Abstract

UHECRs are the highest energy particles in the universe, yet very little is known about them. Their composition, sources, acceleration, and propagation details are all wholly unknown. The first step to addressing this problem is determining the sources, which requires measuring an anisotropy. Above $E \sim 55$ EeV, anisotropies are expected to appear due to the GZK horizon, yet no definitive signal has been seen. Here I overview the current experimental status and present a discussion of anisotropy reconstruction techniques, along with their strengths and weaknesses. I use spherical harmonics as a general tool to detect large scale anisotropies in a low statistics environment. I compare the benefits of a full sky experiment such as JEM-EUSO to ground based partial sky experiments such as the Pierre Auger Observatory and Telescope Array. I show that while Auger can reconstruct a quadrupole without a partial sky penalty, partial sky exposure generally leads to a loss of precision beyond that just from lower statistics compared to a full sky experiment.

Cosmic Ray Anisotropy with Partial Sky Exposure

Peter B. Denton

CCAPP

November 20, 2015

ApJ 802 (2015) 25, arXiv:1409.0883
JHEAp 8 (2015) 1-9, arXiv:1505.03922

Fermilab

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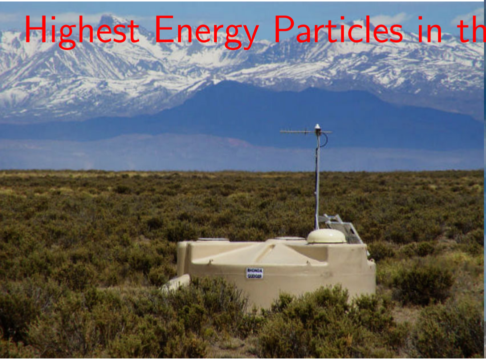
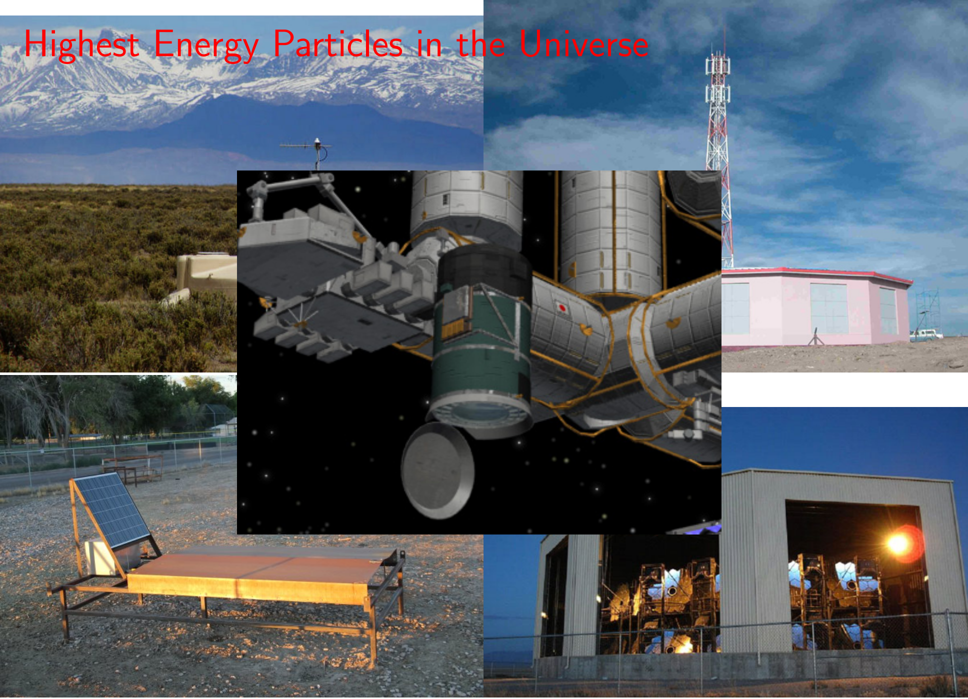
Highest Energy Particles in the Universe



Highest Energy Particles in the Universe



UHECR Anisotropy: What is Known

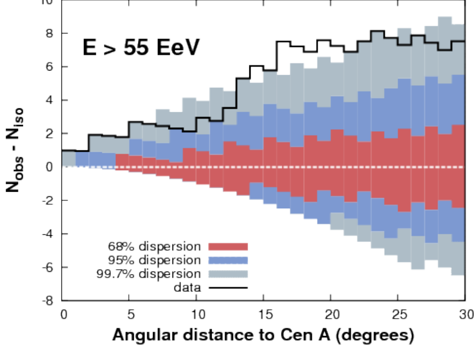
The magnetic field in the Milky Way cannot contain UHECRs.

UHECRs with energies above ~ 55 EeV lose energy due to interactions with the CMB. Greisen; Zatsepin, Kuzmin (1966).

UHECR sources must be close $\Rightarrow$ anisotropies.

UHECRs bend in galactic and extragalactic magnetic fields.

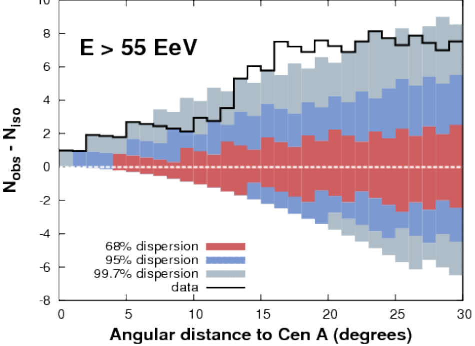
No anisotropies found yet*.

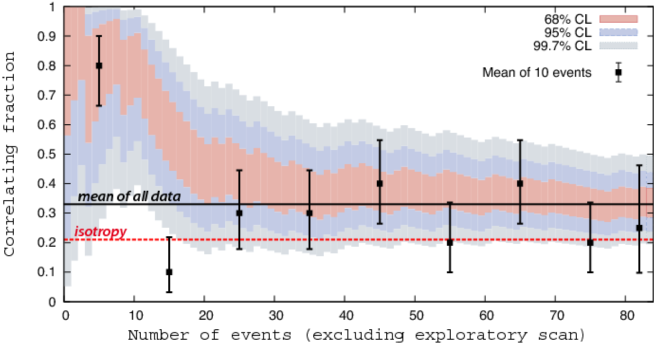


UHECR Anisotropy Hints

Auger, 1106.3048

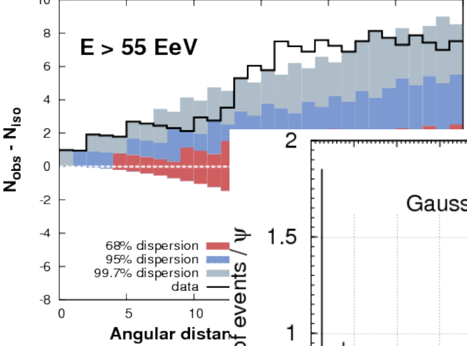
UHECR Anisotropy Hints

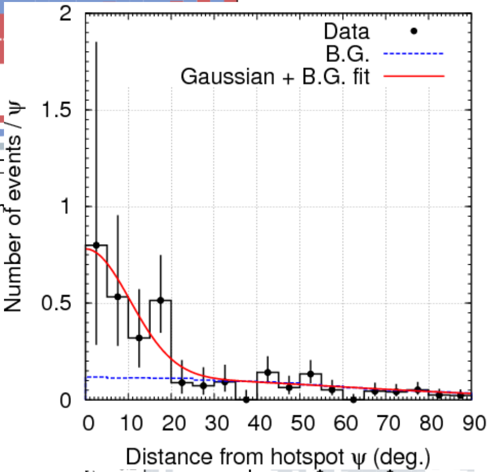


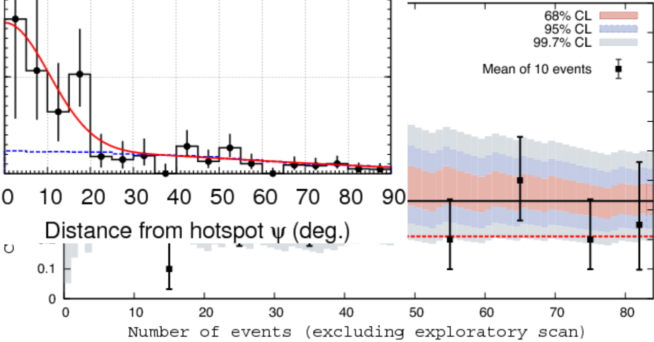


Auger, 1207.4823

UHECR Anisotropy Hints







Telescope Array, 1404.5890

Outline

1. UHECR overview
2. Anisotropy hints
3. Spherical harmonics: general anisotropy tool
4. Partial sky coverage: problems and solutions
5. Quadrupole at Auger: fortuitous
6. Partial vs. full sky coverage: costs

Spherical Harmonics: Distributions on the Sky

General structure can be quantified in terms of Y_ℓ^m's which provide an orthogonal expansion of the sky.

