Spherical Harmonics as a Tool for Finding Anisotropies in UHECR and Astrophysical Neutrino Fluxes

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DAPMe

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The Niels Bohr International Academy

VILLUM FONDEN

VANDERBILT

UHECR Anisotropy Unknowns

- How strong are the magnetic fields inside and between galaxies?

  Pshirkov et. al.: 1103.0814

  Jansson, Farrar: 1204.3662

- What is the composition of UHECRs? Protons? Iron nuclei?
- What is(are) the source(s) of UHECRs?
- How are UHECRs accelerated to such extreme energies?

  Gunn, Ostriker: PRL 22 (1969)

  Pruet, Guiles, Fuller: astro-ph/0205056

  Groves, Heckman, Kauffmann: astro-ph/0607311

  Fang, Kotera, Olinto: 1201.5197

- What is the cause of the suppression at the end of the spectrum?
- New physics...?

UHECR Anisotropy Knowns

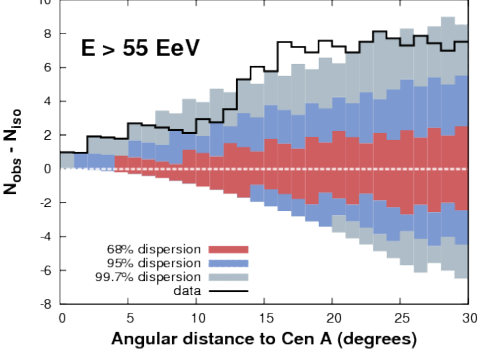
- The magnetic field in the Milky Way cannot contain UHECRs.
- UHECRs with energies above ~ 50 EeV lose energy via the CMB.

Greisen: PRL 16 (1966)

Zatsepin, Kuzmin: JETP Lett. 4 (1966)

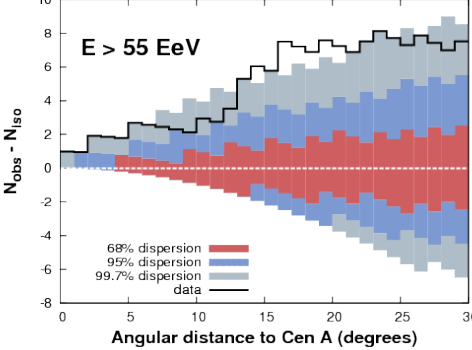
- UHECR sources must be close $\Rightarrow$ anisotropies.
- UHECRs bend in galactic and extragalactic magnetic fields.
- No conclusive anisotropies found yet.

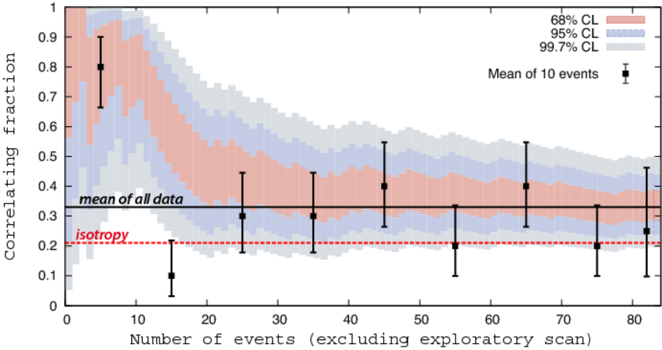
UHECR Anisotropy Searches



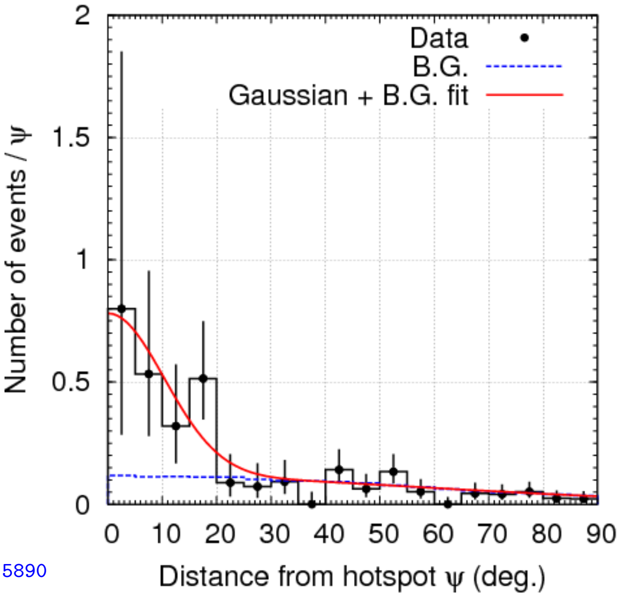
Auger: 1106.3048

UHECR Anisotropy Searches





Auger: 1207.4823



Data
B.G.
Gaussian + B.G. fit
Number of events / ψ
Distance from hotspot ψ (deg.)
0
0.5
1
1.5
2
0
10
20
30
40
50
60
70
80
90

TA: 1404.5890

Spherical Harmonics: Distributions on the Sky

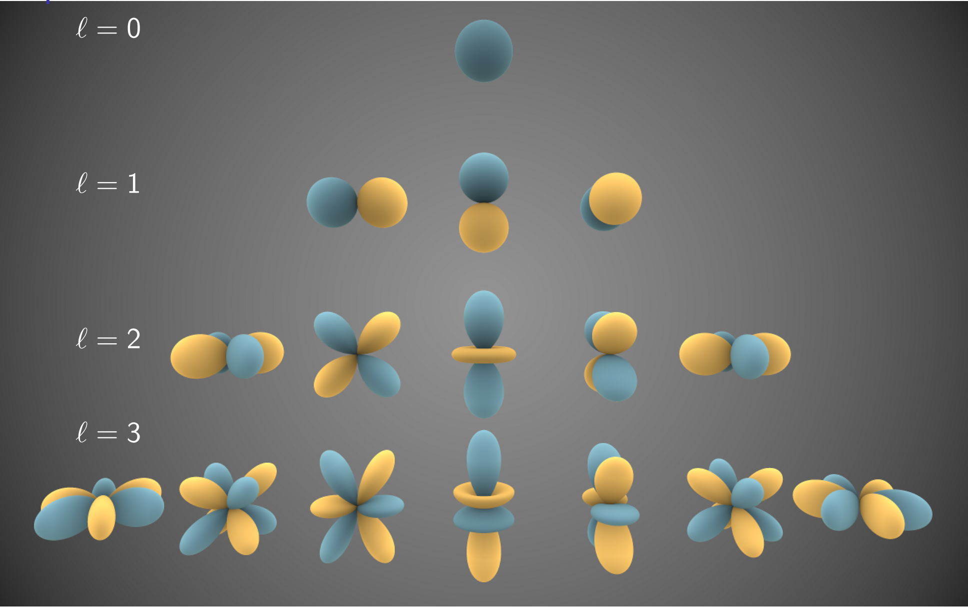
- Y_ℓ^m's provide an orthogonal expansion of any distribution on the sky.
- Useful in low statistics, high uncertainty experiments:
  - UHECR's: unknown angular uncertainties from magnetic fields.
  - IC's HESE: dominated by cascades with $\mathcal{O}(10^\circ)$ uncertainties.
- The true distribution as seen at earth:

$$I(\Omega) = \sum_{\ell,m} a_\ell^m Y_\ell^m(\Omega)\,.$$

- The power spectrum is rotational invariant.

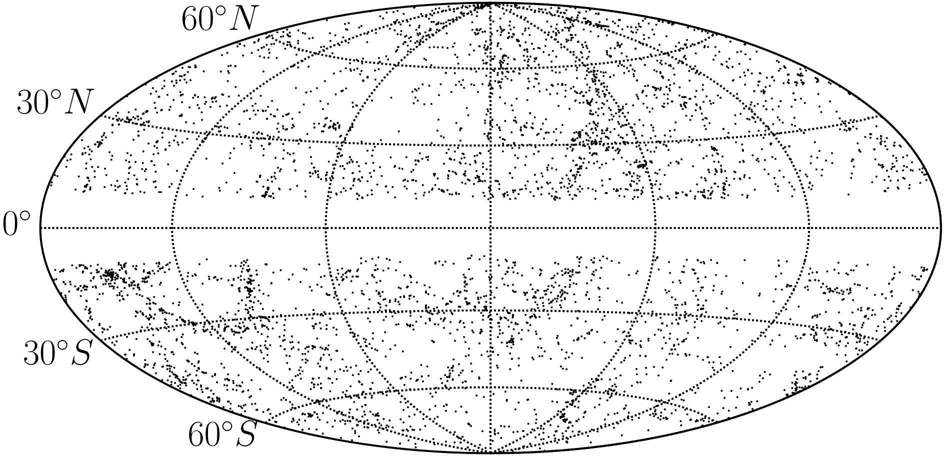
$$C_\ell = \frac{1}{2\ell+1}\sum_m |a_\ell^m|^2\,.$$

Spherical Harmonics Visualizations



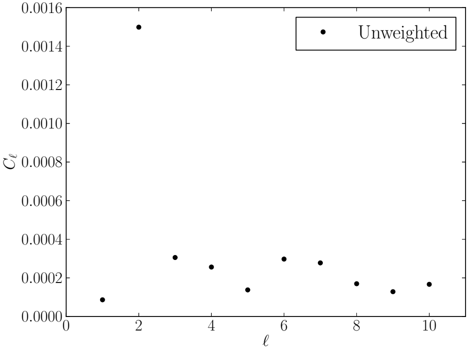
en.wikipedia.org/wiki/Spherical_harmonics

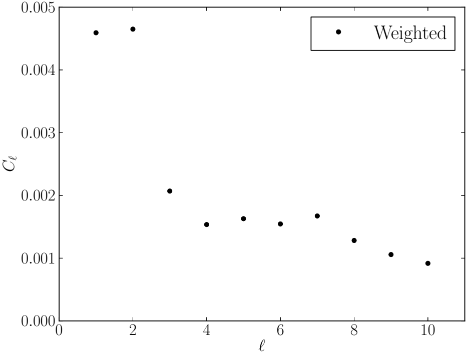
2MRS Sky Map



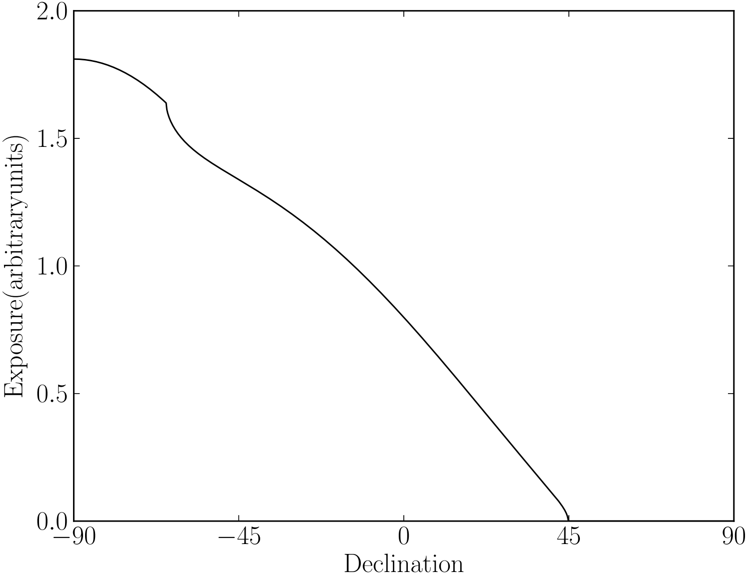
2MRS: 1108.0669

Spherical Harmonics: Possible Sources





Auger's Nonuniform Partial Sky Coverage



Reconstructing a_ℓ^m's for Nonuniform Partial Sky Coverage

Nonuniform exposure is a manageable problem:

$$\bar{a}_\ell^m = \frac{1}{N}\sum_i^N Y_\ell^{m*}(\Omega_i) \to \frac{1}{\mathcal{N}}\sum_i^N \frac{Y_\ell^{m*}(\Omega_i)}{\omega(\Omega_i)}\,,$$

where $\mathcal{N} = \sum_i^N \frac{1}{\omega(\Omega_i)}$,
ω is the exposure function.

Sommers: astro-ph/0004016

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where $\mathcal{N} = \sum_i^N \frac{1}{\omega(\Omega_i)}$,
ω is the exposure function.

Sommers: astro-ph/0004016

Partial sky is more challenging: no information from part of the sky.

