Abstract

In particle physics there exist two regions: the Standard Model which is fairly complete and the new physics sector which is completely unknown. In between and overlapping with both of these is neutrino physics. Neutrinos exist within the Standard Model but are not explained by it due to the discovery of neutrino oscillations. In this talk I will discuss where we stand with neutrino oscillations, where we might go with them, and how we might learn about the nature of neutrinos.

Testing the Three-Flavor Paradigm

Peter B. Denton

Colorado State University

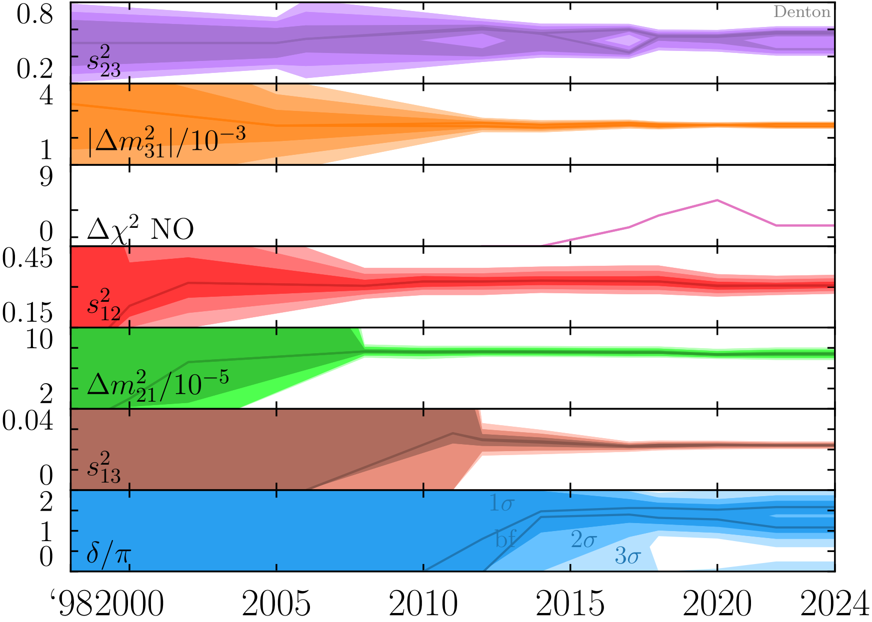
May 28, 2025

Brookhaven™ National Laboratory

Brookhaven NDI Neutrino Discovery Initiative

See also talks by:

- Elizabeth Worcester and Michael Mooney yesterday morning: how DUNE will measure this stuff
- Callum Wilkinson next: DUNE and CPV
- Laura Fields later today: DUNE flux
- André de Gouvêa tomorrow morning: BSM
- And most of the rest of the talks this week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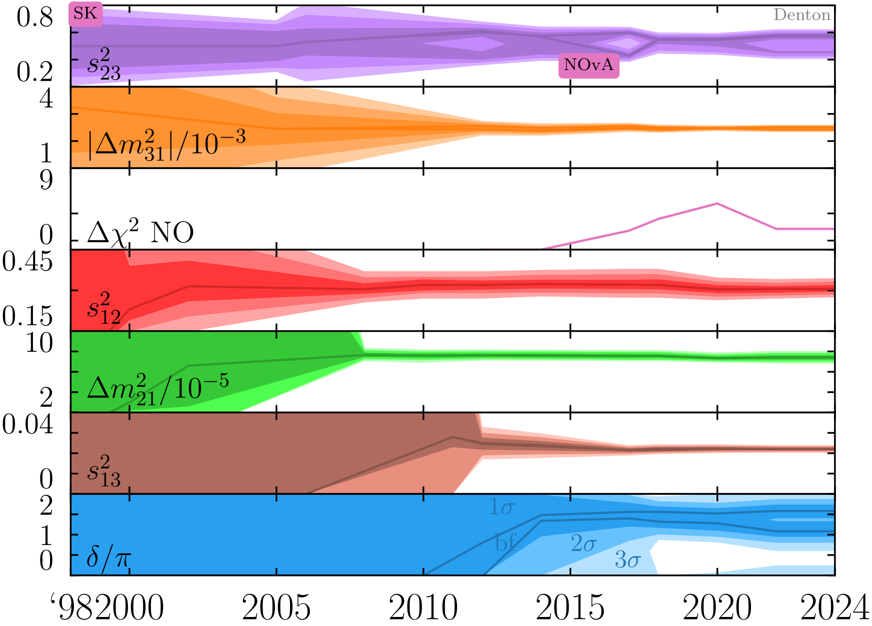
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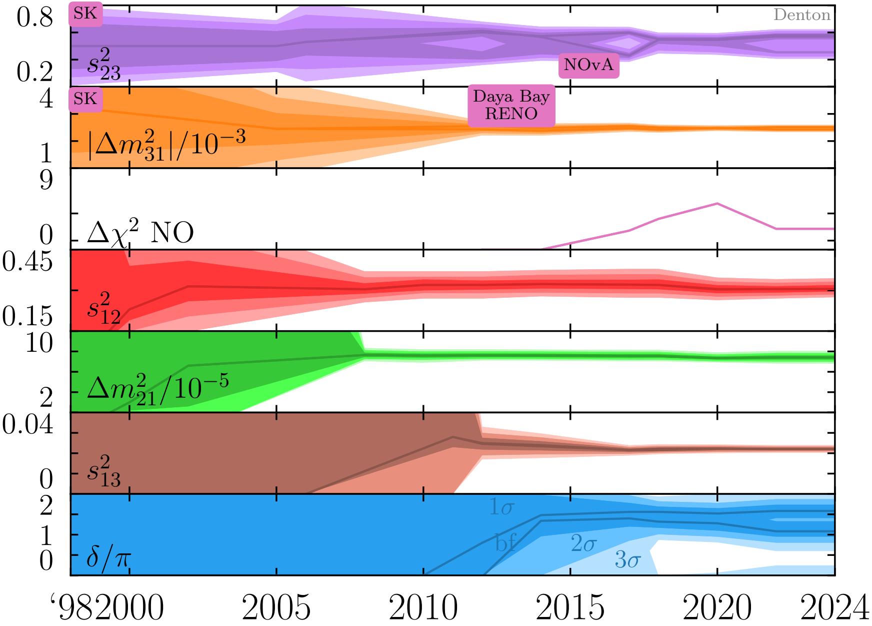

