CP odd terms

Disappearance:

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\alpha) &= 1 - 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2 \Delta_{21} \\ &\quad - 4|U_{\alpha 1}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{31} \\ &\quad - 4|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{32} \\ &= P_{\alpha\alpha}^{CP+} \end{aligned}$$

$$\Delta_{ij} \equiv \Delta m_{ij}^2 L / 4E$$

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Appearance:

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= -4\Re[U_{\alpha 1} U_{\beta 1}^* U_{\alpha 2}^* U_{\beta 2}] \sin^2 \Delta_{21} \\ &\quad - 4\Re[U_{\alpha 1} U_{\beta 1}^* U_{\alpha 3}^* U_{\beta 3}] \sin^2 \Delta_{31} \\ &\quad - 4\Re[U_{\alpha 3} U_{\beta 3}^* U_{\alpha 2}^* U_{\beta 2}] \sin^2 \Delta_{32} \\ &\quad \pm 8J_{CP} \sin \Delta_{21} \sin \Delta_{31} \sin \Delta_{32} \\ &= P_{\alpha\beta}^{CP+} + P_{\alpha\beta}^{CP-} \end{aligned}$$

$$\Delta_{ij} \equiv \Delta m_{ij}^2 L / 4E$$

Sign depends on α, β

Conventional Wisdom

1. Appearance is sensitive to CPV [True]

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1. Appearance is sensitive to CPV [True]
2. Disappearance has no CPV sensitivity [False]
3. Any δ dependence in disappearance is in ν_μ not ν_e [Confusing/False]

$$\begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Correct Statements

- ▶ Appearance is the best way to measure δ and CPV
 - ... given known oscillation parameters, systematics, and realistic experiments
 - ▶ Probes mostly $\sin \delta$ not $\cos \delta$
 - ▶ Don't need both ν and $\bar{\nu}$ (but systematics)
- ▶ **Disappearance can measure δ**
 - ▶ CPV can be discovered with only disappearance measurements
 - ▶ Probes mostly $\cos \delta$ not $\sin \delta$
 - ▶ Requires measurements of two flavors
 - ▶ "Works through unitarity" (as do nearly all oscillation measurements)

Parameter Counting

1. Four parameters in the PMNS matrix

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$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - 4 \sum_{i>j} C_{ij}^\alpha \sin^2 \Delta_{ij}$$

$$C_{ij}^\alpha = |U_{\alpha i}|^2 |U_{\alpha j}|^2$$

$$|U_{\alpha i}| = \left(\frac{C_{ij}^\alpha C_{ik}^\alpha}{C_{jk}^\alpha} \right)^{1/4}$$

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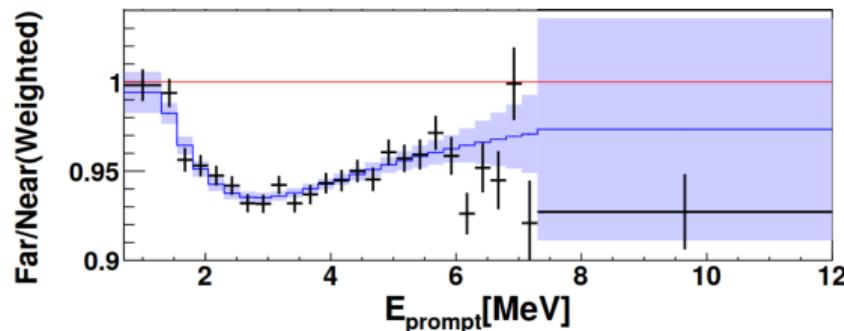
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Daya Bay [1809.02261](#)

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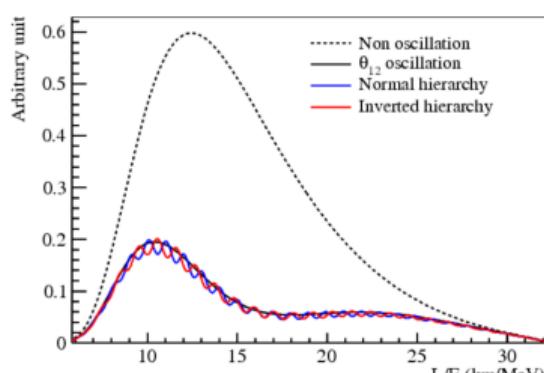
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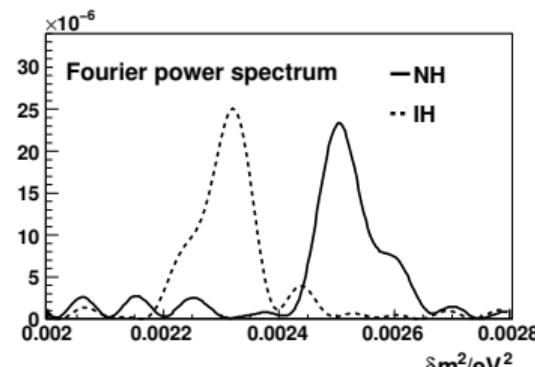
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JUNO [1507.05613](#)



L. Zhan, et al. [0807.3203](#)

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4. Given good measurements of the ν_e and ν_μ disappearance, 4 independent parameters will be measured

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6. If we determine $\cos \delta \neq \pm 1$ \Rightarrow CP is violated!

Direct Analytic Calculation

Disappearance experiments measure various $|U_{\alpha i}|^2$ terms

Suppose 4 are measured: $|U_{e2}|^2, |U_{e3}|^2, |U_{\mu 2}|^2, |U_{\mu 3}|^2$

Actually this gives all 9 magnitudes by unitarity

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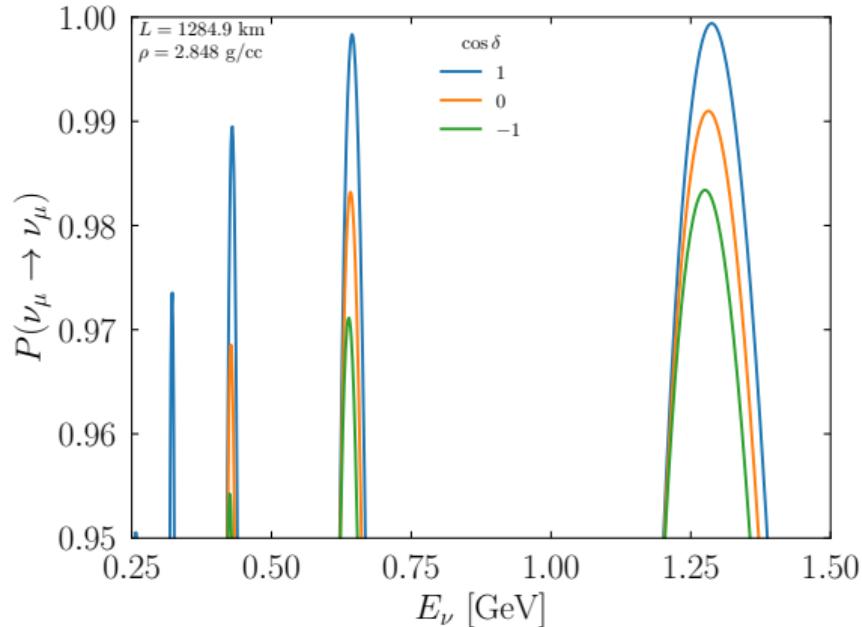
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$$J_{CP}^2 = |U_{e2}|^2 |U_{\mu 2}|^2 |U_{e3}|^2 |U_{\mu 3}|^2 - \frac{1}{4} (1 - |U_{e2}|^2 - |U_{\mu 2}|^2 - |U_{e3}|^2 - |U_{\mu 3}|^2 + |U_{e2}|^2 |U_{\mu 3}|^2 + |U_{e3}|^2 |U_{\mu 2}|^2)^2$$