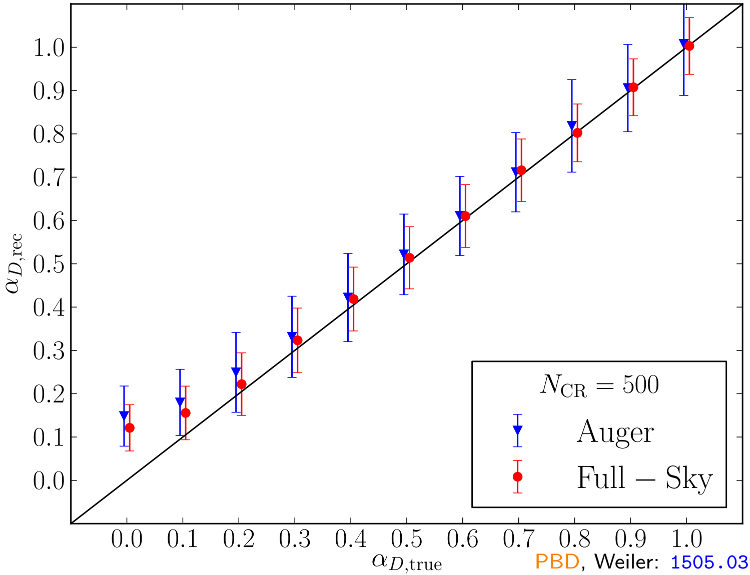
$$[K]_{\ell m}^{\ell' m'} \equiv \int d\Omega\, \omega(\Omega) Y_\ell^m(\Omega) Y_{\ell'}^{m'}(\Omega)$$

$$b_\ell^m = \sum_{\ell' m'} [K]_{\ell m}^{\ell' m'} a_{\ell'}^{m'} \quad \Rightarrow \quad a_\ell^m = \sum_{\ell' m'}^{\ell_{\max}} [K^{-1}]_{\ell m}^{\ell' m'} b_{\ell'}^{m'}$$

$b_\ell^m \to$ uncorrected (observed on earth),
$a_\ell^m \to$ nature's true anisotropy.

Billoir, Deligny: 0710.2290

Dipole Reconstruction Effectiveness



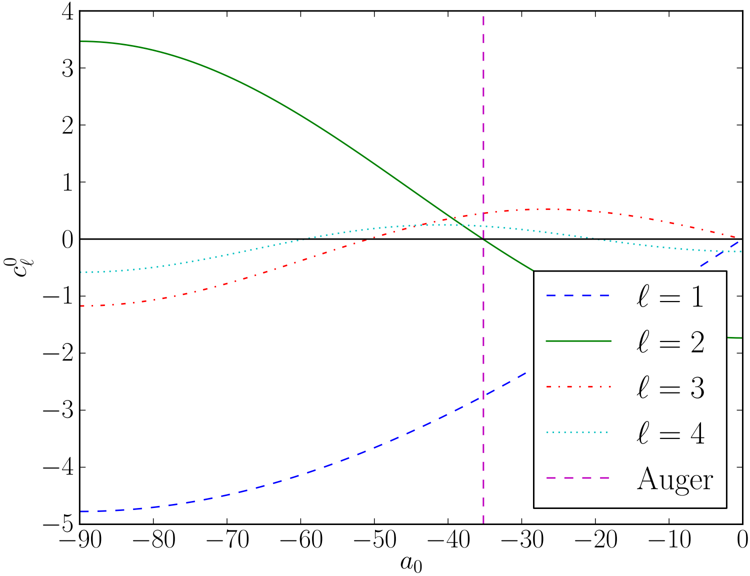
PBD, Weiler: 1505.03922

Reconstructing a_ℓ^m's for Nonuniform Partial Sky Coverage

- An alternative formalism to the K-matrix approach.
- Expand the exposure $\omega(\Omega)=\sum_{\ell,m}c_\ell^mY_\ell^m(\Omega)$.
- ω does not depend on RA $\Rightarrow$ only $m=0$ coefficients are nonzero.
- Fortuitously, $c_2^0=0$ for Auger's exposure (nearly equal to zero for Telescope Array).

PBD, Weiler: 1409.0883

Quadrupole Component of Exposure



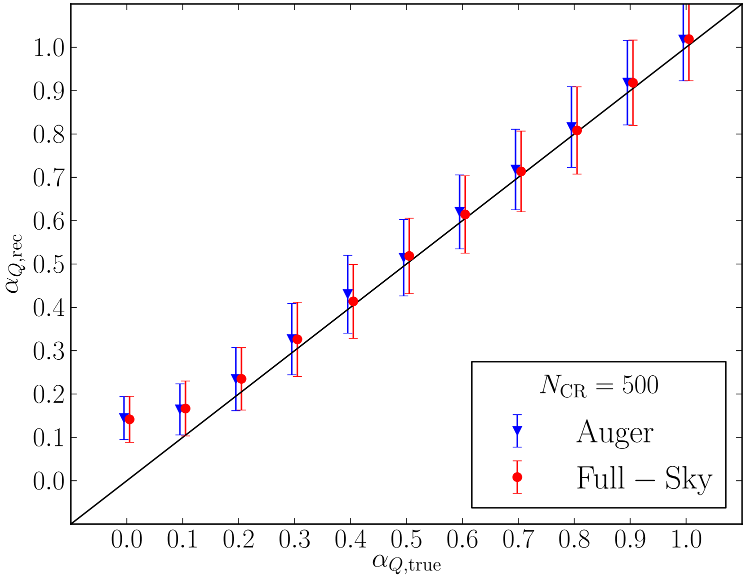
Reconstructing a_ℓ^m's for Nonuniform Partial Sky Coverage