The true distribution of UHECRs as seen at earth follows

$$I(\Omega)=\sum_{\ell,m}a_\ell^mY_\ell^m(\Omega)\,.$$

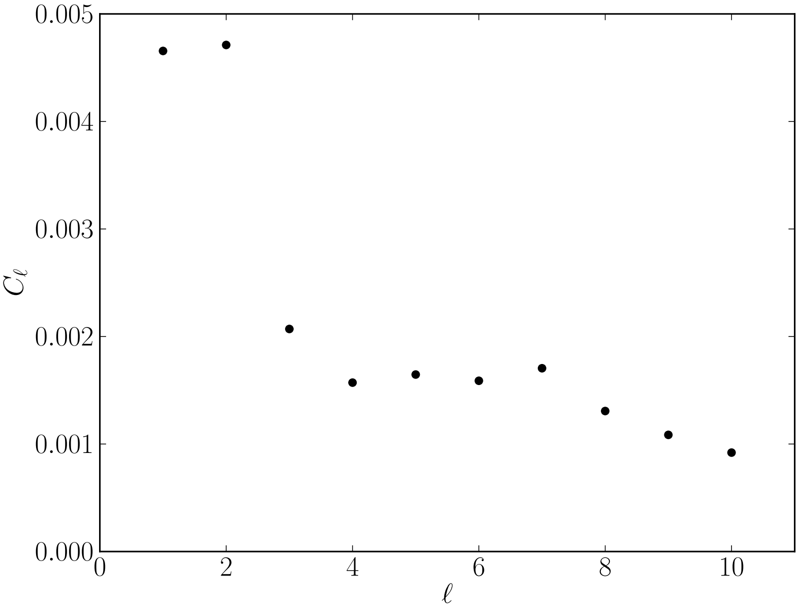
All the anisotropy information is encoded in the a_ℓ^m.

The true distribution may be estimated by $\bar{a}_\ell^m=\frac{1}{N}\sum_i^N Y_\ell^m(\Omega_i)$.

The power spectrum is rotationally invariant.

$$C_\ell=\frac{1}{2\ell+1}\sum_m|a_\ell^m|^2$$

Spherical Harmonics: Possible Sources



Spherical Harmonics: Possible Sources

Identifiable sources: Cen A, Supergalactic plane, etc. use specific Y_ℓ^m's.

Point source	$\Rightarrow$	dipole:	$I_D \propto a_0^0 Y_0^0 + a_1^0 Y_1^0$.
Planar source	$\Rightarrow$	quadrupole:	$I_Q \propto a_0^0 Y_0^0 + a_2^0 Y_2^0$.

Each Y_ℓ^m partitions the sky into $\sim (\ell+1)^2/2$ regions,
so $\ell_{\text{max}} < \lfloor\sqrt{2N}-1\rfloor$.

Simple Anisotropy Measures

A general anisotropy measure:

$$\alpha \equiv \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \in [0,1]\,.$$

Define

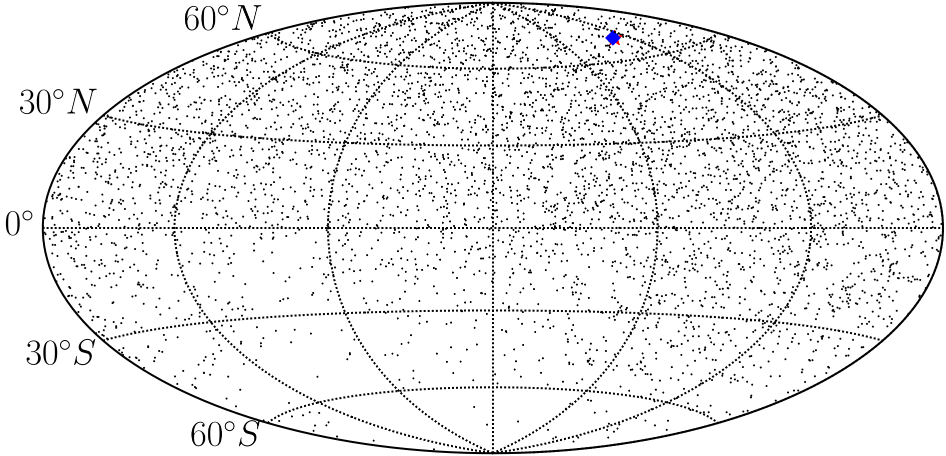
$$\alpha_D \equiv \sqrt{3}\frac{|a_1^0|}{a_0^0} \qquad \alpha_Q \equiv \frac{-3\sqrt{\frac{5}{4}\frac{a_2^0}{a_0^0}}}{2+\sqrt{\frac{5}{4}\frac{a_2^0}{a_0^0}}} \quad (\text{`New' later})\,,$$

$$\alpha_Q = \frac{1-\xi}{1+\xi}\,, \quad \xi = \frac{2-10\Delta}{2+5\Delta}$$

P. Sommers, astro-ph/0004016

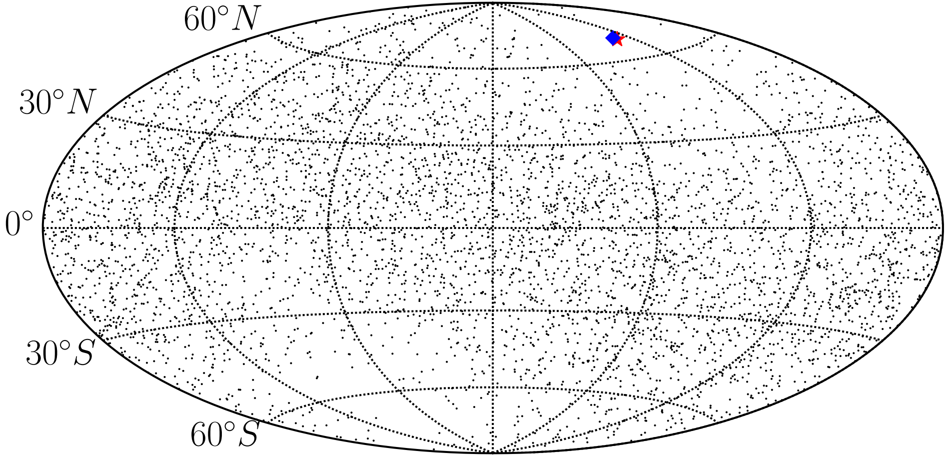
Then $\alpha_D = \alpha$ for a purely dipolar distribution and $\alpha_Q = \alpha$ for a purely quadrupolar distribution.

Sample Dipole



5000 events, $\alpha_D = 1$.
$\theta = 0.3^\circ$, $\Delta\alpha_D = 0.03$.

Sample Quadrupole



5000 events, $\alpha_Q = 1$.
$\theta = 0.8^\circ$, $\Delta\alpha_Q = 0.02$.