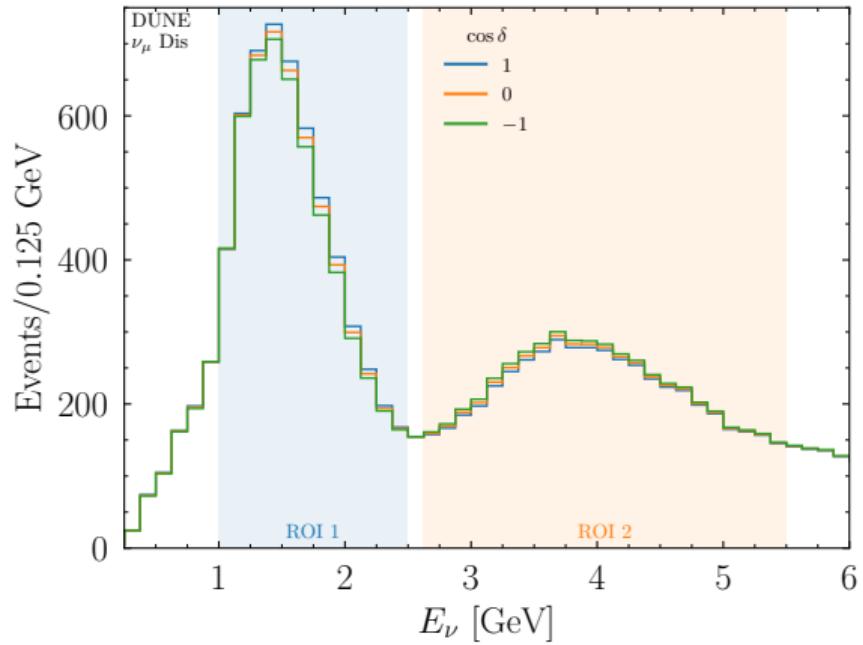
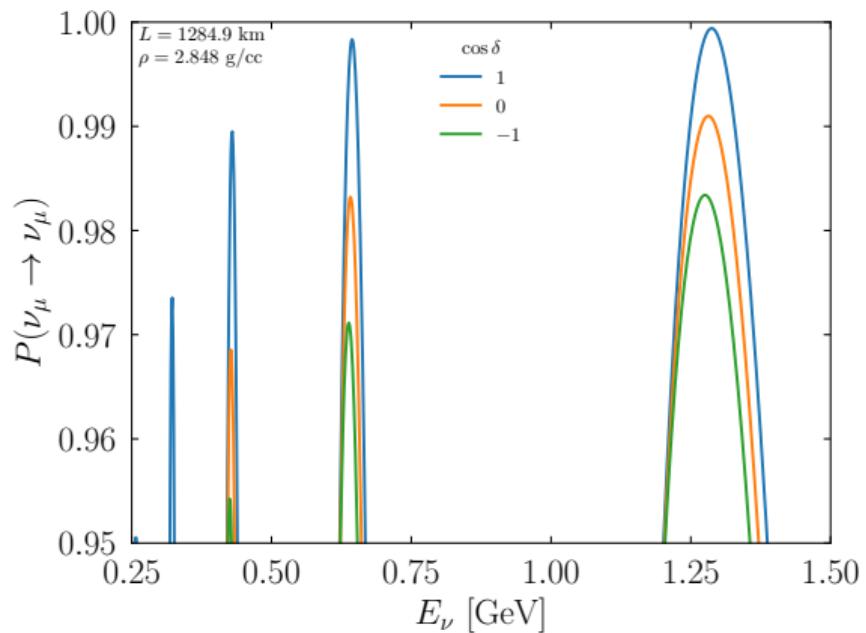
Can show that if any one $|U_{\alpha i}|^2 = 0 \Rightarrow J = 0$

Disappearance can tell us if CP is violated,
but not if nature prefers ν 's or $\bar{\nu}$'s

Where is $|U_{\mu 2}|^2$?



Where is $|U_{\mu 2}|^2$?



	$\cos \delta$	ROI 1	ROI 2
6.5 yrs ν_μ rates	1	5506	5038
	0	5418	5115
	-1	5334	5193

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Approximate size of $|U_{\mu 2}|^2$ signal: (21) sector

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PBD, J. Gehrlein [2302.08513](#)

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- ▶ This term is

$$\approx -4c_{23}^2 (s_{12}^2 c_{12}^2 + s_{23} c_{23} s_{13} \sin 2\theta_{12} \cos 2\theta_{12} \cos \delta) \sin^2 \Delta_{21}$$

$$\approx -2 (0.21 + 0.03 \cos \delta) \left(\frac{\pi}{33} \right)^2$$

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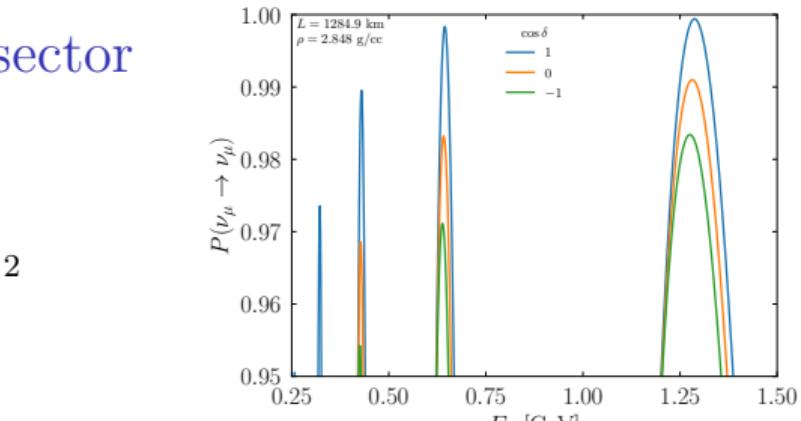
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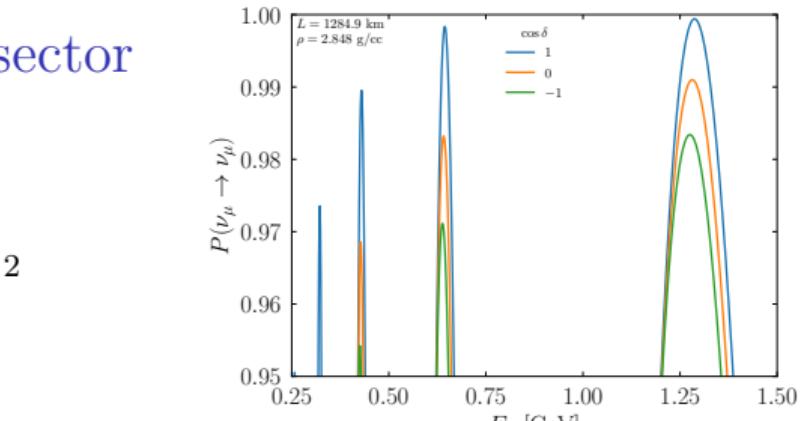
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Sign is wrong

Magnitude is ~ 16 too small

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Matter effects matter: (21) sector

- ▶ Let's start again at

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- ▶ Solar splitting modified by

$$\frac{\Delta m_{21}^2 \rightarrow \Delta m_{21}^2 \mathcal{S}_\odot}{\mathcal{S}_\odot \approx \sqrt{(\cos 2\theta_{12} - c_{13}^2 a / \Delta m_{21}^2)^2 + \sin^2 2\theta_{12}} \approx 3.4}$$

at $E = 1.3$ GeV
PBD, S. Parke [1902.07185](#)

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- ▶ Let's start again at

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- ▶ Solar splitting modified by

$$\frac{\Delta m_{21}^2 \rightarrow \Delta m_{21}^2 \mathcal{S}_\odot}{\mathcal{S}_\odot \approx \sqrt{(\cos 2\theta_{12} - c_{13}^2 a / \Delta m_{21}^2)^2 + \sin^2 2\theta_{12}} \approx 3.4}$$

at $E = 1.3$ GeV
PBD, S. Parke [1902.07185](#)

- ▶ Mixing angle is modified

$$\cos 2\theta_{12} = 0.37 \rightarrow \frac{\cos 2\theta_{12} - c_{13}^2 a / \Delta m_{21}^2}{\mathcal{S}_\odot} \approx -0.96 < 0$$

$$a \propto \rho E$$

Matter effects matter: (21) sector

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$$\sin 2\theta_{12} \cos 2\theta_{12} = 0.35 \rightarrow -0.26$$

- ▶ Also s_{13} increases in matter $\sim 15\%$: total effect is $0.004 \cos \delta$
- ▶ This gets us **half** of the effect, and the correct sign

Matter effects matter: (32) sector

- ▶ $\frac{\Delta m_{\mu\mu}^2 L}{4E}$ in matter at the maximum is $\sim \pi$

H. Nunokawa, S. Parke, R. Funchal [hep-ph/0503283](#)
PBD, S. Parke [2401.10326](#)

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PBD, S. Parke [2401.10326](#)

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- ▶ The Δm_{32}^2 component is a bit off π at max
- ▶ Leading order in s_{13} :