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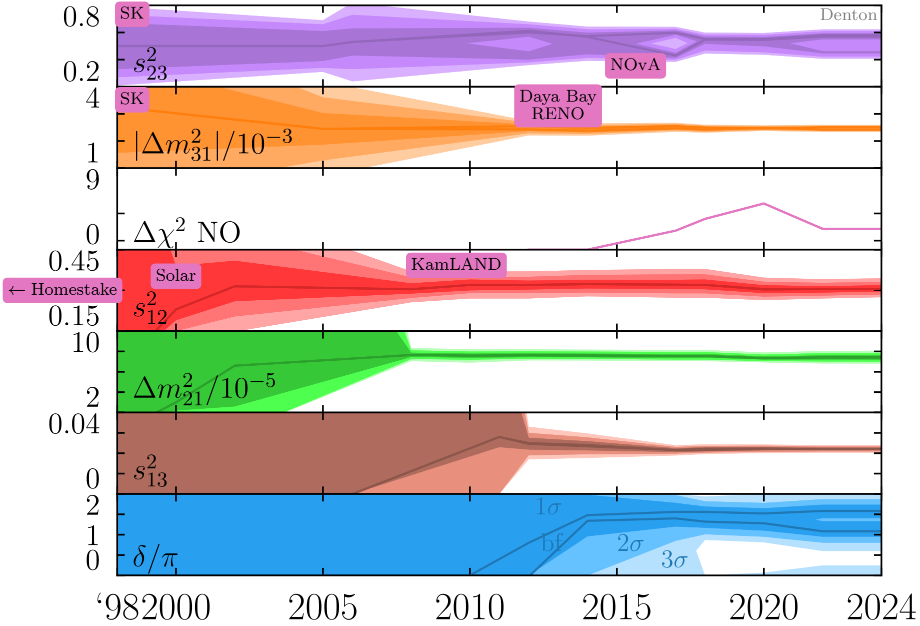

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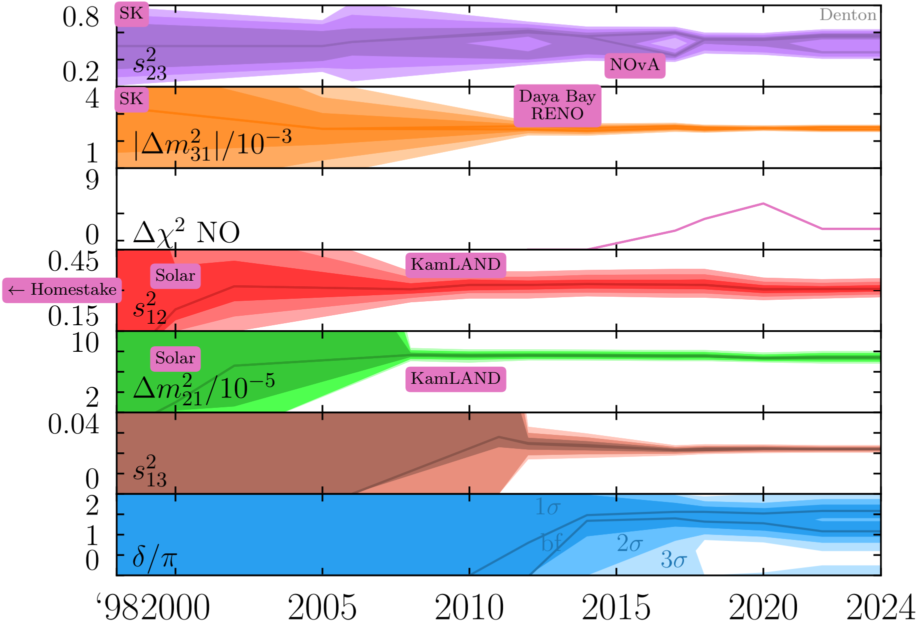
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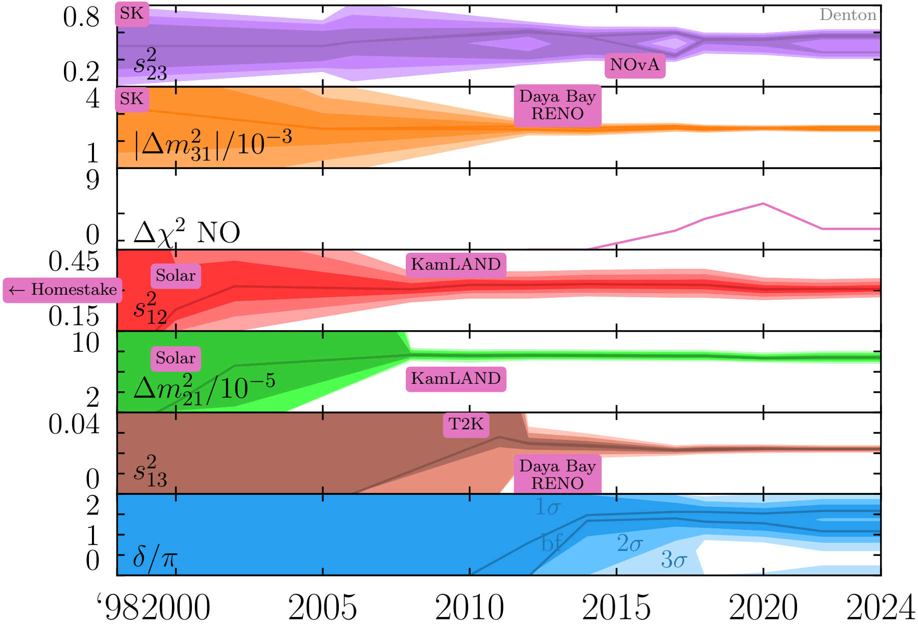