Nonuniform Partial Sky Coverage

For an experiment at latitude a_0 with maximum zenith angle θ_m, the relative exposure ω at declination δ is,

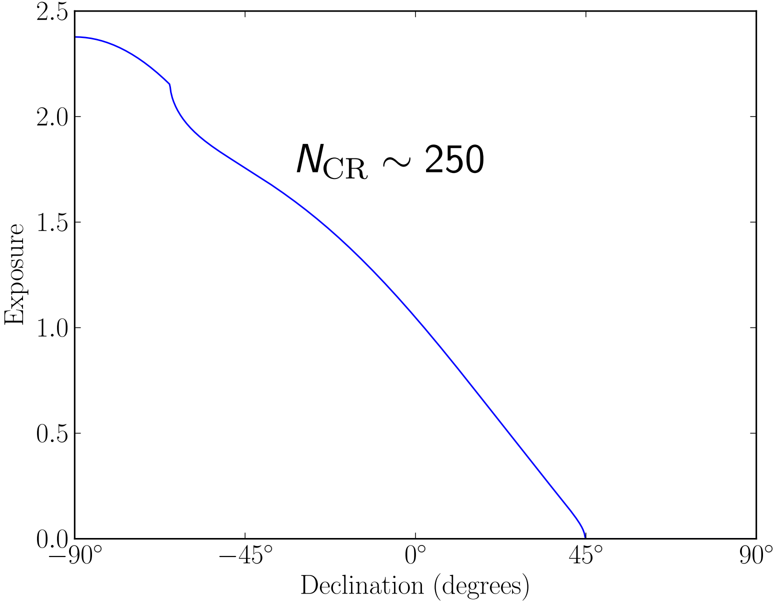
$$\omega(\delta) \propto \cos a_0 \cos\delta \sin\alpha_m + \alpha_m \sin a_0 \sin\delta$$

$$\alpha_m = \begin{cases} 0 & \text{for}\, \xi > 1 \\ \pi & \text{for}\, \xi < -1 \\ \cos^{-1}\xi & \text{otherwise} \end{cases}$$

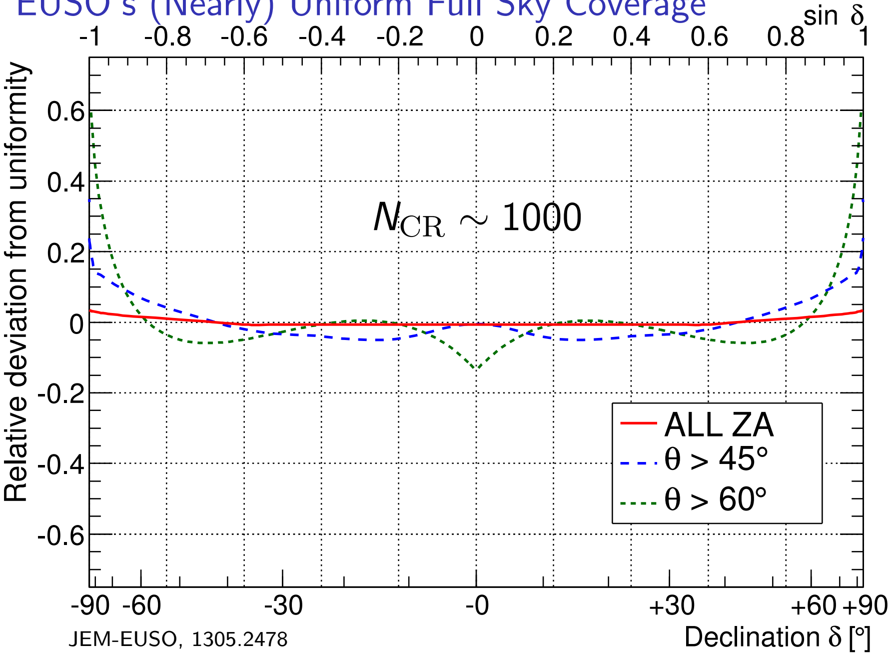
$$\xi = \frac{\cos\theta_m - \sin a_0 \sin\delta}{\cos a_0 \cos\delta}$$

P. Sommers, astro-ph/0004016

Auger's Nonuniform Partial Sky Coverage



EUSO's (Nearly) Uniform Full Sky Coverage



Reconstructing a_ℓ^m's for Nonuniform Partial Sky Coverage

Nonuniform full sky exposure is readily handled:

$$\bar{a}_\ell^m = \frac{1}{N}\sum_i^N Y_\ell^m(\Omega_i) \to \frac{1}{\mathcal{N}}\sum_i^N \frac{Y_\ell^m(\Omega_i)}{\omega(\Omega_i)}\,,$$

where $\mathcal{N} = \sum_i^N \frac{1}{\omega(\Omega_i)}$,
ω is the exposure function.

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where $\mathcal{N} = \sum_i^N \frac{1}{\omega(\Omega_i)}$,
ω is the exposure function.

Partial sky is more challenging: no information from part of the sky.

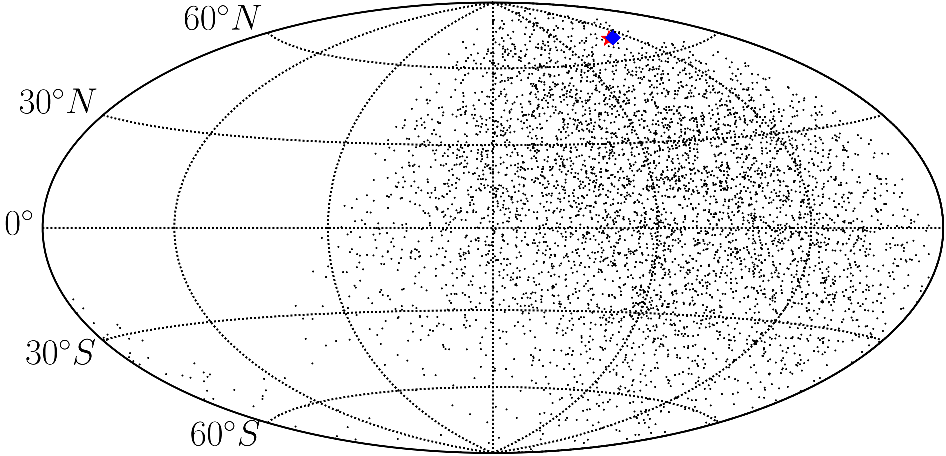
$$[K]_{\ell m}^{\ell' m'} \equiv \int_{\Delta\Omega} d\Omega\omega(\Omega) Y_\ell^m(\Omega) Y_{\ell'}^{m'}(\Omega)$$

$$b_\ell^m = \sum_{\ell' m'} [K]_{\ell m}^{\ell' m'} a_{\ell'}^{m'} \quad \Rightarrow \quad a_\ell^m = \sum_{\ell' m'}^{\ell_{\max}} [K^{-1}]_{\ell m}^{\ell' m'} b_{\ell'}^{m'}$$

$b_\ell^m \to$ observed on earth,
$a_\ell^m \to$ nature's true anisotropy.

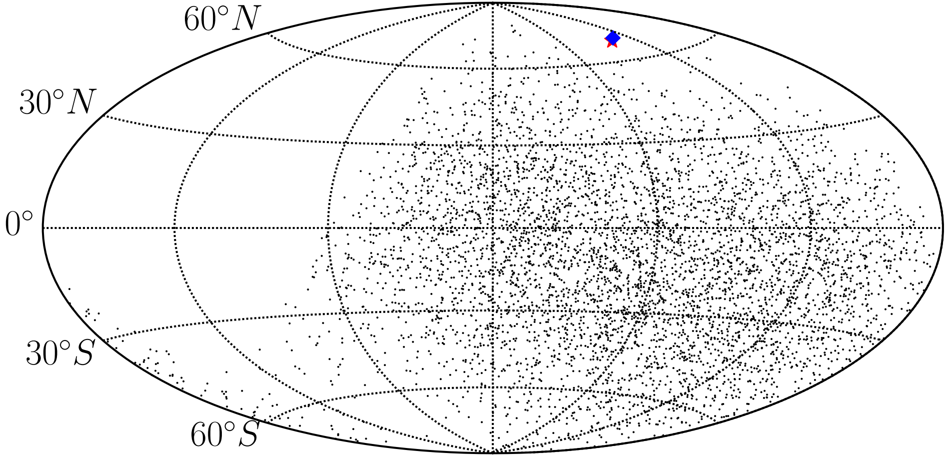
P. Billoir, O. Deligny, 0710.2290

Sample Dipole with Auger's Exposure



5000 events, $\alpha_D = 1$.
$\theta = 2^\circ$, $\Delta\alpha_D = 0.002$.