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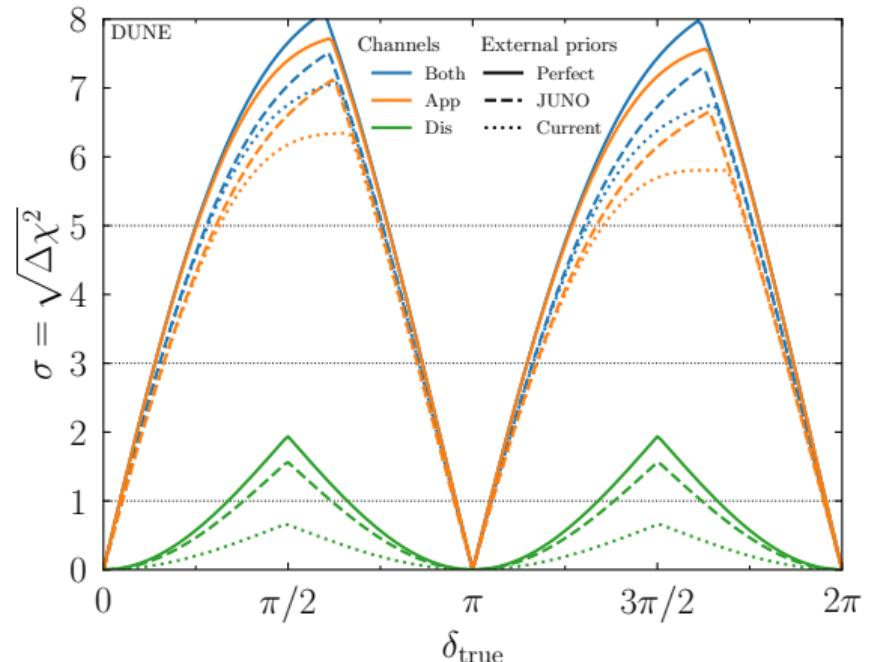
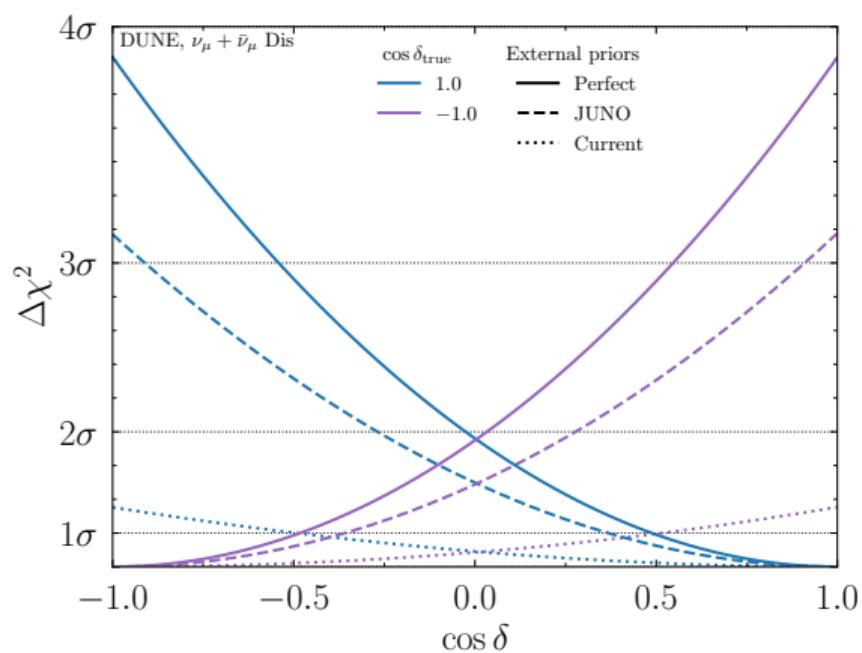
- ▶ Adds in another $\approx 0.004\cos\delta$ effect
- ▶ Total is $\approx 0.008\cos\delta$ which agrees with exact calculation

Numerical Studies

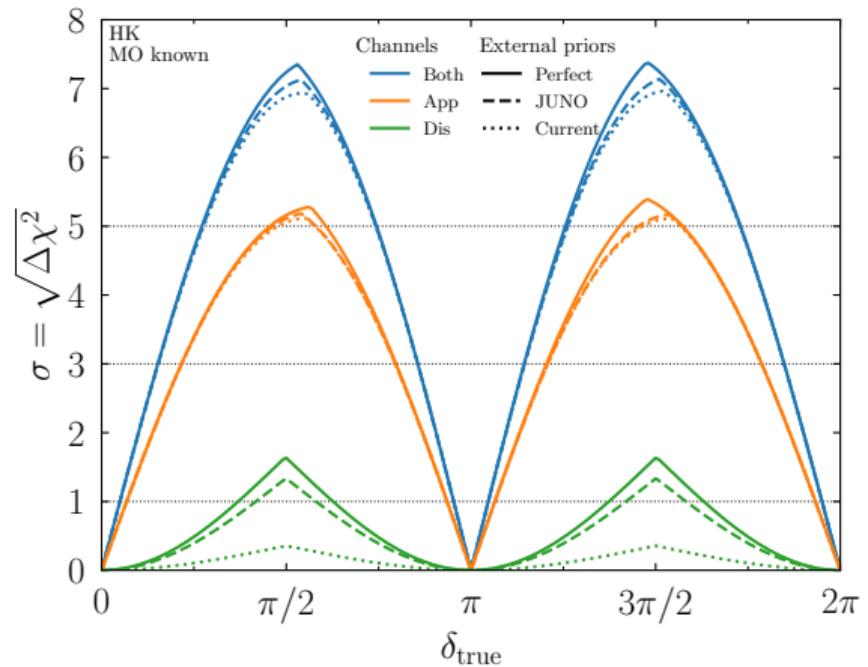
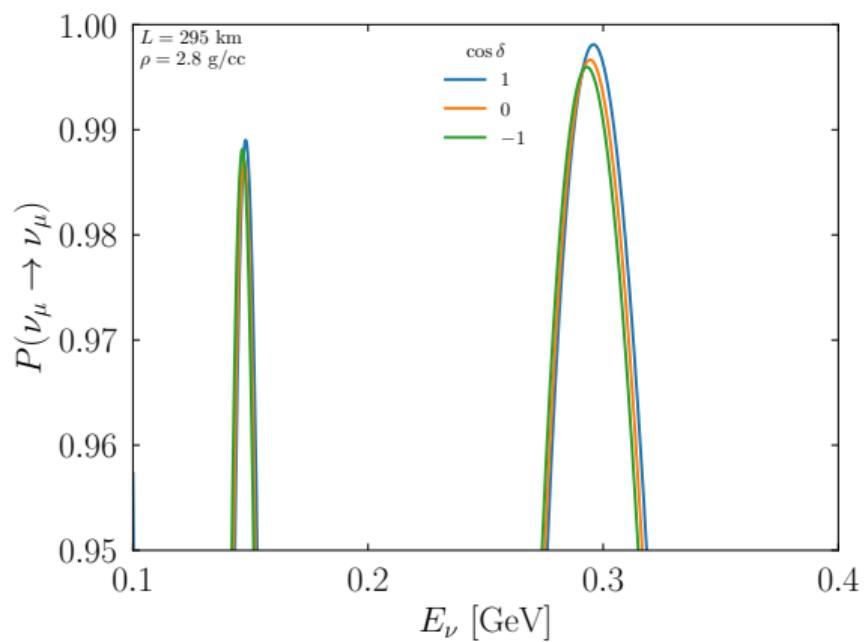
Inputs are *only*:

- ▶ Daya Bay data for θ_{13} [1809.02261](#)
- ▶ KamLAND data for θ_{12} and Δm_{21}^2 [1303.4667](#)
- ▶ JUNO 6 yrs precision sensitivity on θ_{12} , Δm_{21}^2 , Δm_{31}^2 [2204.13249](#)
- ▶ DUNE 6.5+6.5 yrs disappearance channels sensitivity only [2103.04797](#)