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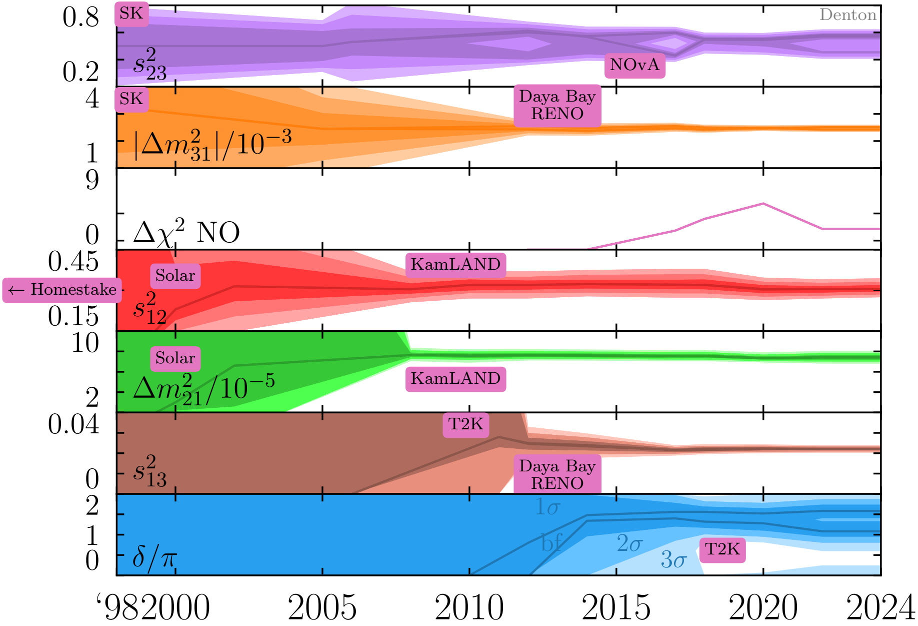

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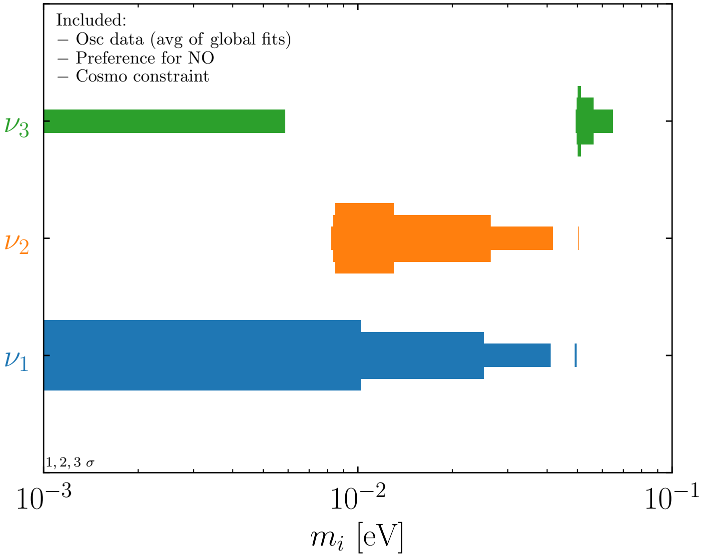
Absolute masses