Sample Quadrupole with Auger's Exposure



5000 events, $\alpha_Q = 1$.
$\theta = 2^\circ$, $\Delta\alpha_Q = 0.02$.

Reconstructing a_ℓ^m's for Nonuniform Partial Sky Coverage

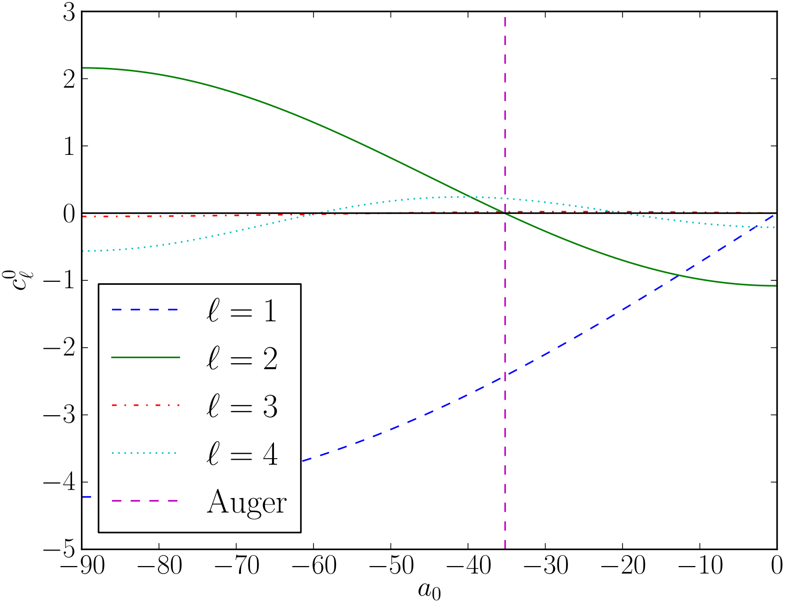
An alternative formalism to the K-matrix approach:

Expand the exposure $\omega(\Omega)=\sum_{\ell,m}c_\ell^mY_\ell^m(\Omega)$.

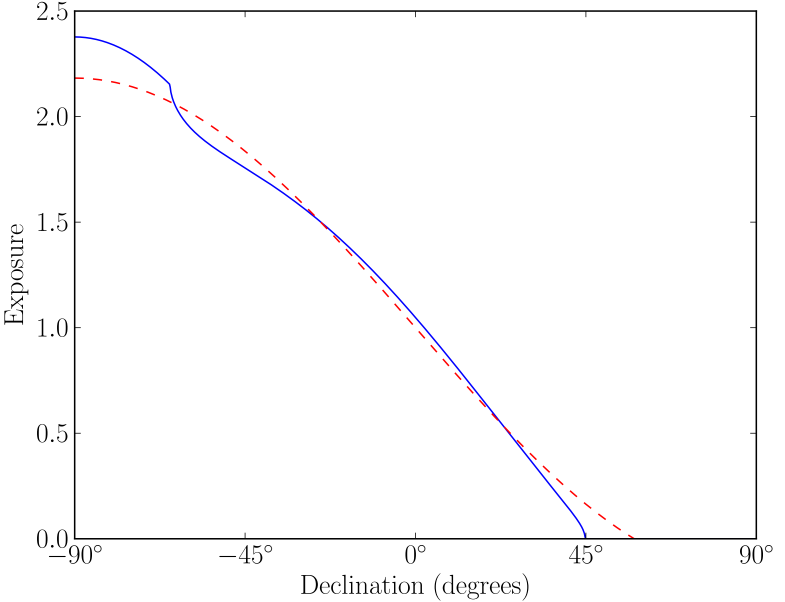
ω does not depend on RA $\Rightarrow$ only $m=0$ coefficients are nonzero.

Fortuitously, $c_2^0=0$ for Auger's exposure
(nearly equal to zero for Telescope Array).

Quadrupole Component of Exposure



Auger's Nonuniform Partial Sky Coverage with $c_{10} Y_{10}$



Bracket Mechanics

The observed coefficients are then

$$b_\ell^m = (-1)^m \sum_{\ell_1,m_1,\ell_2,m_2} a_{\ell_1}^{m_1} c_{\ell_2}^{m_2} \begin{bmatrix} \ell_1 & \ell_2 & \ell \\ m_1 & m_2 & -m \end{bmatrix},$$

where

$$\begin{bmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{bmatrix} \equiv \int d\Omega\, Y_{\ell_1}^{m_1}(\Omega) Y_{\ell_2}^{m_2}(\Omega) Y_{\ell_3}^{m_3}(\Omega)$$

(related to the Wigner 3-j symbol).

This bracket is invariant under permutations of columns,
and is nonzero only when

1. $m_1 + m_2 + m_3 = 0$,
2. $|\ell_i - \ell_j| \leq \ell_k \leq \ell_i + \ell_j$,
3. $\ell_i + \ell_j + \ell_k$ is even.

Bracket Mechanics for Quadrupole

Applying $c_\ell^m = 0$ for $m \neq 0$ and the previous rules,

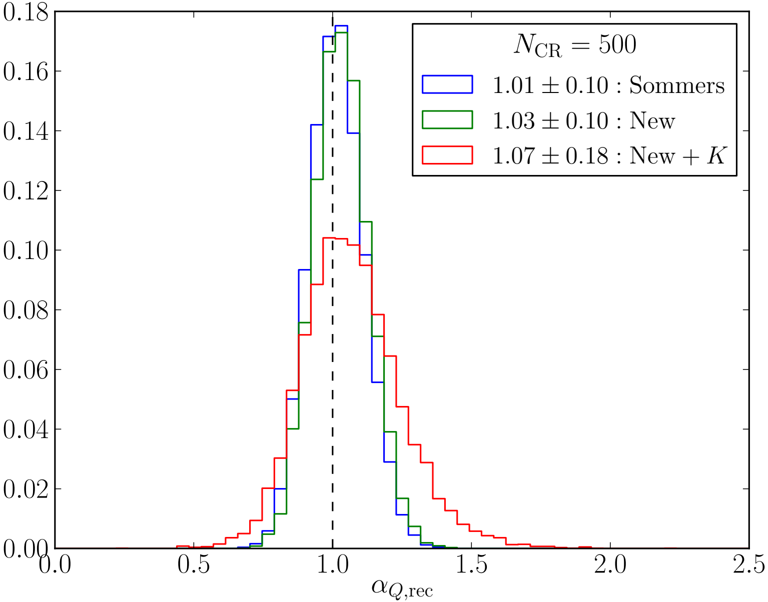
$$b_2^m = (-1)^m \sum_{\ell_1} \sum_{Z=0,\pm 2} a_{\ell_1}^m c_{\ell_1+Z}^0 \begin{bmatrix} 2 & \ell_1 & \ell_1 + Z \\ m & -m & 0 \end{bmatrix}$$

For a quadrupolar distribution,

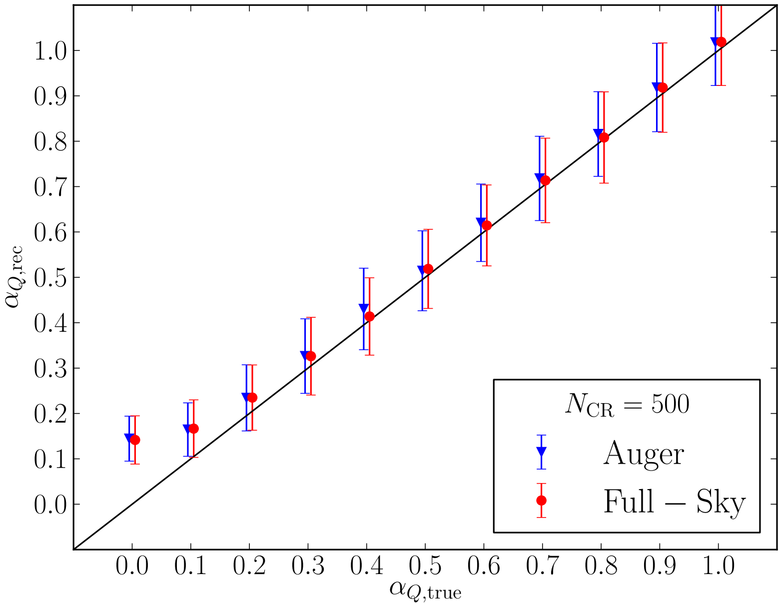
$$b_2^m = a_2^m \left[1 + \frac{(-1)^m c_4^0 f(m)}{7\sqrt{4\pi}}\right]$$

A correction of $0.053, -0.035, 0.0088$ for $|m| = 0, 1, 2$.