JUNO and DUNE disappearance Sensitivities

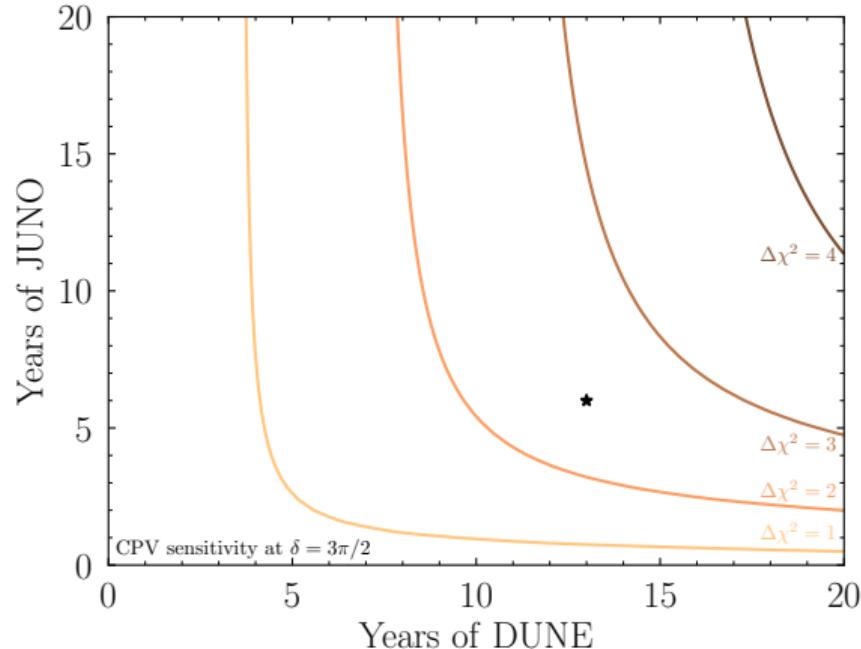


JUNO and HK disappearance Sensitivities



Varying Runtime/Power

Significance to disfavor $|\cos \delta| = 1$ at $\cos \delta = 0$



Improvement requires **both** experiments!

NuFast

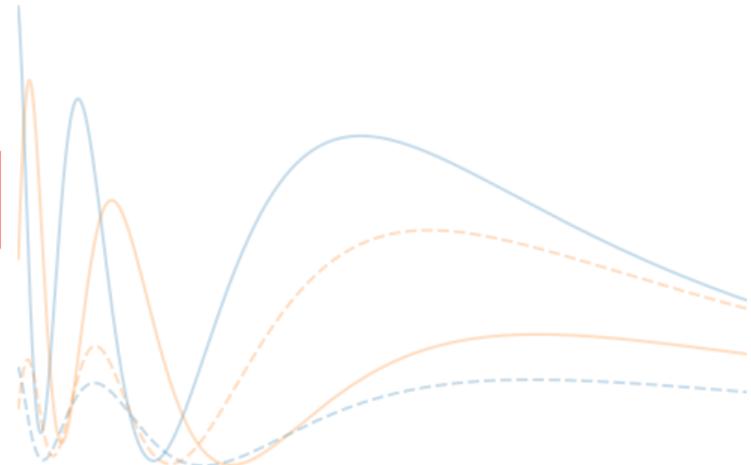
A fast code for long-baseline
neutrino oscillation probabilities in matter

[2405.02400](#) with S. Parke

The problem

$\Delta m_{21}^2, \Delta m_{31}^2$
 $s_{23}^2, s_{13}^2, s_{12}^2$
 δ

\Rightarrow
in matter



Many approaches

Solve the Schrödinger equation

$$i\frac{d}{dt}|\nu\rangle = H(t)|\nu\rangle$$

If $H(t) = H$ (constant density)

$$\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta) = [e^{-iHL}]_{\beta\alpha} \quad P = |\mathcal{A}|^2$$

Exponential requires computing eigenvalues and eigenvectors of H

Many approaches

Modify vacuum probabilities

- ▶ Get the eigenvalues by solving the cubic

Cardano 1545
V. Barger, et al. PRD 22 (1980) 2718

- ▶ Get the eigenvectors

H. Zaglauer, K. Schwarzer Z.Phys. C40 (1988) 273
K. Kimura, A. Takamura, H. Yokomakura [hep-ph/0205295](#)
[PBD](#), S. Parke, X. Zhang [1907.02534](#)
A. Abdulahi, S. Parke [2212.12565](#)

Other approaches?
Are approximations useful?
Optimal hybrids?
What is the goal?

Fermilab computing experts bolster NOvA evidence, 1 million cores consumed

July 3, 2018 | Marcia Teckenbrock

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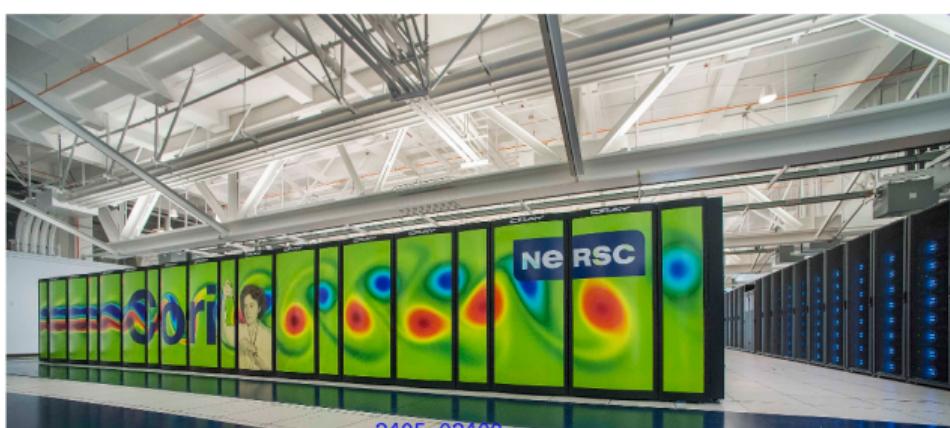
Array

How do you arrive at the physical laws of the universe when you're given experimental data on a renegade particle that interacts so rarely with matter, it can cruise through light-years of lead? You call on the power of advanced computing.

The NOvA neutrino experiment, in collaboration with the Department of Energy's Scientific Discovery through Advanced Computing (SciDAC-4) program and the HEPCloud program at DOE's Fermi National Accelerator Laboratory, was able to perform the largest-scale analysis ever to support the [recent evidence of antineutrino oscillation](#), a phenomenon that may hold clues to how our universe evolved.

Using Cori, the newest supercomputer at the [National Energy Research Scientific Computing Center \(NERSC\)](#), located at Lawrence Berkeley National Laboratory, NOvA used over 1 million computing cores, or CPUs, between May 14 and 15 and over a short timeframe one week later. This is the largest number of CPUs ever used concurrently over this duration — about 54 hours — for a single high-energy physics experiment. This unprecedented amount of computing enabled scientists to carry out some of the most complicated techniques used in neutrino physics, allowing them to dig deeper into the seldom seen interactions of neutrinos. This Cori allocation was more than 400 times the amount of Fermilab computing allocated to the NOvA experiment and 50 times the total computing capacity at Fermilab allocated for all of its rare-physics experiments. A continuation of the analysis was performed on NERSC's Cori and Edison supercomputers one week later. In total, [nearly 35 million core-hours were consumed by NOvA](#) in the 54-hour period. Executing the same analysis on a single desktop computer would take 4,000 years.

[FNAL Newsroom](#)



2405.02400

Monte-Carlo estimates of statistical significances

Wilks' theorem is often wrong

At each point in parameter space, simulate the experiment many times

“many” means $\gg 1/p$ for a desired p -value

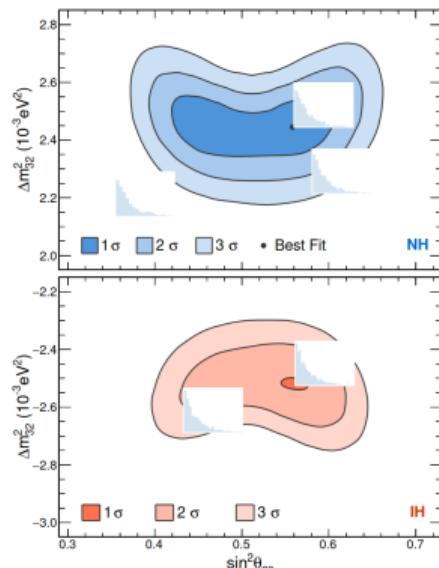
This is sometimes called Feldman-Cousins

G. Feldman, R. Cousins [physics/9711021](#)
This isn't actually what was novel in the FC paper

Study found most of
the time was spent
computing probabilities

NOvA/T2K are $\sim 3\sigma$ experiments,
but DUNE/HK will be $\gtrsim 5\sigma$ experiments!

DUNE sensitivities require computing
the probabilities “a zillion times”



Start at the end

What is needed for experiments?