When reconstructing a pure quadrupole, Auger and TA's exposures may be ignored,

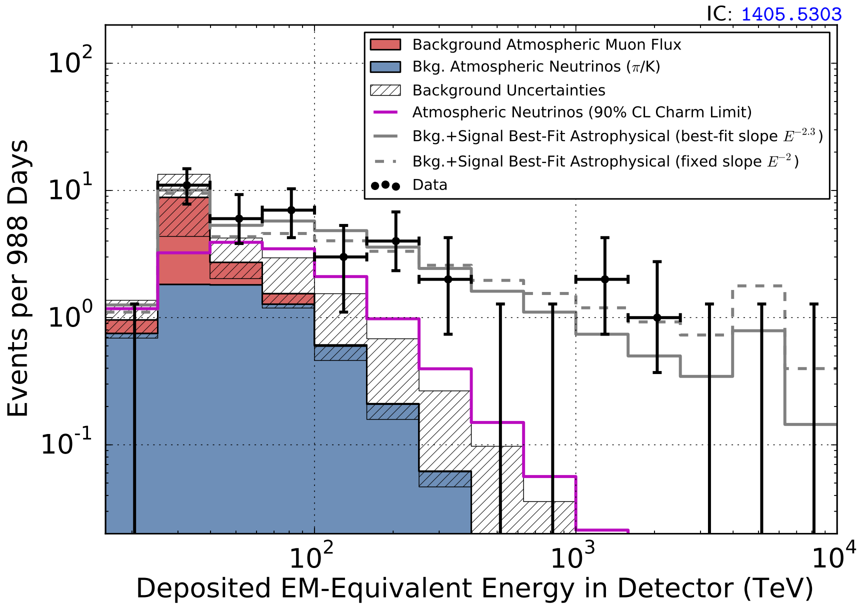
$$b_2^m = a_2^m\left[1+\frac{(-1)^m c_4^0 f(m)}{7\sqrt{4\pi}}\right]$$

A correction of $0.05, -0.04, 0.009$ for $|m| = 0, 1, 2$.

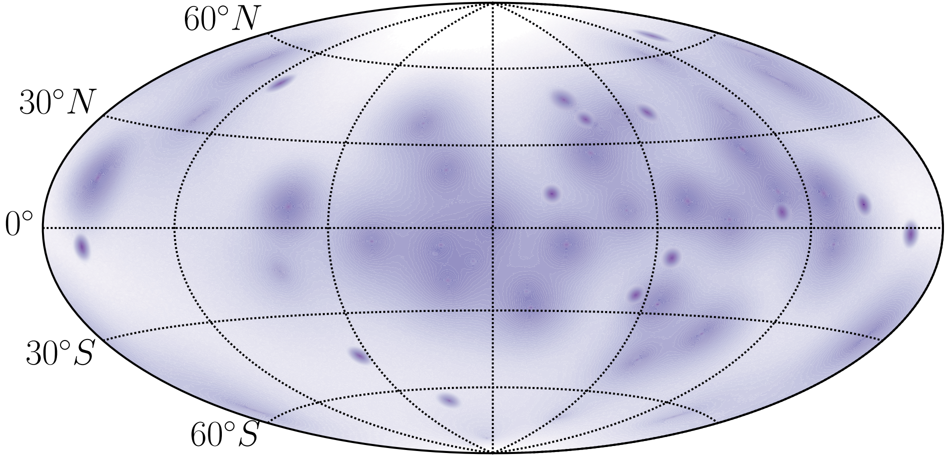
Quadrupole Reconstruction Effectiveness



Astrophysical Neutrinos

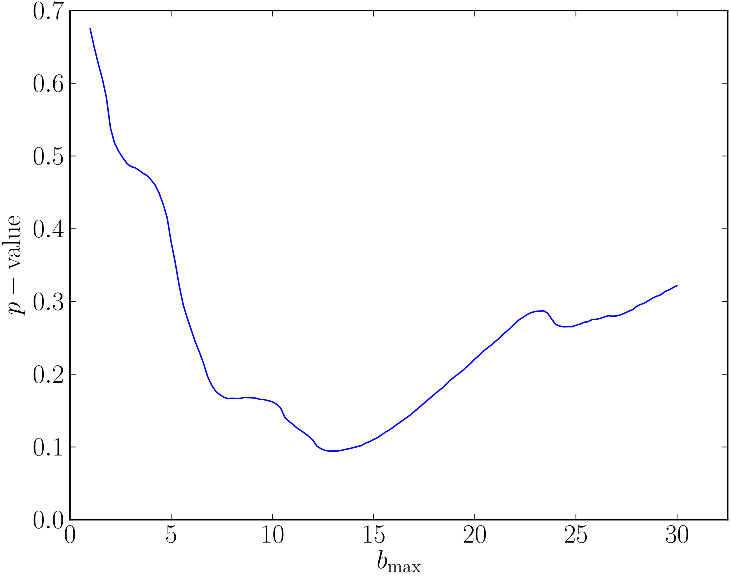


The Astrophysical Neutrino Flux Measured by IceCube



(54-1) events from IC 4 year HESE

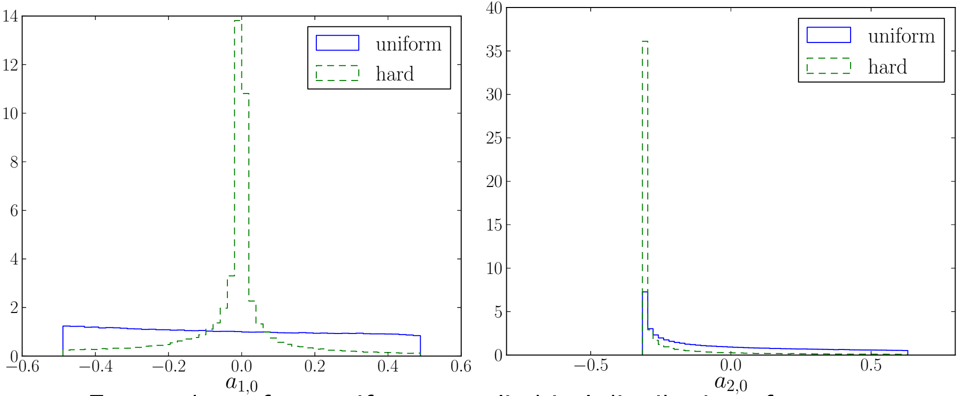
A Simple Test of Galactic Origin



More Information

- Low energy events are more likely to be atmospherics.
- Flavor information.
- Earth absorption.
- We know the shape of the galaxy (pre- vs. post-).