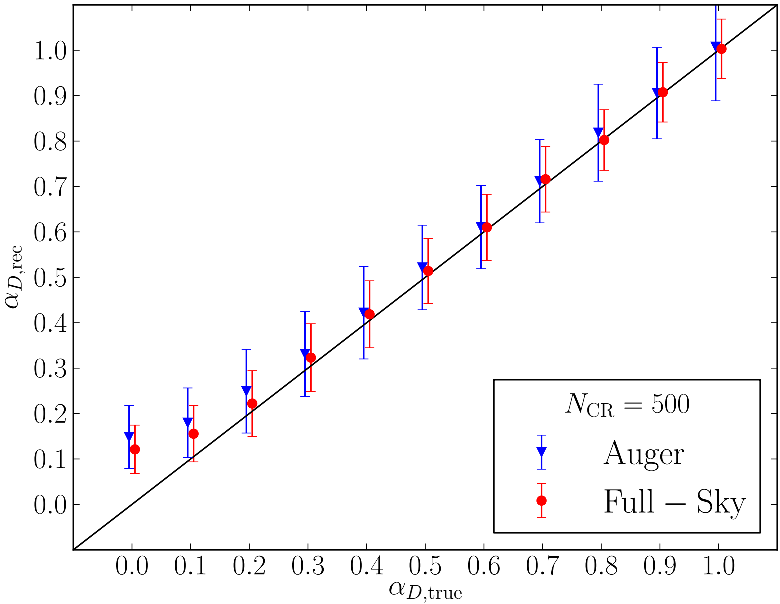
Auger Quadrupole Reconstruction Technique Effectiveness



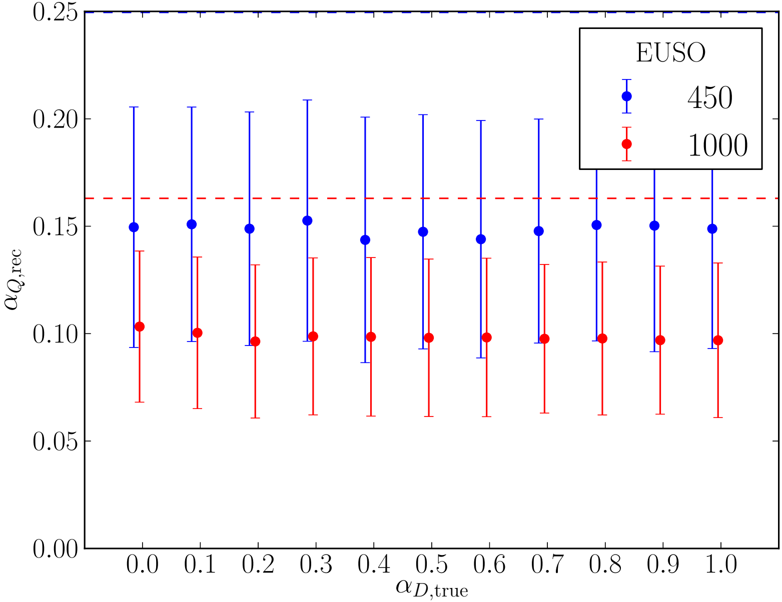
Quadrupole Reconstruction Effectiveness



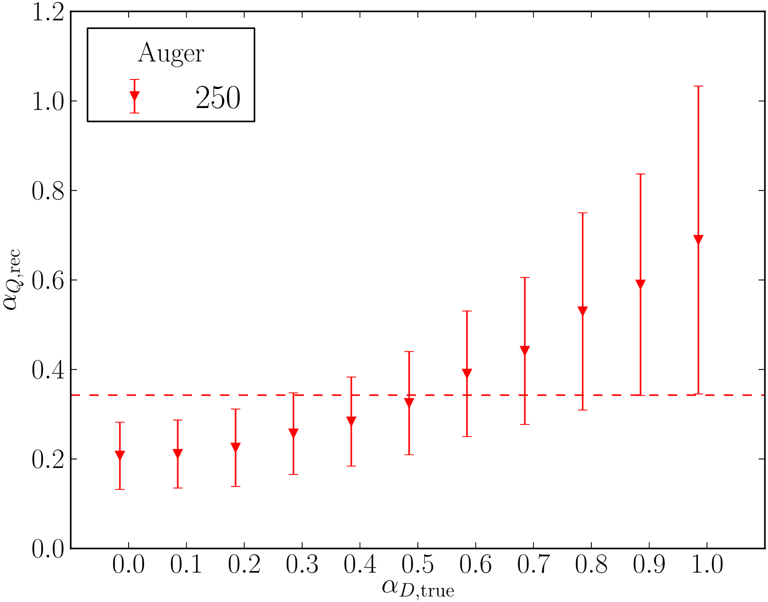
Dipole Reconstruction Effectiveness



Spherical Harmonic Mixing



Spherical Harmonic Mixing



Conclusions

The source(s) of UHECRs is still very much an open question.

TA has evidence of a warm-spot.

Spherical harmonics provide a general purpose anisotropy tool.

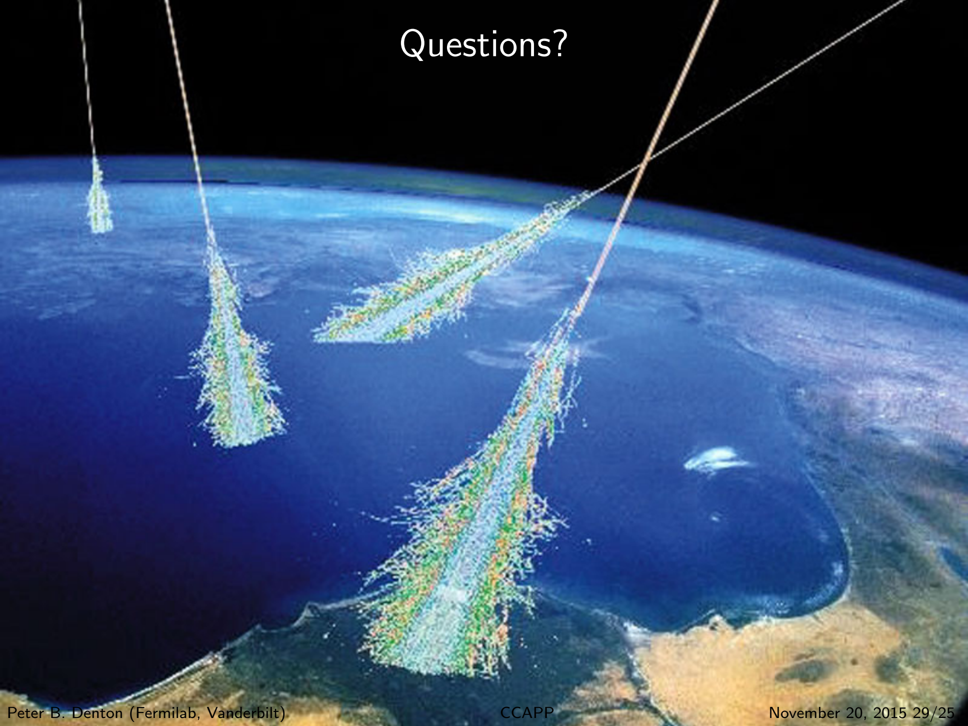
EUSO would collect significantly more statistics than Auger, TA.