1. All 9 channels ($\nu_\alpha \rightarrow \nu_\beta$)

- ▶ DUNE will certainly do ν_τ appearance

See e.g. P. Machado, H. Schulz, J. Turner [2007.00015](#)

- ▶ $\nu_\tau \rightarrow \nu_\beta$ channels are not needed, but come from free from unitarity
- ▶ JUNO only needs $\nu_e \rightarrow \nu_e$

2. Different energies, baselines, and densities

3. ν and $\bar{\nu}$

4. NO and IO

5. Oscillation parameters are mostly known

- ▶ Don't need to consider e.g. $\Delta m_{21}^2 > |\Delta m_{31}^2|$ or $\theta_{23} \sim 10^\circ$

How to achieve speed

1. Avoid costly operations

- ▶ `sin`, `cos` (and inverse functions) are very slow
- ▶ `sqrt` is quite slow, but not as bad as trigs
- ▶ Division is slower than multiplication ($0.2x$ may be faster than $x/5$)

2. Reduce repeated calculations

- ▶ Compute $\frac{L}{4E}$ in the correct units once
- ▶ Compute each of the three $\sin \frac{\Delta m_{ij}^2 L}{4E}$ once

All of these are compiler dependent

Optimal structure of the probability

1. Amplitude requires four trig functions of kinematic variables ($\Delta m_{ij}^2 L / 4E$) ✗
2. Writing the probabilities out requires three trig functions ✓
3. Disappearance structure is straightforward:

$$P_{\alpha\alpha} = 1 - 4 \sum_{i>j} |V_{\alpha i}|^2 |V_{\alpha j}|^2 \sin^2 \frac{\Delta \lambda_{ij} L}{4E}$$

H in matter has eigenvalues λ_i and eigenvectors $V_{\alpha i}$

Optimal structure of the probability

4. Appearance structure:

$$P_{\mu e}^{CP+} = 2 \sum_{i>j} (|V_{\tau k}|^2 - |V_{\mu i}|^2 |V_{ej}|^2 - |V_{\mu j}|^2 |V_{ei}|^2) \sin^2 \frac{\Delta \lambda_{ij} L}{4E}$$

Fun fact:

$$\begin{aligned} & 2\Re(V_{\alpha i} V_{\beta j}^* V_{\alpha j}^* V_{\beta i}) \\ &= |V_{\alpha k}|^2 |V_{\beta k}|^2 - |V_{\alpha i}|^2 |V_{\beta i}|^2 - |V_{\alpha j}|^2 |V_{\beta j}|^2 \\ &= |V_{\gamma k}|^2 - |V_{\alpha i}|^2 |V_{\beta j}|^2 - |V_{\alpha j}|^2 |V_{\beta i}|^2 \end{aligned}$$

$$P_{\mu e}^{CP-} = -8J \frac{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2}{\Delta \lambda_{21} \Delta \lambda_{31} \Delta \lambda_{32}} \sin \frac{\Delta \lambda_{21} L}{4E} \sin \frac{\Delta \lambda_{31} L}{4E} \sin \frac{\Delta \lambda_{32} L}{4E}$$

Leverages NHS identity:
V. Naumov [IJMP 1992](#)
P. Harrison, W. Scott [hep-ph/9912435](#)

Note that CP even/odd is actually T even/odd due to matter

Account for matter

1. Need the eigenvalues λ_i
2. For eigenvectors, naively need $\Re(V_{\alpha i} V_{\beta j}^* V_{\alpha j}^* V_{\beta i})$
3. Given our form, need only the $|V_{\alpha i}|^2$ and J
 - ▶ Don't need any phase information of the eigenvectors!

Leverages PBD, S. Parke, X. Zhang [1907.02534](#)

4. Can compute the $|V_{\alpha i}|^2$ from the λ_i and submatrix eigenvalues (requires only a square root) using Eigenvector-Eigenvalue Identity

$$|V_{\alpha i}|^2 = \frac{\prod_{k=1}^{n-1} (\lambda_i - \xi_k^\alpha)}{\prod_{k=1; k \neq i}^n (\lambda_i - \lambda_k)}$$

See e.g. PBD, S. Parke, T. Tao, X. Zhang [1908.03795](#)
Can actually avoid the $\sqrt{-}$ in practice

Eigenvalues are hard

The eigenvalues in matter λ_i depend on S :

$$S = \cos \left\{ \frac{1}{3} \cos^{-1} \left[\frac{2A^3 - 9AB + 27C}{2(A^2 - 3B)^{3/2}} \right] \right\}$$

where

$$A = \sum \lambda_i = \Delta m_{21}^2 + \Delta m_{31}^2 + a$$

$$B = \sum_{i>j} \lambda_i \lambda_j = \Delta m_{21}^2 \Delta m_{31}^2 + a[\Delta m_{21}^2(1 - |U_{e2}|^2) + \Delta m_{31}^2(1 - |U_{e3}|^2)]$$

$$C = \prod \lambda_i = a \Delta m_{21}^2 \Delta m_{31}^2 |U_{e1}|^2$$

Approximate eigenvalues

1. Instead, approximate one eigenvalue
 - ▶ λ_3 is best because is never parametrically small and easy to approximate
2. From DMP:

$$\lambda_3 \approx \Delta m_{31}^2 + \frac{1}{2} \Delta m_{ee}^2 \left(x - 1 + \sqrt{(1-x)^2 + 4x s_{13}^2} \right)$$

$$x \equiv \frac{a}{\Delta m_{ee}^2} \quad \Delta m_{ee}^2 \equiv \Delta m_{31}^2 - s_{12}^2 \Delta m_{21}^2$$

H. Minakata, S. Parke [1505.01826](#)
PBD, H. Minakata, S. Parke [1604.08167](#)
H. Nunokawa, S. Parke, R. Funchal [hep-ph/0503283](#)

3. Get other two eigenvalues by picking two of A, B, C conditions

Requires one more $\sqrt{}$

The approximation

- ▶ This is the only approximation used in the entire approach
- ▶ In vacuum the approximation returns to the correct value

Many approximations in the literature are not correct in vacuum limit

See G. Barenboim, PBD, S. Parke, C. Ternes [1902.00517](#)

- ▶ In fact can iteratively improve λ_3 with rapid convergence via Newton's method:

$$\lambda_3 \rightarrow \lambda_3 - \frac{X(\lambda_3)}{X'(\lambda_3)}$$

$$X(\lambda) = \lambda^3 - A\lambda^2 + B\lambda - C = 0$$

- ▶ Precision improvement starts at 10^{-5} for the first step
- ▶ The improvement is quadratic thereafter
- ▶ One line of code, just loop as many times as desired

All 9 channels

Given P_{ee} , $P_{\mu\mu}$, $P_{\mu e}^{CP+}$, and $P_{\mu e}^{CP-}$:

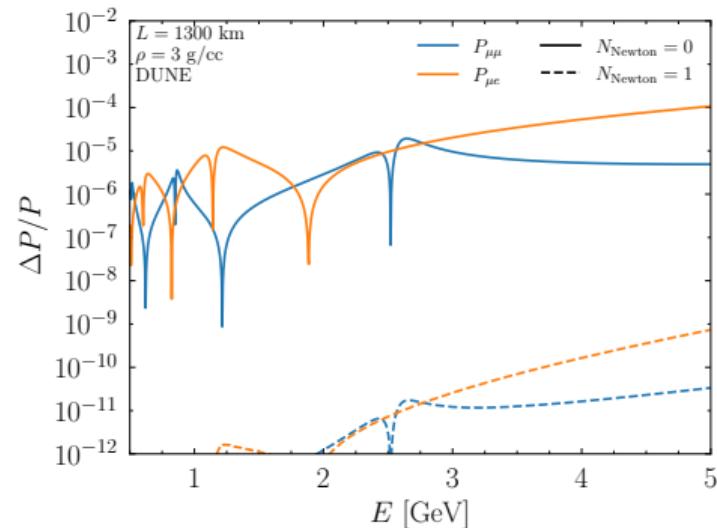
	$P_{\alpha e}$	$P_{\alpha \mu}$	$P_{\alpha \tau}$
$P_{e\beta}$	P_{ee}	$P_{\mu e}^{CP+} - P_{\mu e}^{CP-}$	$1 - P_{ee} - P_{\mu e}^{CP+} + P_{\mu e}^{CP-}$
$P_{\mu\beta}$	$P_{\mu e}^{CP+} + P_{\mu e}^{CP-}$	$P_{\mu\mu}$	$1 - P_{\mu\mu} - P_{\mu e}^{CP+} - P_{\mu e}^{CP-}$
$P_{\tau\beta}$	$1 - P_{ee} - P_{\mu e}^{CP+} - P_{\mu e}^{CP-}$	$1 - P_{\mu\mu} - P_{\mu e}^{CP+} + P_{\mu e}^{CP-}$	$-1 + P_{ee} + P_{\mu\mu} + 2P_{\mu e}^{CP+}$