Four known unknown in particle physics: all neutrinos

Atmospheric mass ordering

θ_{23} octant

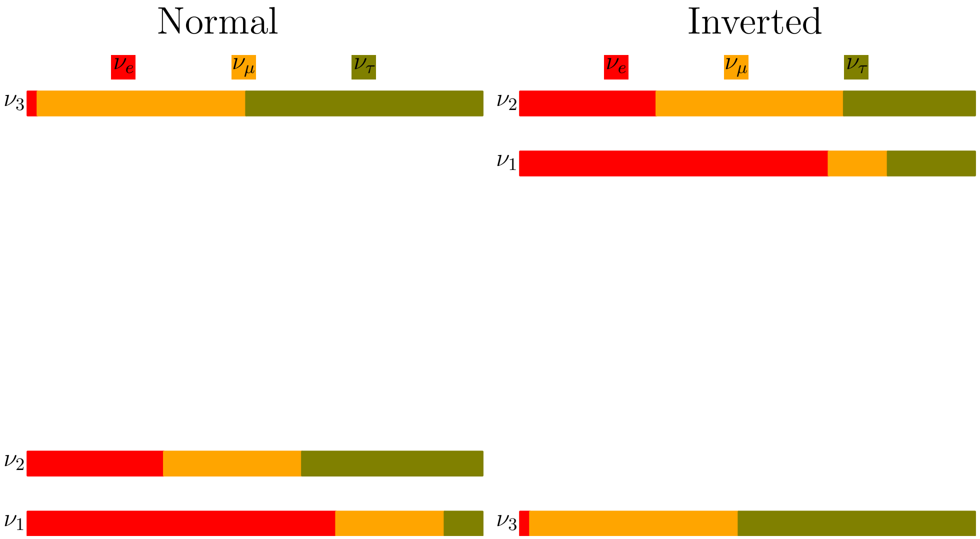
Complex phase

Absolute mass scale

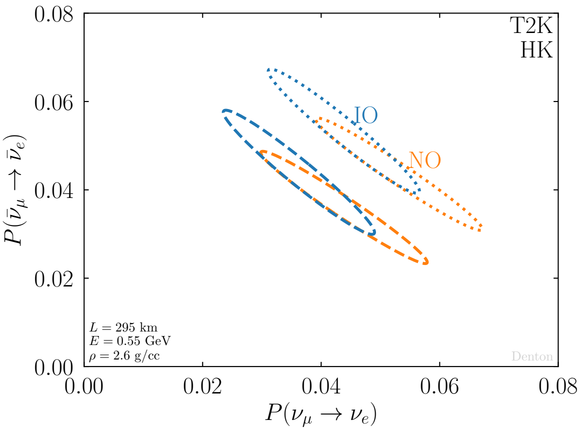
Cosmology, scattering, $0\nu\beta\beta$, ...

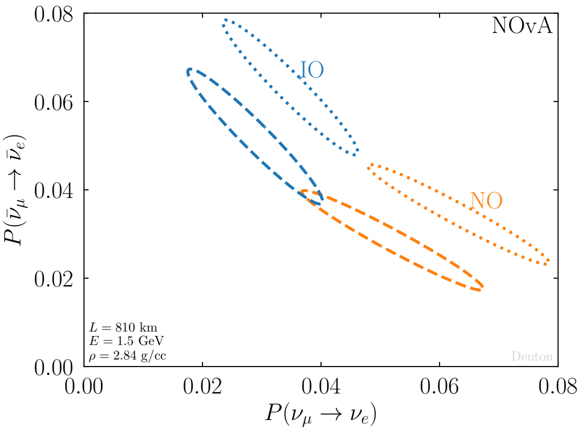
Atmospheric mass ordering

Mass ordering: what is it?



Mass ordering: what is it really?





Requires the matter effect

Mass ordering current status: oscillations

1. NOvA and T2K both prefer NO over IO
2. NOvA+T2K prefers IO over NO
3. SK still prefers NO over IO – statistics complicated
4. NOvA+T2K+SK still prefers NO over IO
5. + Daya Bay & RENO $\Rightarrow$ slight preference NO
6. = no significant preference either way; with SK $\sim 2\sigma$

PBD, J. Gehrlein, R. Pestes 2008.01110
K. Kelly, et al. 2007.08526
I. Esteban, et al. 2007.14792
F. Capozzi, et al. 2107.00532
P. de Salas, et al. 2006.11237
I. Esteban, et al. 2410.05380

Mass ordering current status: all

From oscillations:

$$\text{Normal}: \quad m_1 + m_2 + m_3 > 60 \text{ meV} \qquad \text{Inverted}: \quad m_1 + m_2 + m_3 > 100 \text{ meV}$$

Cosmology: $m_1 + m_2 + m_3 < 90$ meV at 95% CL

E. Valentino, S. Gariazzo, O. Mena 2106.15267

$\to$ 20 meV precision with DESI, EUCLID, ...

Pushing to very low (negative?) masses!?

N. Craig, et al. 2405.00836
Many caveats: T. Bertólez-Martínez, et al. 2411.14524

See also KATRIN 2406.13516

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Many caveats: T. Bertólez-Martínez, et al. 2411.14524

See also KATRIN 2406.13516

PRIORS?