Spherical Harmonics for IceCube



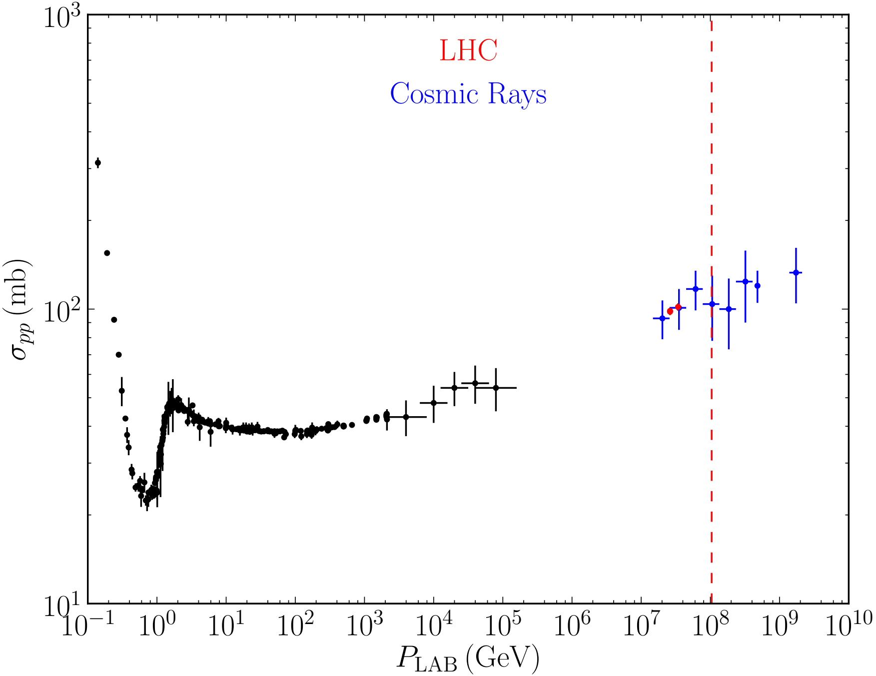
Expected $a_{\ell m}$ for a uniform vs. cylindrical distribution of sources including absorption.

PBD, Weiler in prep.

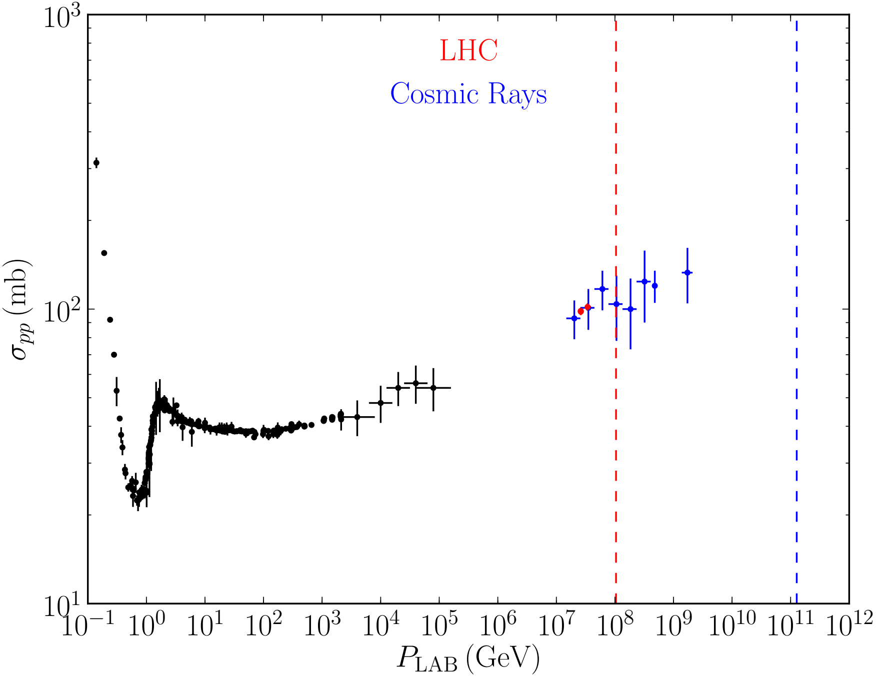
Conclusions

- The sources of UHECRs and astrophysical ν's is still an open question.
- Handling partial sky exposure analytically can be useful.
- A single kind of source of IceCube is becoming problematic.
- Spherical harmonics are useful for large or unknown angular uncertainties.

Backups

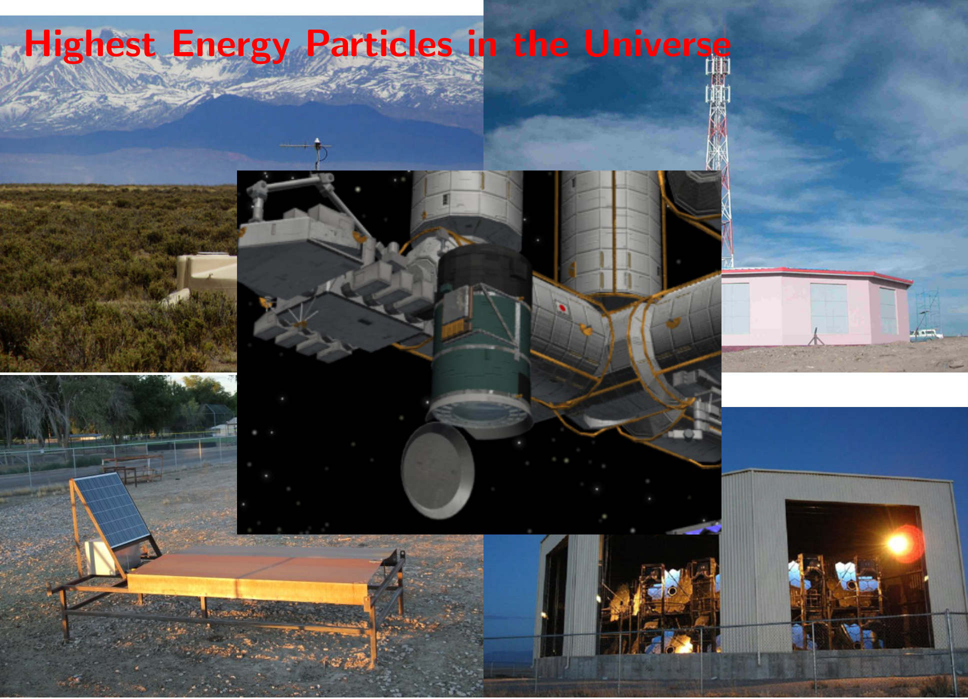


LHC
Cosmic Rays
σ_{pp} (mb)
10^3
10^2
10^1
10^{-1} 10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9 10^{10}
$P_{\rm LAB}$ (GeV)