Auger and TA can reconstruct a quadrupole without a penalty.

Partial sky, in general, implies an additional penalty factor.

Partial sky $\to$ mixing can result in misidentification of anisotropies.

Questions?



Backups

Catalogs

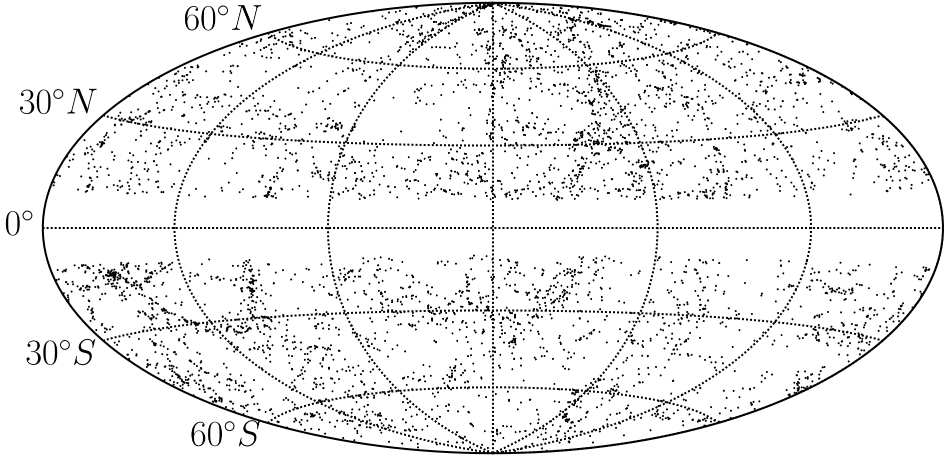
A future step is to consider galactic catalogs.

The catalog used is the SDSS 2MRS.

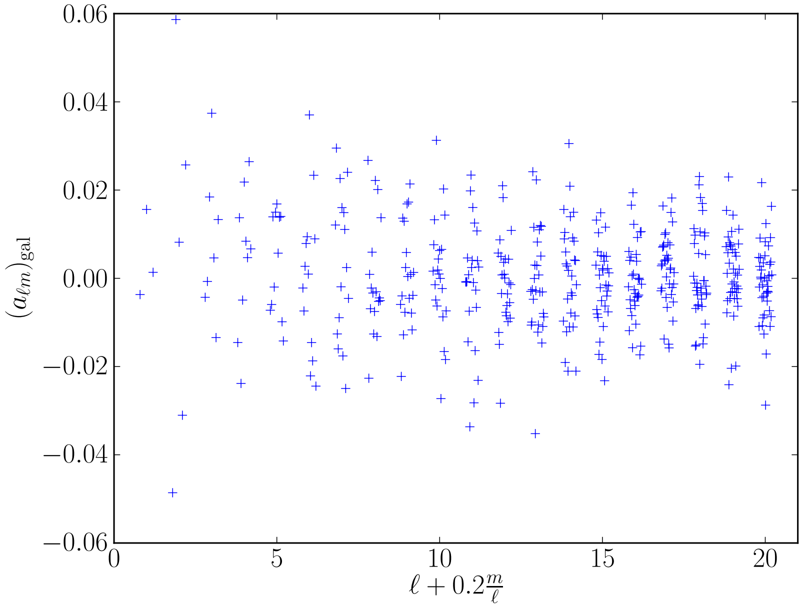
Contains 5310 galaxies out to redshift 0.03: 120 Mpc.

Nearby galaxies need their distances adjusted for peculiar velocities.

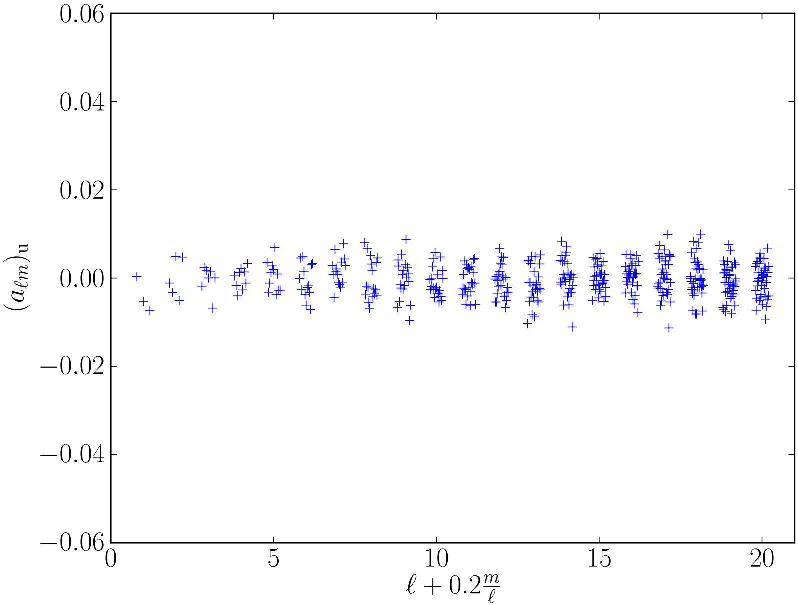
2MRS Sky Map



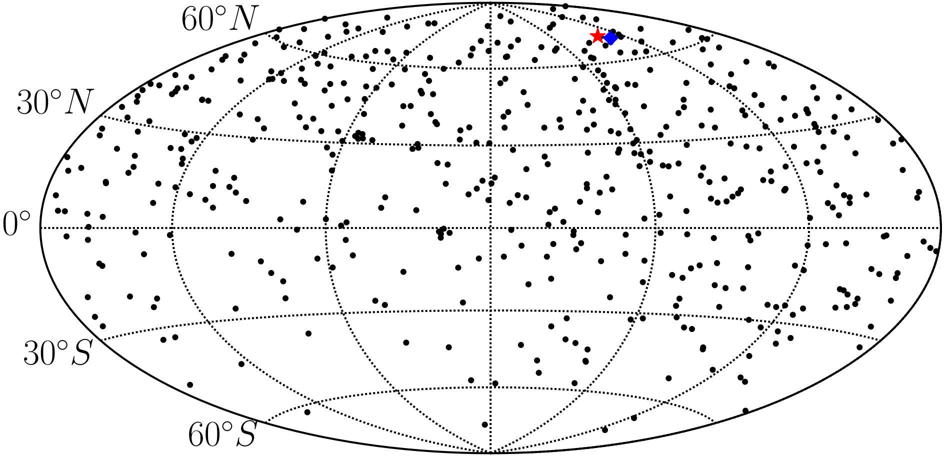
Spherical Harmonic Coefficients: Galaxies