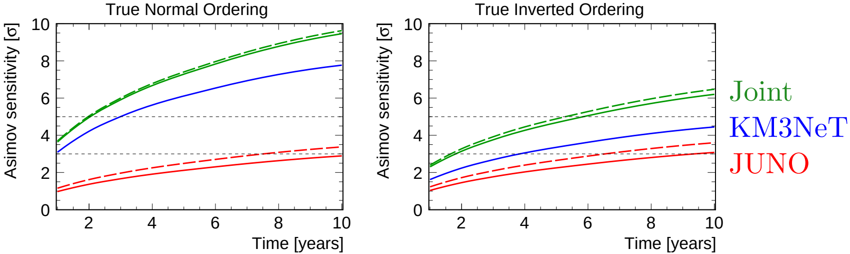
Some claim "decisive" Bayesian evidence for normal

R. Jimenez, et al. 2203.14247

More general prior assumptions $\Rightarrow$ no significant information from cosmology

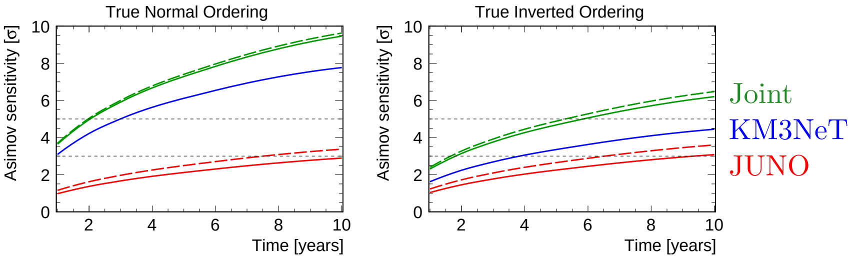
S. Gariazzo, et al. 1801.04946
S. Gariazzo, et al. 2205.02195

Mass ordering: future sensitivities



JUNO, KM3NeT 2108.06293
JUNO, IceCube 1911.06745
Note: if lower octant, KM3NeT is less sensitive

Mass ordering: future sensitivities



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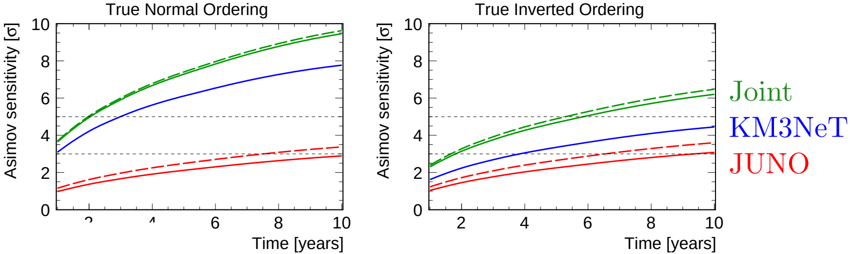
Note: if lower octant, KM3NeT is less sensitive

$$\Delta m^2_{ee} = c^2_{12}\Delta m^2_{31} + s^2_{12}\Delta m^2_{32}$$
$$\Delta m^2_{\mu\mu} = s^2_{12}\Delta m^2_{31} + c^2_{12}\Delta m^2_{32} + \mathcal{O}(s_{13}\Delta m^2_{21})$$

Differ by $\pm \sim 1.1\%$ in each mass ordering

H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283

Mass ordering: future sensitivities



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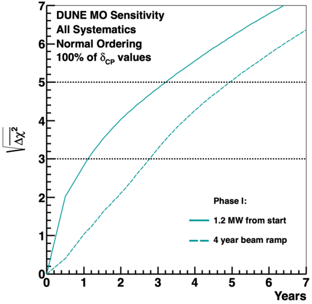


Matter effect ⇒ DUNE 2203.06100

$$\Delta m^2_{ee} = c^2_{12}\Delta m^2_{31} + s^2_{12}\Delta m^2_{32}$$

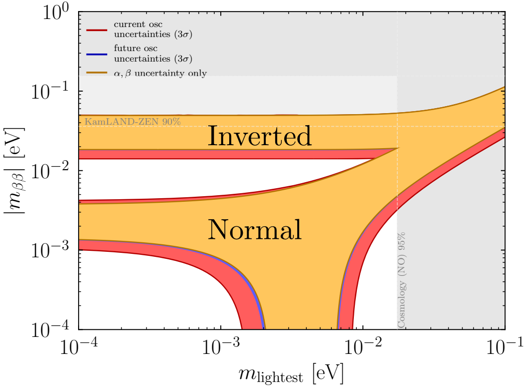
$$\Delta m^2_{\mu\mu} = s^2_{12}\Delta m^2_{31} + c^2_{12}\Delta m^2_{32} + \mathcal{O}(s_{13}\Delta m^2_{21})$$

Differ by $\pm\sim 1.1\%$ in each mass ordering

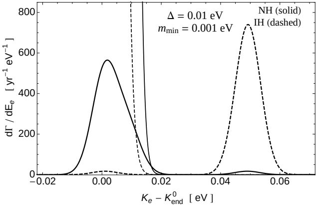
H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283

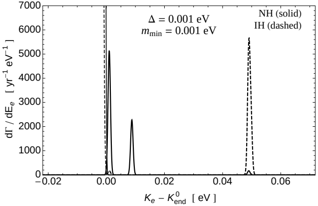
Mass ordering: broad implications

- Affects cosmology
- Affects galactic SN signal
- Affects $0\nu\beta\beta$
- Affects flavor models
- Affects end point measurements
- Affects CνB



PBD, J. Gehrlein 2308.09737





A. Long, C. Lunardini, E. Sabancilar 1405.7654

Mass ordering: new physics degeneracies

In the presence of new physics such as NSI we have:

$$[\text{NO}] + [\epsilon = 0] \quad \equiv \quad [\text{IO}] + [\epsilon_{ee} = -2]$$

$$[\text{IO}] + [\epsilon = 0] \quad \equiv \quad [\text{NO}] + [\epsilon_{ee} = -2]$$

Equivalences hold even if all oscillation probabilities are *perfectly* measured

P. Bakhti, Y. Farzan 1403.0744
P. Coloma, T. Schwetz 1604.05772
P. Coloma, PBD, et al. 1701.04828
PBD, S. Parke 2106.12436
PBD, J. Gehrlein 2204.09060

This is known as the **LMA-Dark** solution

Is the mass ordering robust?