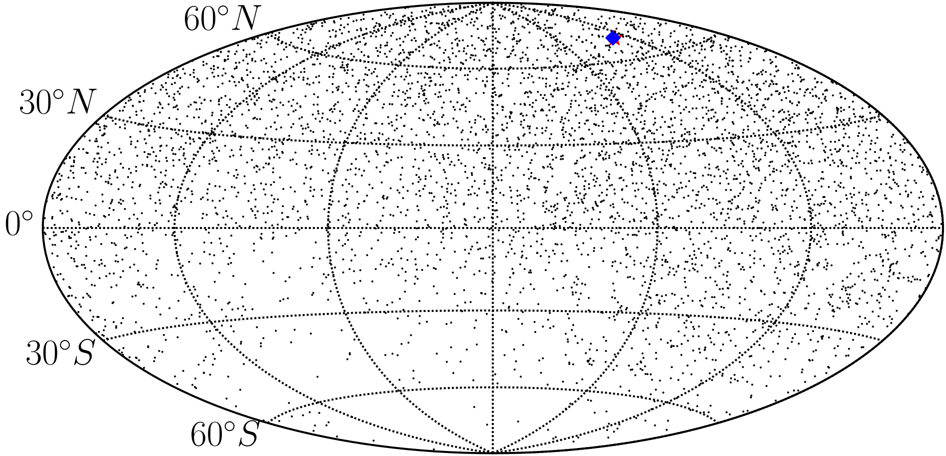

LHC
Cosmic Rays
σ_{pp} (mb)
$P_{\rm LAB}$ (GeV)

Highest Energy Particles in the Universe

Highest Energy Particles in the Universe

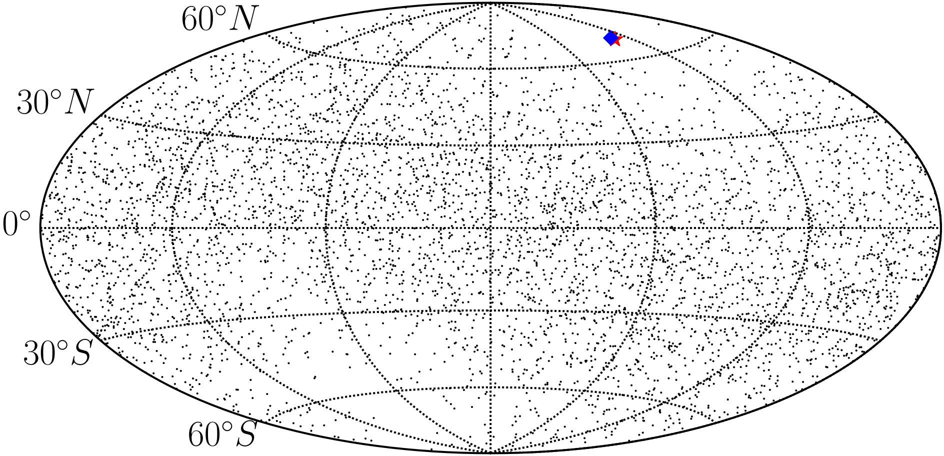


Sample Dipole



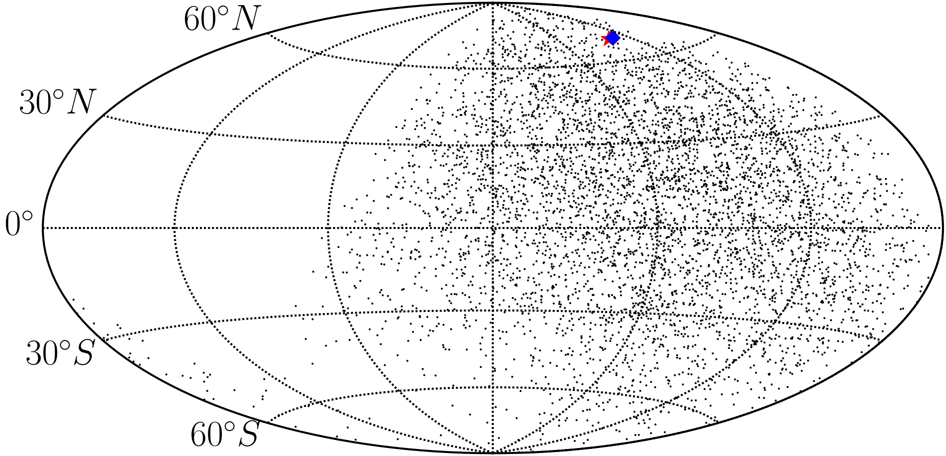
5000 events, $\alpha_D = 1$.
$\theta = 0.3°$, $\Delta\alpha_D = 0.03$.

Sample Quadrupole



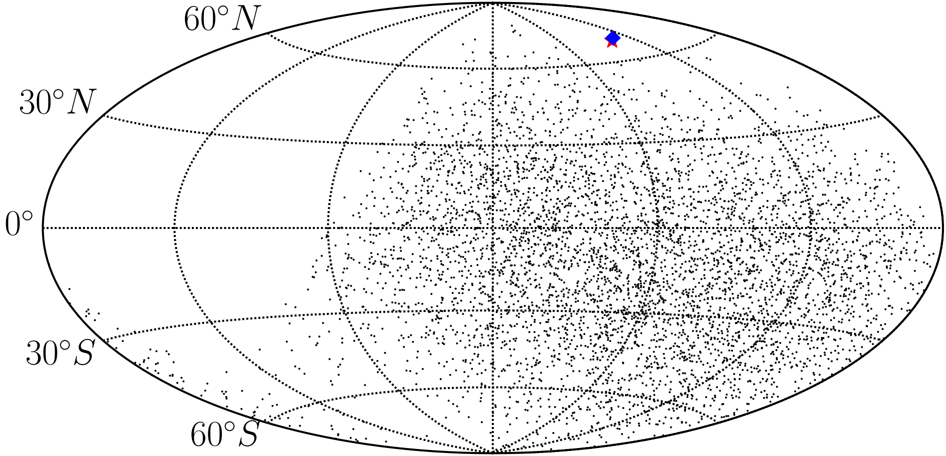
5000 events, $\alpha_Q = 1$.
$\theta = 0.8^\circ$, $\Delta\alpha_Q = 0.02$.

Sample Dipole with Auger's Exposure



5000 events, $\alpha_D = 1$.
$\theta = 2^\circ$, $\Delta\alpha_D = 0.002$.