Total approach

1. Inputs: 6 oscillation parameters, experimental details (L, E, ρ, Y_e)
2. Calculate λ_3 approximately
 - ▶ Iteratively improve with Newton's method, if desired
3. Calculate the $|V_{\alpha i}|^2$'s with the Eigenvector-Eigenvalue Identity
4. Calculate the sines of the kinematic terms
5. Calculate the CP odd term with the NHS identity
6. Calculate key probabilities: P_{ee} , $P_{\mu\mu}$, and $P_{\mu e}$
7. Calculate remaining probabilities

Precision

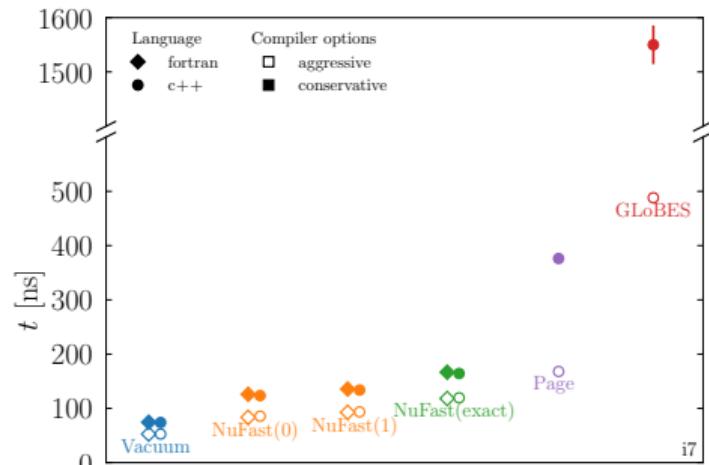
Is this approximation okay?
DUNE requires $\lesssim 1\%$ level precision



Slightly better for HK
 $\sim 10^{-10}$ for JUNO
See backups

Speed

Is this algorithm fast?
How does it compare?



- ▶ “Conservative” = default, -O0
- ▶ “Aggressive” = -Ofast and -ffast-math
- ▶ Some variation expected due to architecture

See also J. Page [2309.06900](#)
and P. Huber, et al. [hep-ph/0701187](#)

NuFast: the code

- ▶ Code is on github: github.com/PeterDenton/NuFast
- ▶ Implementations in c++ and f90
- ▶ Easy to use and there are comments!

```
// NuFast.cpp
// Calculate the probabilities:
Probability_Matter_LBL(s12sq, s13sq, s23sq, delta,
    Dmsq21, Dmsq31, L, E, rho, Ye, N_Newton,
    &probs_returned);
// Print out the probabilities to terminal
for (int alpha = 0; alpha < 3; alpha++)
{
    for (int beta = 0; beta < 3; beta++)
    {
        printf("%d %d %g\n", alpha, beta,
            probs_returned[alpha][beta]);
    } // beta, 3
} // alpha, 3
```

- ▶ $\bar{\nu}$: $E < 0$; IO: $\Delta m_{31}^2 < 0$
- ▶ Folder called **Benchmarks** to make the plots and speed tests in the paper

Other talks at CETUP to Look Out For

TALK: Dark Matter Raining on DUNE and Other Large Volume Detectors

⌚ 45m

Authors: Javier Acevedo, Joshua Berger, Peter Denton

Direct detection is a powerful means of searching for particle physics evidence of dark matter (DM) heavier than about a GeV with volume, low-threshold detectors.

In many scenarios, some fraction of the DM may be boosted to large velocities enhancing and generally modifying possible detection signatures. We investigate the scenario where 100% of the DM may be boosted at the Earth due to new attractive long-range forces. This opens up two main improvements in detection capabilities: 1) the detection signatures are stronger opening up large-volume neutrino detectors, such as DUNE, Super-K, Hyper-K, and JUNO, as possible DM detectors, and 2) the large boost allows for detectable signatures of sub-GeV DM. At lower boosts, a modified, higher-than-usual energy signal could be accessible at direct detection experiments such as LZ. In addition, the model leads to a significant anisotropy in the signal with the DM flowing dominantly vertically at the Earth's surface instead of the typical approximately isotropic DM signal. We develop the theory behind this model and also calculate realistic constraints using a detailed GENIE simulation of the signal inside detectors.

Speaker: Joshua Berger (Colorado State University)

TALK: A Modern Look at the Oscillation Physics Case for a Neutrino Factory

Author: Julia Gehrlein

The next generation of neutrino oscillation experiments, JUNO, DUNE, and HK, are under construction now and will collect data over the next decade and beyond. As there are no approved plans to follow up this program with more advanced neutrino oscillation experiments, we consider here one option that had gained considerable interest more than a decade ago: a neutrino factory. Such an experiment uses stored muons in a racetrack configuration with extremely well characterized decays reducing systematic uncertainties and providing for more oscillation channels. Such a machine could also be one step towards a high energy muon collider program. We consider a long-baseline configuration to SURF using the DUNE far detectors or modifications thereof, and compare the expected sensitivities to the three-flavor oscillation parameters to the anticipated results from DUNE and HK. We find that a neutrino factory can improve our understanding of CP violation and also aid in disentangling the complicated flavor puzzle.

Speaker: Julia Gehrlein (Colorado State University)

Discussion and Conclusions

- ▶ **Disappearance can discover CPV**
- ▶ Requires two good measurements: JUNO and DUNE/HK
- ▶ Can rule out some values of δ at $> 3\sigma$
- ▶ Works in vacuum or matter; matter slightly minimizes HK's effect
- ▶ Subject to BSM degeneracies, as are most other oscillation measurements
- ▶ Analyses already exist but...

Discussion and Conclusions

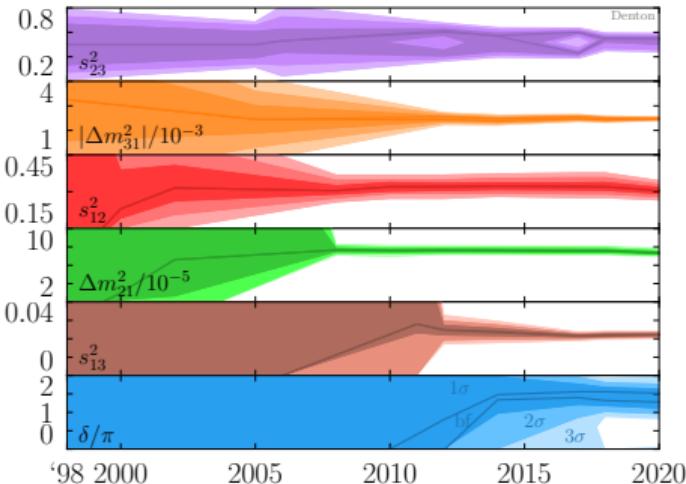
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- ▶ Since systematics are different, provides a good cross check

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- ▶ **LBL Experiments should break down δ analyses into app vs. dis**
- ▶ Since systematics are different, provides a good cross check
- ▶ NuFast for fast oscillation probabilities
- ▶ Atmospherics and nighttime solar in the works!