Spherical Harmonic Coefficients: Uniform

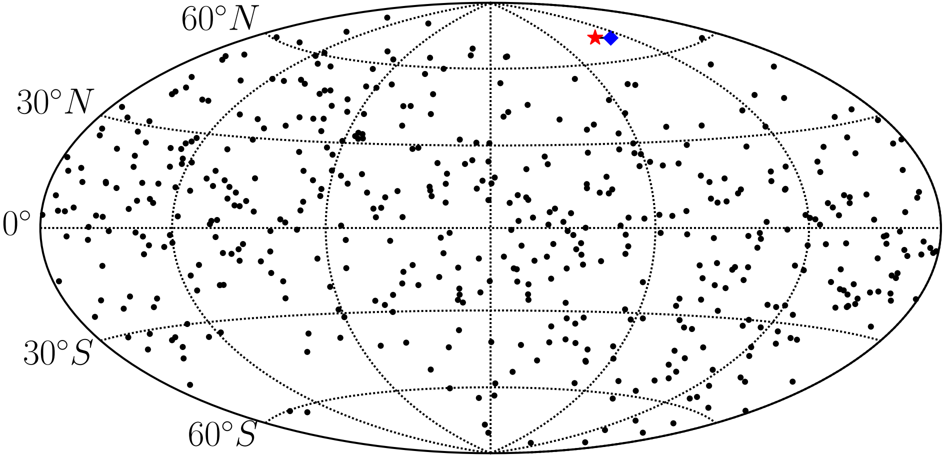


Sample Dipole



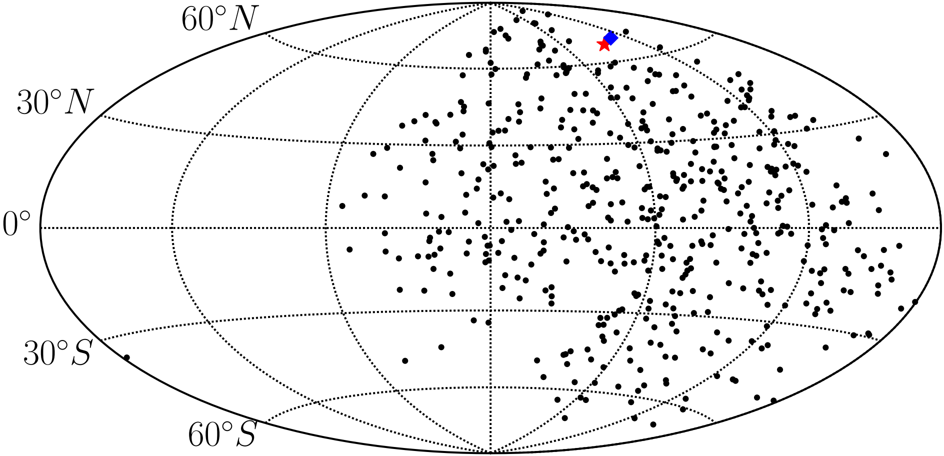
500 events, $\alpha_D = 1$.
$\theta = 3^\circ$, $\Delta\alpha_D = 0.04$.

Sample Quadrupole



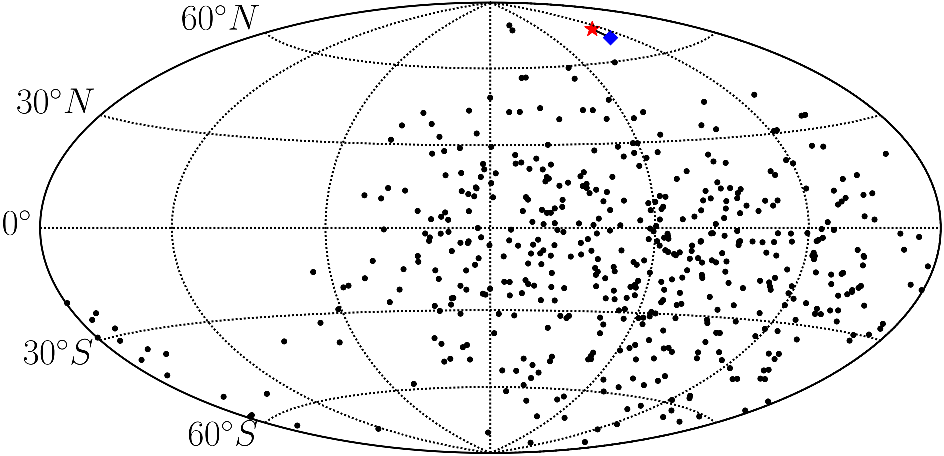
500 events, $\alpha_Q = 1$.
$\theta = 4^\circ$, $\Delta\alpha_Q = 0.06$.

Sample Dipole with Auger's Exposure



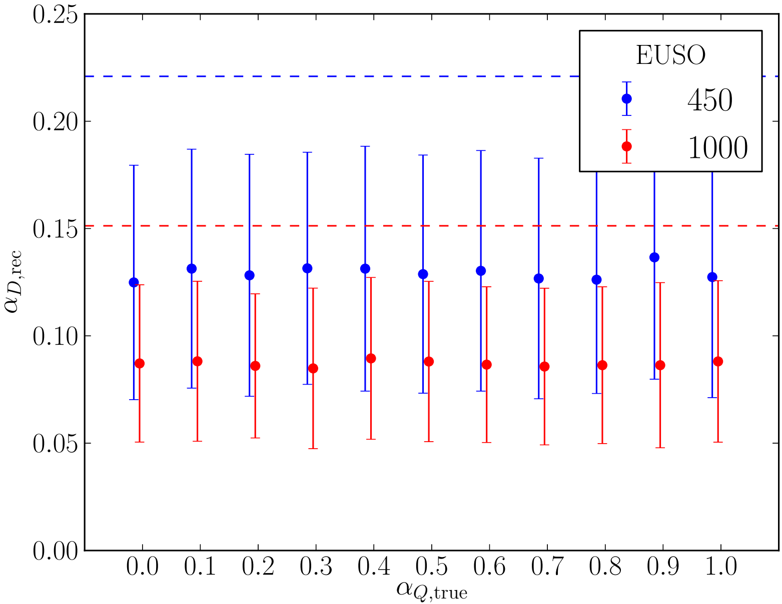
500 events, $\alpha_D = 1.$
$\theta = 6^\circ$, $\Delta\alpha_D = 0.004.$

Sample Quadrupole with Auger's Exposure

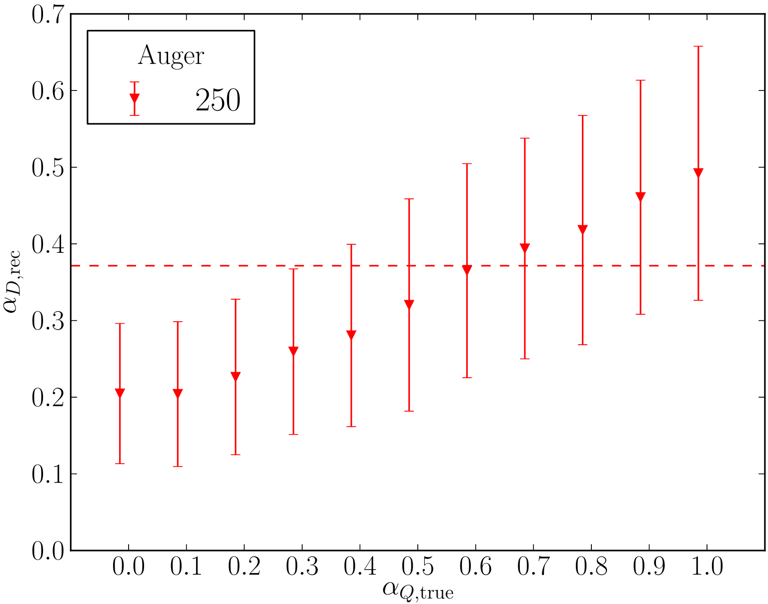


500 events, $\alpha_Q = 1$.
$\theta = 4^\circ$, $\Delta\alpha_Q = 0.09$.

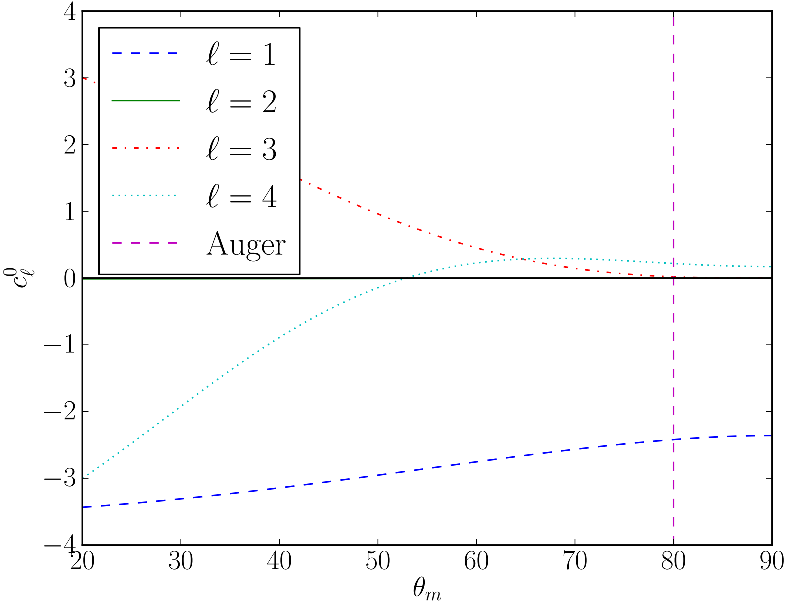
Spherical Harmonic Mixing



Spherical Harmonic Mixing



Quadrupole Component of Exposure



Auger and TA Exposure Combination

Combined exposure:

$$\omega(\Omega, b) = \omega_{\rm TA}(\Omega) + b\omega_{\rm Auger}(\Omega)$$

Fudge factor:

$$\bar{b}^{(0)} = \frac{\Delta N_{\rm Auger}}{\Delta N_{\rm TA}} \frac{\int_{\Delta\Omega} d\Omega \omega_{\rm TA}(\Omega)}{\int_{\Delta\Omega} d\Omega \omega_{\rm Auger}(\Omega)}$$