Sample Quadrupole with Auger's Exposure



5000 events, $\alpha_Q = 1$.
$\theta = 2^\circ$, $\Delta\alpha_Q = 0.02$.

Simple Anisotropy Measures

A general anisotropy measure:

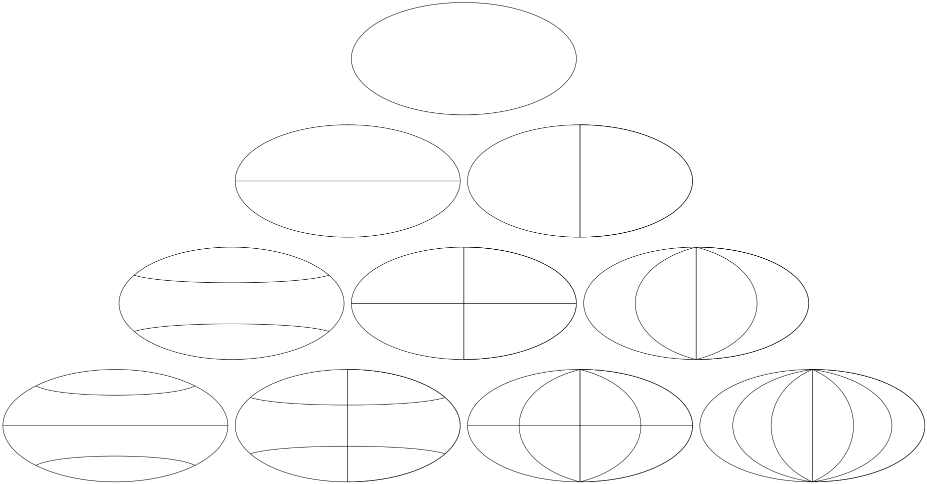
$$\alpha \equiv \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \in [0, 1] \, .$$

Define

$$\alpha_D \equiv \sqrt{3}\frac{|a_1^0|}{a_0^0} \qquad \alpha_Q \equiv \frac{-3\sqrt{\frac{5}{4}\frac{a_2^0}{a_0^0}}}{2 + \sqrt{\frac{5}{4}\frac{a_2^0}{a_0^0}}} \quad \text{('New' later)} \, ,$$

Then $\alpha_D = \alpha$ for a purely dipolar distribution and $\alpha_Q = \alpha$ for a purely quadrupolar distribution.

Spherical Harmonics Visualizations



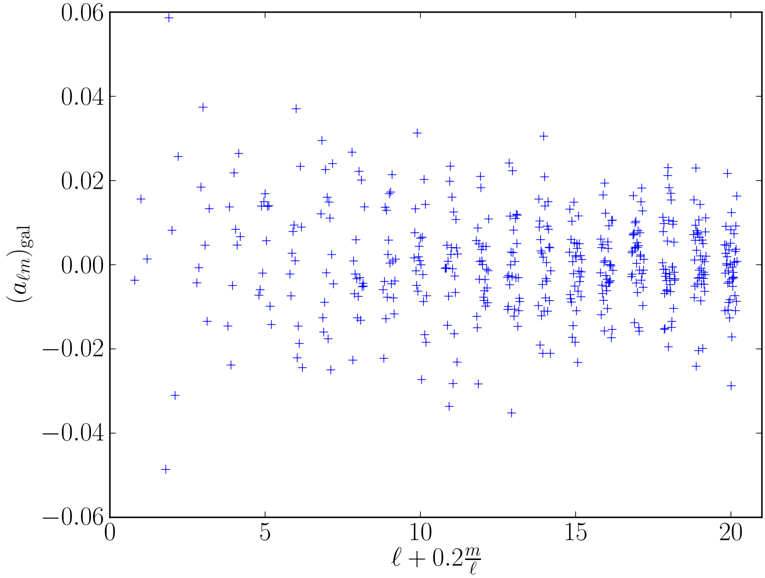
Spherical Harmonics: Possible Sources

Identifiable sources: Cen A, supergalactic plane, etc. use specific Y_ℓ^m's.

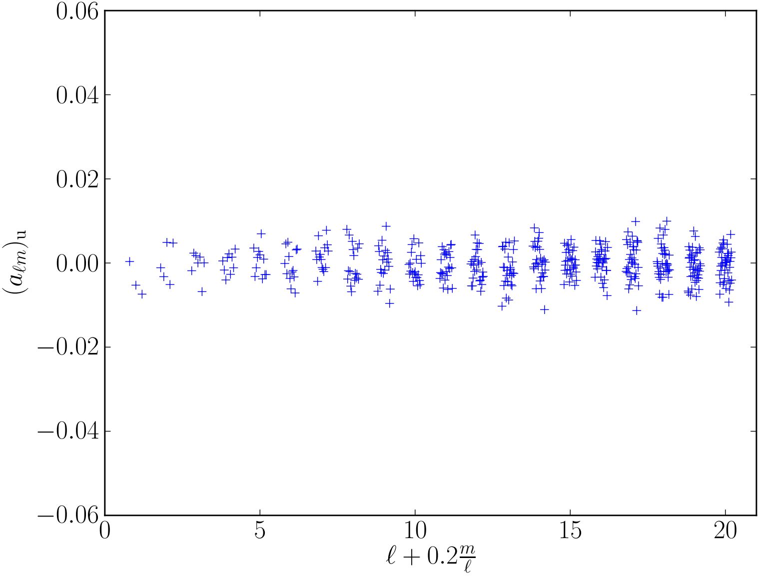
Point source	$\Rightarrow$	dipole:	$I_D \propto a_0^0 Y_0^0 + a_1^0 Y_1^0$.
Planar source	$\Rightarrow$	quadrupole:	$I_Q \propto a_0^0 Y_0^0 + a_2^0 Y_2^0$.

Each Y_ℓ^m partitions the sky into $\sim \ell^2/3$ zones, so $\ell_{\max} \approx \sqrt{3N}$.