Backups

References



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D. Forero, M. Tortola, J. Valle [1205.4018](#)

D. Forero, M. Tortola, J. Valle [1405.7540](#)

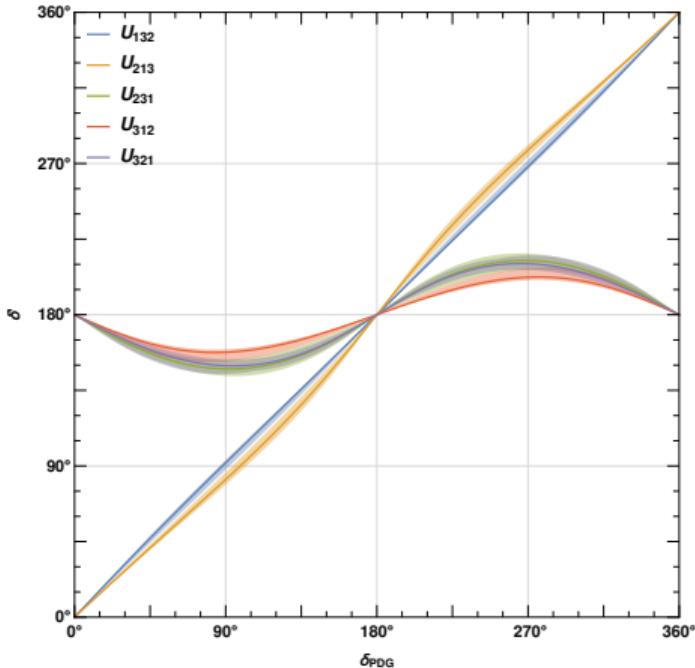
P. de Salas, et al. [1708.01186](#)

F. Capozzi et al. [2003.08511](#)

Complex phase in different parameterizations

- ▶ Can relate the complex phase in one parameterization to that in another
- ▶ U_{132} and U_{213} similar to U_{123}
- ▶ δ constrained to $\sim [150^\circ, 210^\circ]$ in U_{231} , U_{312} , U_{321}
- ▶ Bands indicate 3σ uncertainty on θ_{12} , θ_{13} , θ_{23}
- ▶ “50% of possible values of δ ”
⇒ parameterization dependent

DUNE TDR II [2002.03005](#)



Quark mixing

From the PDG, V_{CKM} in the V_{123} parameterization is

$$\theta_{12} = 13.09^\circ \quad \theta_{13} = 0.2068^\circ \quad \theta_{23} = 2.323^\circ \quad \delta_{\text{PDG}} = 68.53^\circ$$

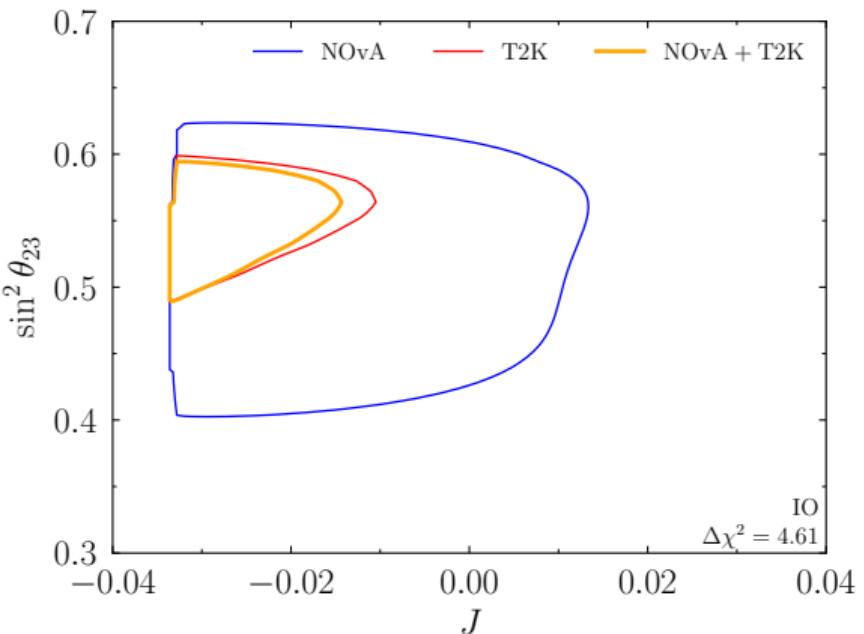
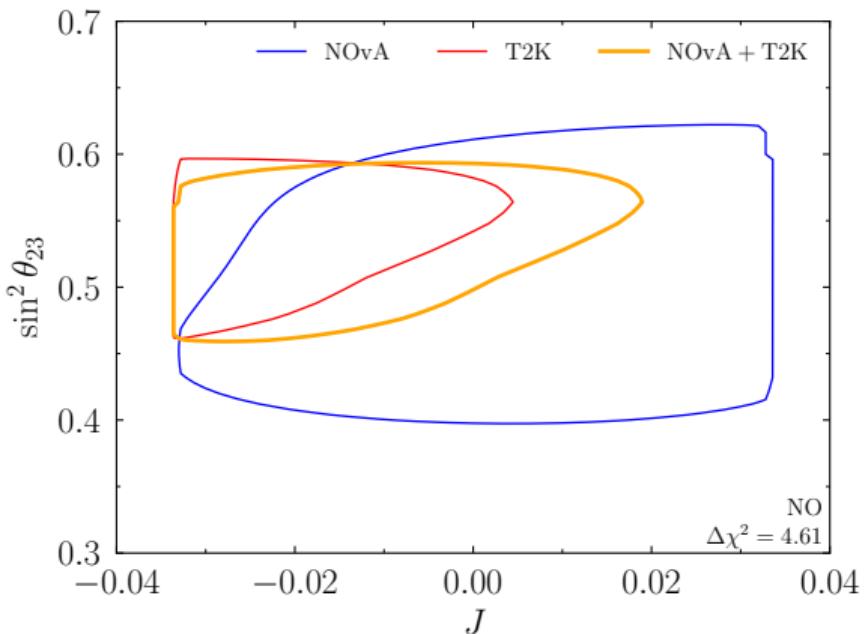
Looks like “large” CPV:

$$\sin \delta_{\text{PDG}} = 0.93 \sim 1$$

yet $J_{\text{CKM}}/J_{\text{max}} = 3 \times 10^{-4}$.

Switch to V_{212} parameterization, $\Rightarrow \delta' = 1^\circ$ and $\sin \delta' = 0.02$.

Standard oscillation parameters



Can see that the combination doesn't like the NO while it does like the IO
IO preferred over NO at $\Delta\chi^2 = 2.3$

CP violation in oscillations

In vacuum at first maximum:

$$P_{\mu e} - \bar{P}_{\mu e} \approx 8\pi J \frac{\Delta m_{21}^2}{\Delta m_{32}^2}$$

$$J \equiv s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23} \sin \delta$$

C. Jarlskog [PRL 55, 1039 \(1985\)](#)

- ▶ Extracting δ from data requires every other oscillation parameter
- ▶ J requires only Δm_{21}^2 (up to matter effects)

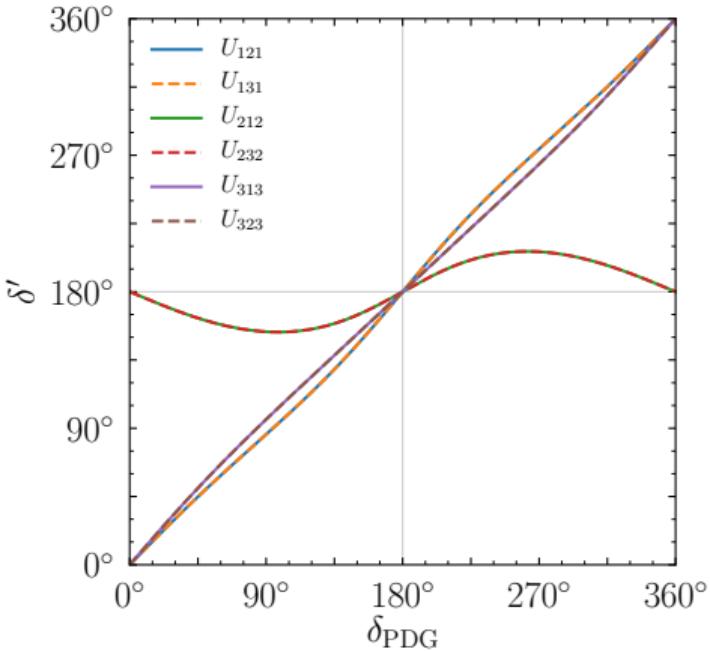
Matter effects are easily accounted for

$$\hat{J} \simeq \frac{J}{\sqrt{(c_{212} - c_{13}^2 a / \Delta m_{21}^2)^2 + s_{212}^2} \sqrt{(c_{213} - a / \Delta m_{ee}^2)^2 + s_{213}^2}}$$

[PBD](#), S. Parke [1902.07185](#)

[PBD](#), H. Minakata, S. Parke [1604.08167](#)

Repeated rotations

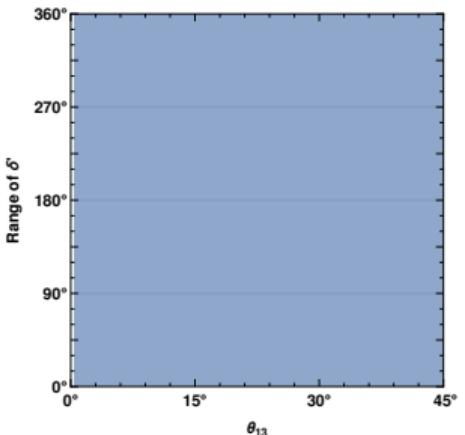


	U_{121}	U_{131}	U_{212}	U_{232}	U_{313}	U_{323}
$ U_{e2} $	✓	✓	✓	✓	✗	✗
$ U_{e3} $	✓	✓	✗	✗	✓	✓
$ U_{\mu 3} $	✗	✗	✓	✓	✓	✓