P. Billoir for Auger, 1403.6314

Problems:

1. Statistics are low in the intersection region
2. $\bar{b}^{(0)}$ is a zeroth order approximation to b under the assumption of isotropy
3. Corrections to $\bar{b}^{(0)}$ need to be fit along along with anisotropy parameters
4. Large systematic energy uncertainty between the experiments

Proof of the Rotational Invariance of C_ℓ (sketch)

$$\bar{a}_\ell^m = \frac{1}{N} \sum_i Y_\ell^{m*}(\mathbf{u}_i)$$

$$\bar{C}_\ell = \frac{1}{2\ell+1} \sum_m |\bar{a}_\ell^m|^2$$

$$\bar{C}_\ell = \frac{1}{N^2(2\ell+1)} \sum_m \left| \sum_i Y_\ell^{m*}(\mathbf{u}_i) \right|^2$$

$$P_\ell(\mathbf{x} \cdot \mathbf{y}) = \frac{4\pi}{2\ell+1} \sum_m Y_\ell^{m*}(\mathbf{x}) Y_\ell^m(\mathbf{y})$$

$$\bar{C}_\ell = \frac{1}{4\pi N} + \frac{1}{2\pi N^2} \sum_{i<j} P_\ell(\mathbf{u}_i \cdot \mathbf{u}_j)$$

Full Definition of Sommers's Quadrupole Technique

$$S_{ab} \equiv \frac{1}{\mathcal{N}} \sum_i \frac{(\mathbf{u}_i \cdot \mathbf{a})(\mathbf{u}_i \cdot \mathbf{b})}{\omega(\mathbf{u}_i)}$$

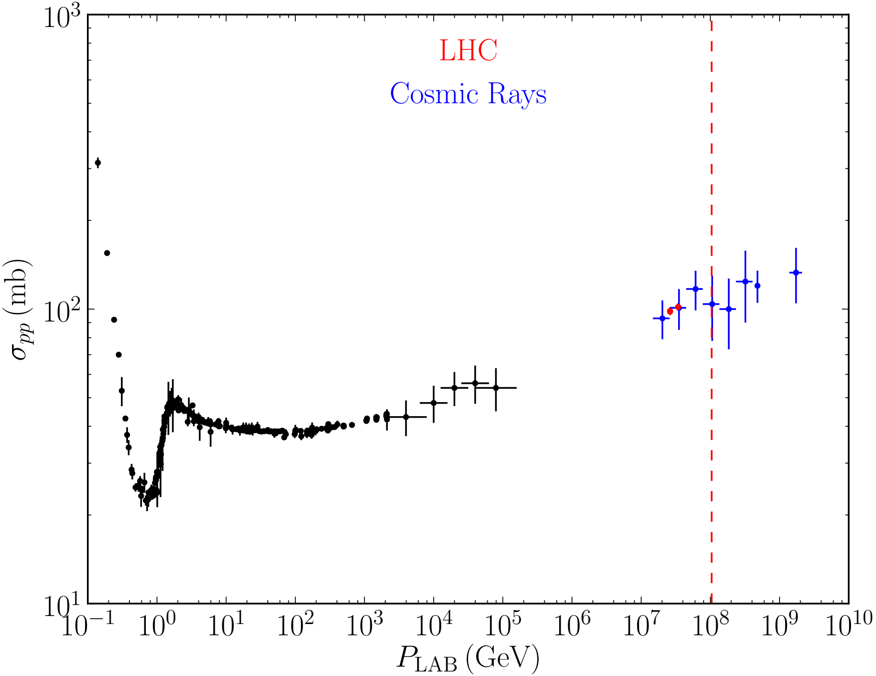
for $a, b \in \{x, y, z\}$.

Let $\lambda_1, \lambda_2, \lambda_3$ be the eigenvalues of S in increasing order.

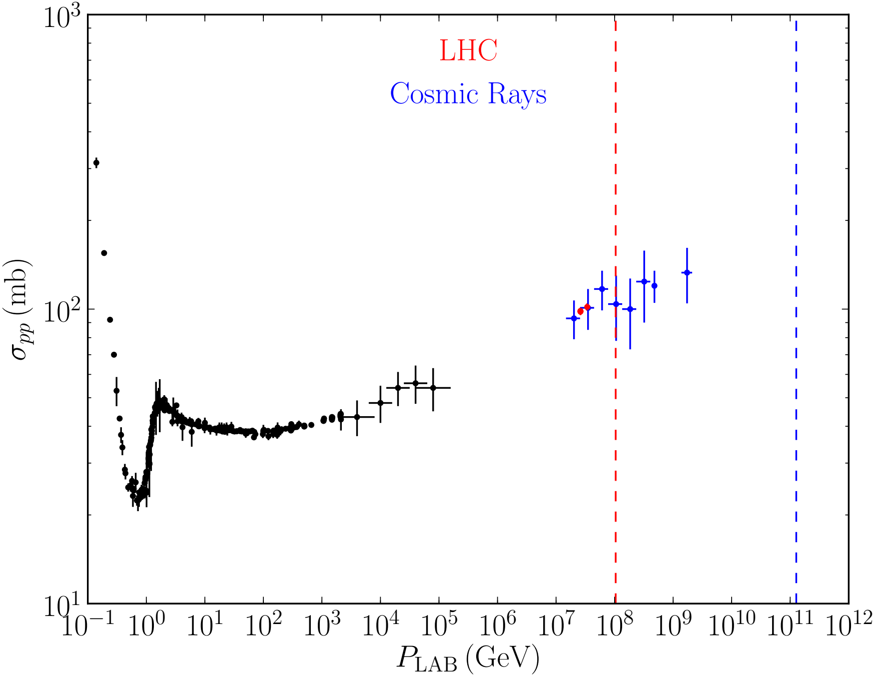
$$\Delta \equiv (\lambda_2 + \lambda_3)/2 - \lambda_1$$

$$\xi \equiv (2 - 10\Delta)/(2 + 5\Delta)$$

$$\alpha_Q = \frac{1 - \xi}{1 + \xi}$$

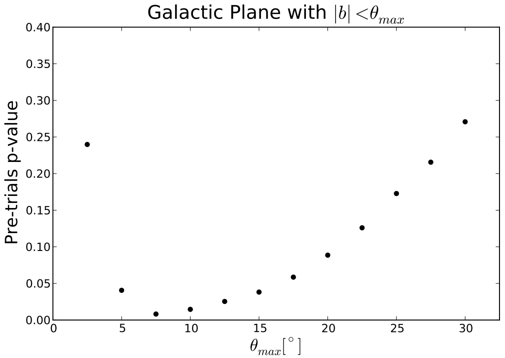


LHC
Cosmic Rays
σ_{pp} (mb)
10^3
10^2
10^1
10^{-1} 10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9 10^{10}
P_{LAB} (GeV)



LHC
Cosmic Rays
10^3
10^2
10^1
σ_{pp} (mb)
10^{-1} 10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9 10^{10} 10^{11} 10^{12}
P_{LAB} (GeV)

IceCube Galactic Plane Anisotropy



"Pre-trials p-value vs. width of galactic plane hypothesis. The width of the galactic plane is varied from ± 2.5 to ± 30 in steps of 2.5. For each width, the pre-trials p-value is calculated by comparing the maximized likelihood to that from scrambled datasets. All results are consistent with the background-only hypothesis."

IceCube, 1405.5303