Need **scattering** to break

Can probe same NC $\epsilon = -2$ process in scattering, but…

1. COHERENT for $M_{Z'} \gtrsim 50$ MeV and cosmology for $M_{Z'} \lesssim 5$ MeV

   PBD, Y. Farzan, I. Shoemaker 1804.03660

2. Dresden-II for ϵ_{ee} for any mediator mass

   PBD, J. Gehrlein 2204.09060

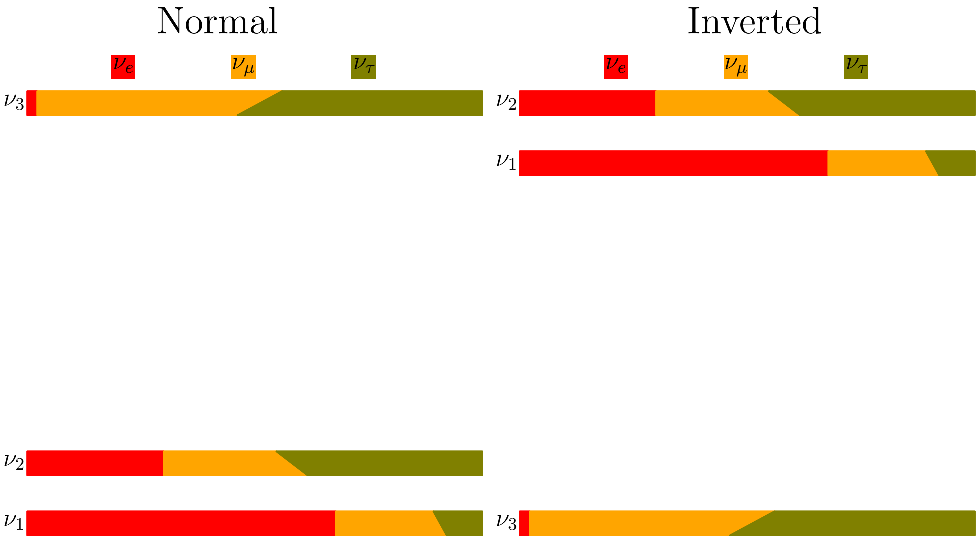
3. Can still evade with specific flavor structures

   $\epsilon_{\mu\mu} = \epsilon_{\tau\tau} = 2$ or certain u / d combinations

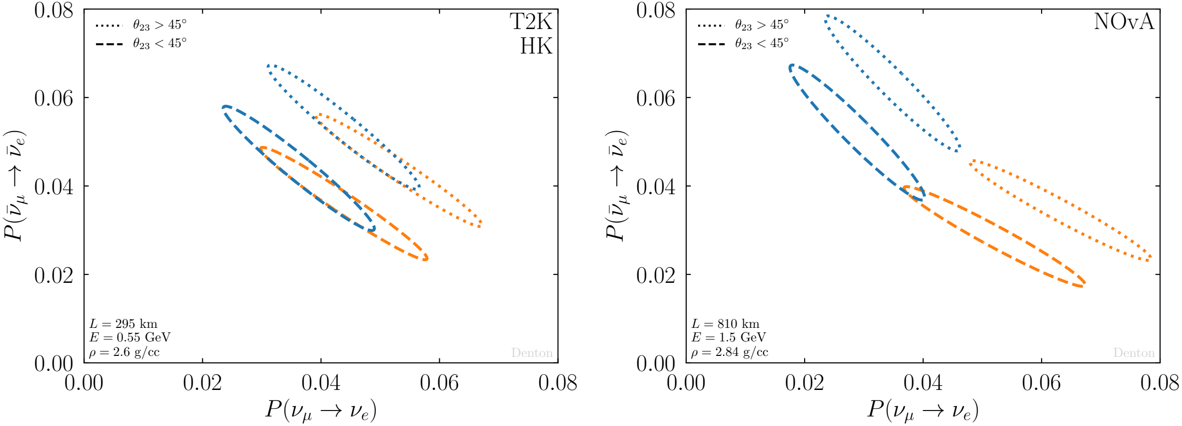
4. CCM & COHERENT can close all loopholes

θ_{23} octant

θ_{23} octant: what is it?

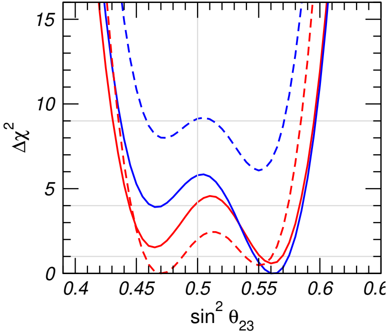


θ_{23} octant: what is it really?