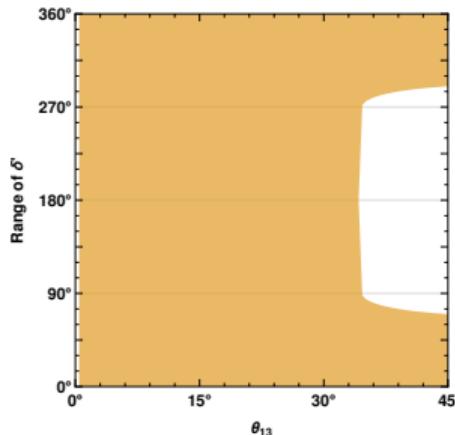
Note that $e^{i\delta}$ must be on first or third rotation

Allowed δ' range

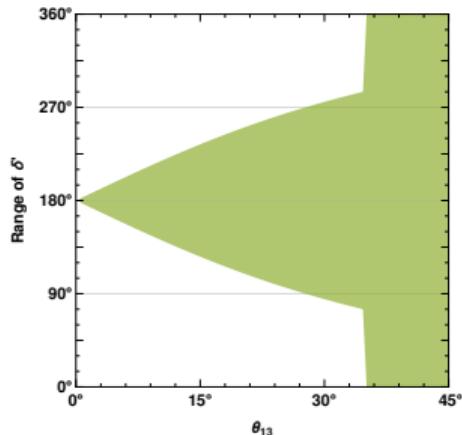
U_{132}



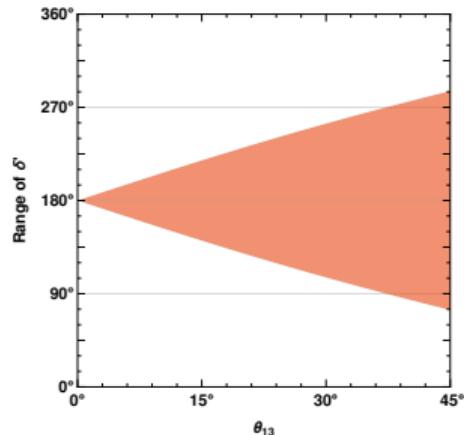
U_{213}



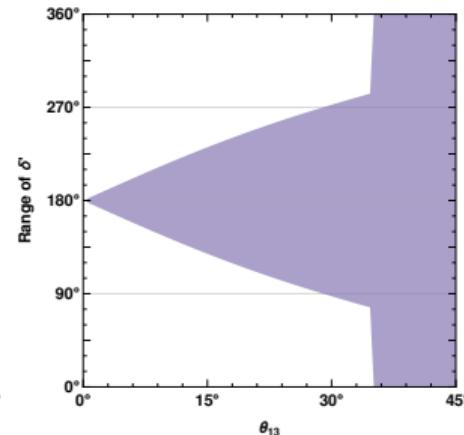
U_{231}



U_{312}



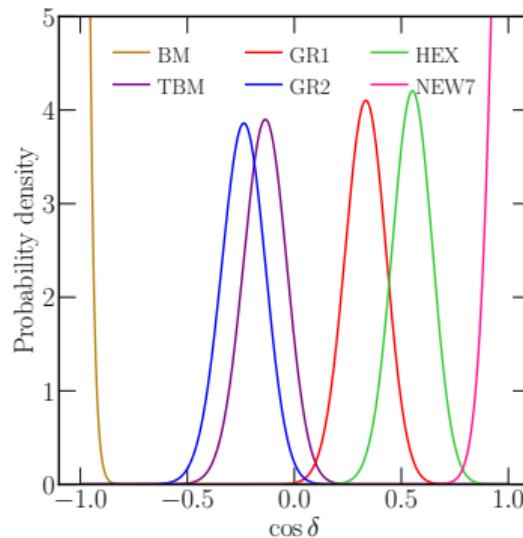
U_{321}



The importance of $\cos \delta$

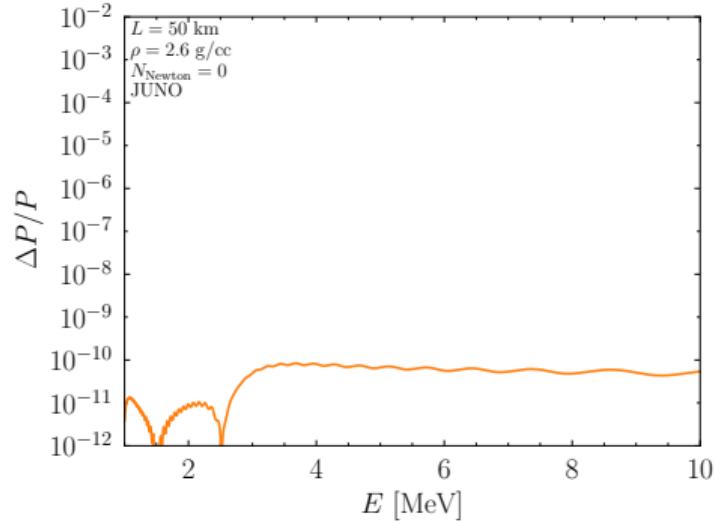
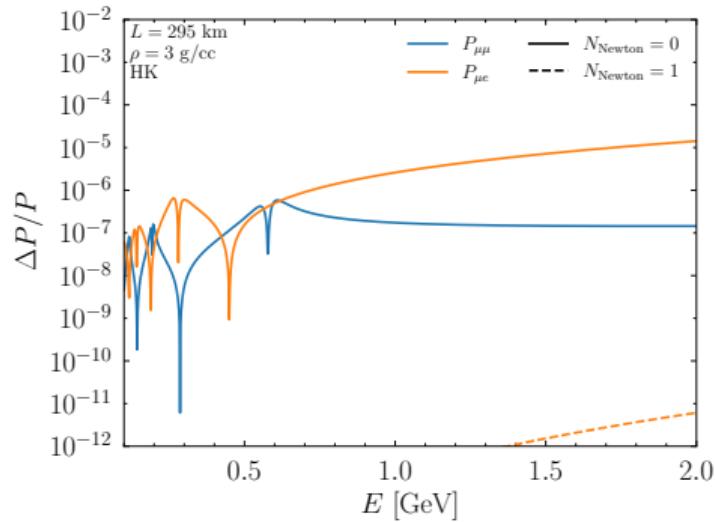
- ▶ If only $\sin \delta$ is measured \Rightarrow sign degeneracy: $\cos \delta = \pm \sqrt{1 - \sin^2 \delta}$
- ▶ Most flavor models predict $\cos \delta$

J. Gehrlein, et al. [2203.06219](#)

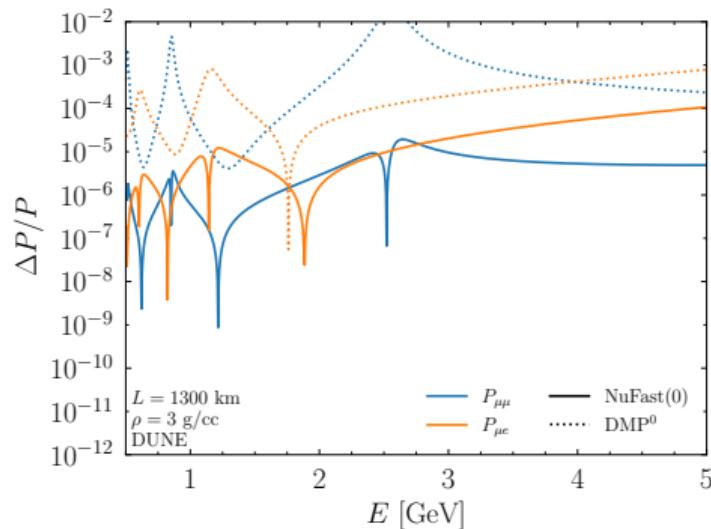


L. Everett, et al. [1912.10139](#)

Precision



Comparison to DMP



DMP: [PBD](#), H. Minakata, S. Parke [1604.08167](#)