Spherical Harmonic Coefficients: Galaxies



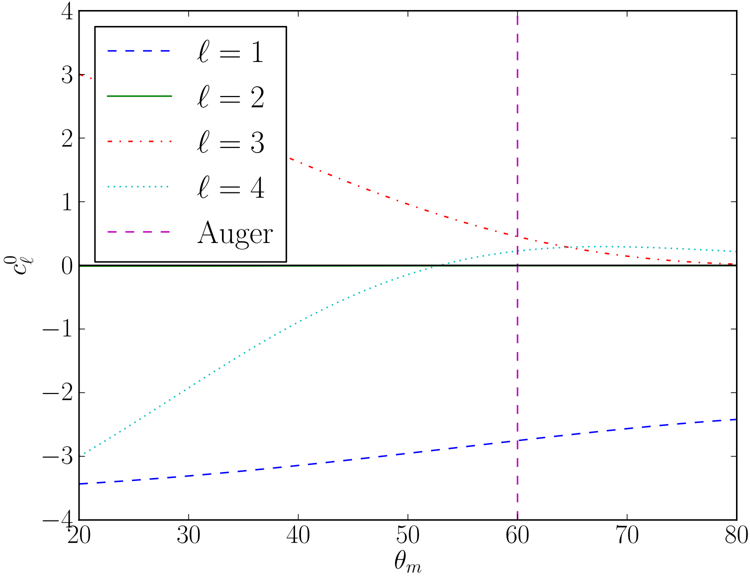
Spherical Harmonic Coefficients: Uniform



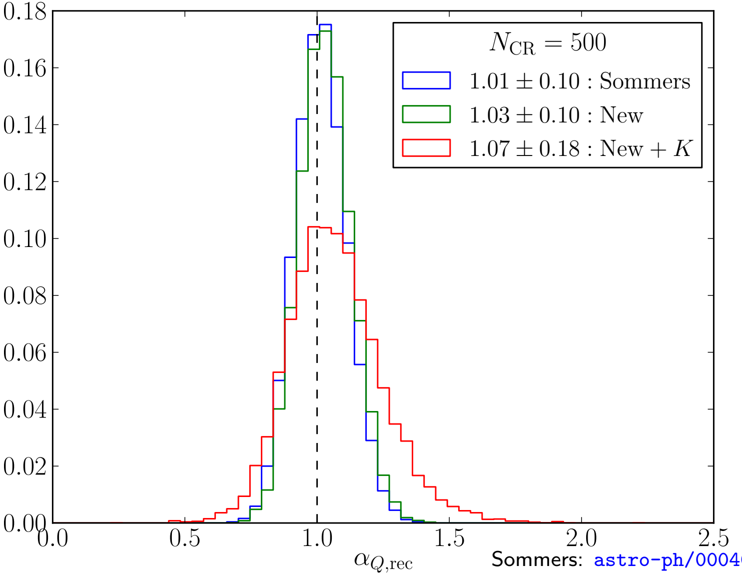
Catalogs

- We consider galactic catalogs.
- The catalog used is the 2MRS.
- Contains 5310 galaxies out to redshift 0.03: 120 Mpc.
- Nearby galaxies need their distances adjusted for peculiar velocities.

Quadrupole Component of Exposure



Quadrupole Reconstruction Technique Effectiveness



Sommers: `astro-ph/0004016`

Rotational Invariance of the Power Spectrum

$$I(\Omega) = \frac{1}{N}\sum_{i=1}^{N}\delta(\mathbf{u}_i,\Omega)\,,$$

$$\bar{a}_\ell^m = \frac{1}{N}\sum_{i=1}^{N} Y_\ell^{m*}(\mathbf{u}_i)\,,$$

$$\bar{C}_\ell = \frac{1}{N^2(2\ell+1)}\sum_{|m|\leq\ell}\left|\sum_{i=1}^{N} Y_\ell^{m*}(\mathbf{u}_i)\right|^2\,.$$

The addition formula for spherical harmonics:

$$P_\ell(\mathbf{x}\cdot\mathbf{y}) = \frac{4\pi}{2\ell+1}\sum_{|m|\leq\ell} Y_\ell^{m*}(\mathbf{x})Y_\ell^m(\mathbf{y})\,.$$

e.g. Arfken, Weber: *Mathematical Methods for Physicists*

$$\bar{C}_\ell = \frac{1}{4\pi N} + \frac{1}{2\pi N^2}\sum_{i<j} P_\ell(\mathbf{u}_i\cdot\mathbf{u}_j)\,.$$

b10-cut: Analytical Derivation

We conservatively fill in the unknown region of the galactic distribution with a uniform distribution,

$$I_g(\Omega) = I_{g,>10}(\Omega) + I_{u,<10}(\Omega)\,.$$

$$(a_\ell^m)_g = (a_\ell^m)_{g,>10} + (a_\ell^m)_{u,<10}\,,$$

Note the following properties of the $(a_\ell^m)_{u,<10}$:

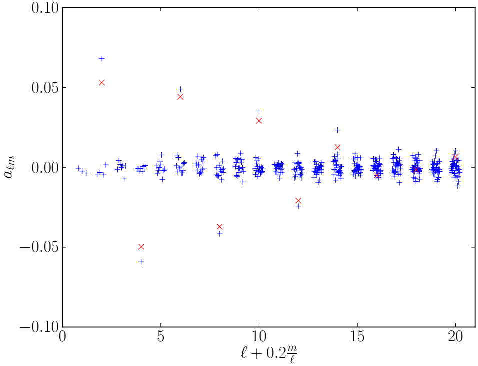
- $I_{u,<10}$ isn't a function of ϕ.

$$a_\ell^m = 2\pi\sqrt{\frac{2\ell+1}{4\pi}}\int P_\ell(x)I(x)d(x)\delta_{m0}\,.$$

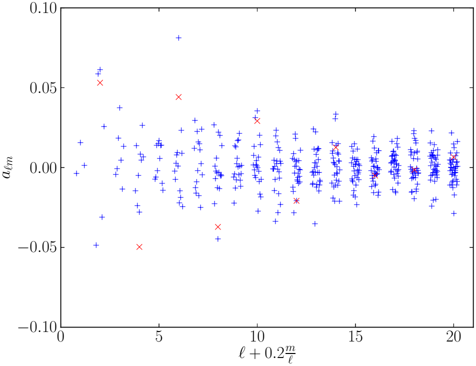
- $I_{u,<10}$ has even parity (the P_ℓ have definite parity).

$$(a_\ell^0)_{u,<10} = \sqrt{\frac{2\ell+1}{4\pi}}\int_0^{\cos(80^\circ)} P_\ell(x)dx\,.$$

b10-cut: Numerical Verification



Uniform



2MRS

Significance of the Galaxy as the Source