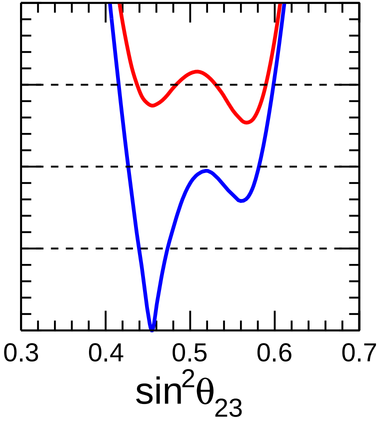
Lower octant more "normal" than upper octant

θ_{23} octant: current status



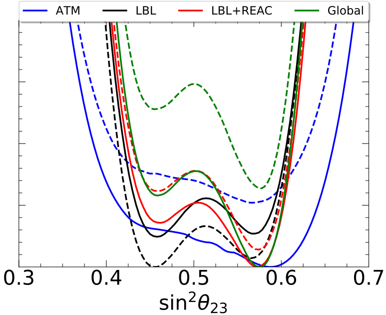
I. Esteban, et al. 2410.05380

Upper/lower at $\sim 1\sigma$



F. Capozzi, et al. 2107.00532

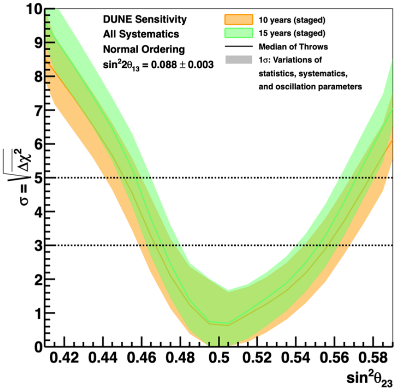
Prefers **lower** at $\sim 1.5\sigma$



P. de Salas, et al. 2006.11237

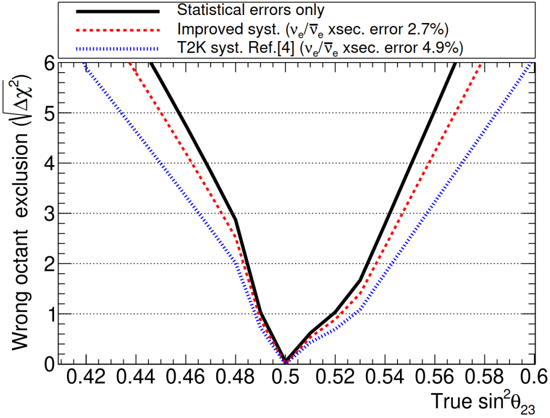
Prefers **upper** at $> 2\sigma$

θ_{23} octant: future sensitivities



$\sim 3-5\sigma$

DUNE 2002.03005



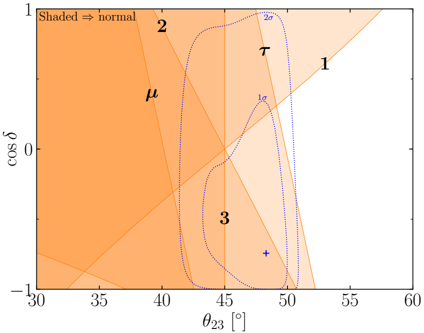
$\sim 3-5\sigma$

HK 2505.15019

θ_{23}: broader implications

Normalcy

Is the heaviest neutrino mostly ν_τ?
Is the lightest neutrino least ν_τ?



Quarks easily satisfy normalcy PBD 2003.04319

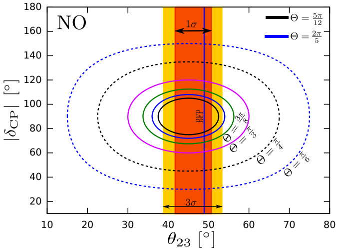
μ-τ interchange/reflection symmetry

$$\nu_\mu \leftrightarrow \nu_\tau$$

$$M_\nu^* = X M_\nu X^T \qquad X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$M_\nu \equiv U D_\nu U^\dagger$$

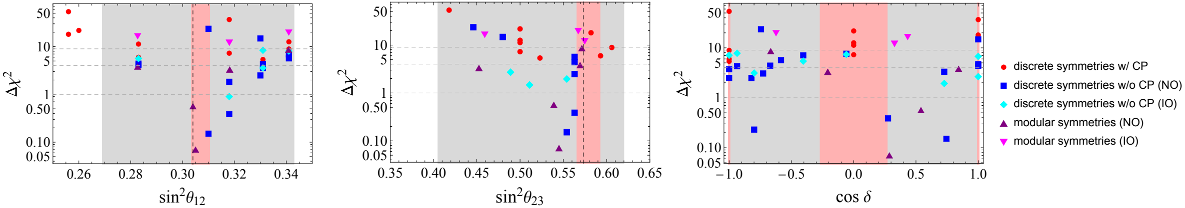
Predicts: $\theta_{23} = 45^\circ$, often $\theta_{13} = 0$



P. Chen, et al. 1512.01551

Parameter interplay

Models predict specific correlations among the parameters



J. Gehrlein, et al. 2203.06219

Precision in all neutrino parameters is key!

Complex phase

δ and CP violation

$$J_{CP} = s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23}\sin\delta$$

C. Jarlskog PRL 55, 1039 (1985)

δ and CP violation

$$J_{CP}=s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23}\sin\delta$$

C. Jarlskog PRL 55, 1039 (1985)

1. Strong interaction: no observed EDM $\Rightarrow$ CP (nearly) **conserved**

$$\frac{\bar\theta}{2\pi}<10^{-11}$$

J. Pendlebury, et al. 1509.04411

2. Quark mass matrix: non-zero but **small** CP violation

$$\frac{|J_{\rm CKM}|}{J_{\max}}=3\times10^{-4}$$

CKMfitter 1501.05013

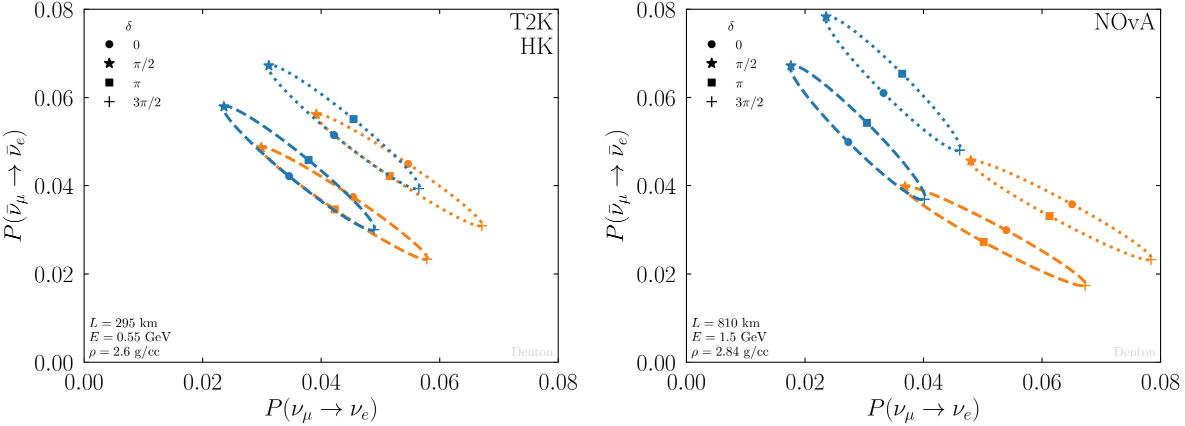
3. Lepton mass matrix: **?**

$$\frac{|J_{\rm PMNS}|}{J_{\max}}<0.34$$

PBD, J. Gehrlein, R. Pestes 2008.01110

$J_{\max}=\frac{1}{6\sqrt3}\approx0.096$

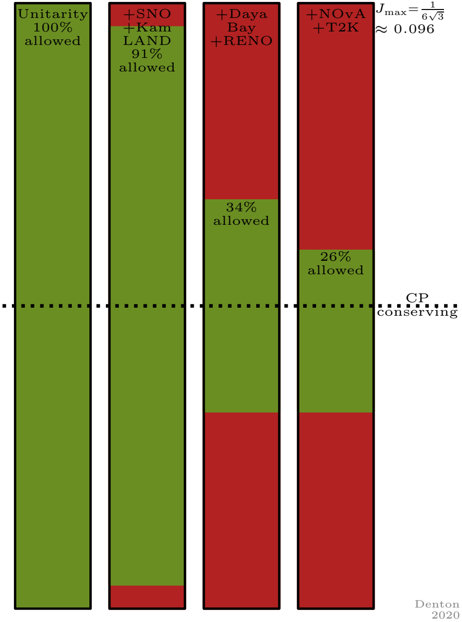
δ: what is it really?



δ, J: current status

Maximal CP violation is already ruled out:

1. $\theta_{12} \neq 45^\circ$ at $\sim 15\sigma$
2. $\theta_{13} \neq \tan^{-1}\frac{1}{\sqrt{2}} \approx 35^\circ$ at many (100) σ
3. $\theta_{23} = 45^\circ$ allowed at $\sim 1\sigma$
4. $|\sin\delta| = 1$ allowed



CP violation in oscillations

In vacuum at first maximum:

$$P_{\mu e} - \bar{P}_{\mu e} \approx 8\pi J \frac{\Delta m^2_{21}}{\Delta m^2_{32}}$$

$$J \equiv s_{12}c_{12}s_{13}c^2_{13}s_{23}c_{23}\sin\delta$$

C. Jarlskog PRL 55, 1039 (1985)

- Extracting δ from data requires every other oscillation parameter
- J requires only Δm^2_{21} (up to matter effects)
- Instead of asymmetry, can be determined via triple sine dependence

Matter effects in triple sine term can be accounted for

$$\hat{J} \simeq \frac{J}{\sqrt{(c_{212} - c^2_{13}a/\Delta m^2_{21})^2 + s^2_{212}}\sqrt{(c_{213} - a/\Delta m^2_{ee})^2 + s^2_{213}}}$$

PBD, S. Parke 1902.07185
PBD, H. Minakata, S. Parke 1604.08167

When δ and when J?

If the goal is **CP violation** the Jarlskog invariant should be used

however

If the goal is **measuring the parameters** one must use δ

Given θ_{12}, θ_{13}, θ_{23}, and J, I can't determine the sign of $\cos\delta$ which is physical

e.g. $P(\nu_\mu \to \nu_\mu)$ depends on $\cos\delta$
PBD 2309.03262

- T2K/HK are mostly sensitivity to $\sin\delta$; they should focus on J

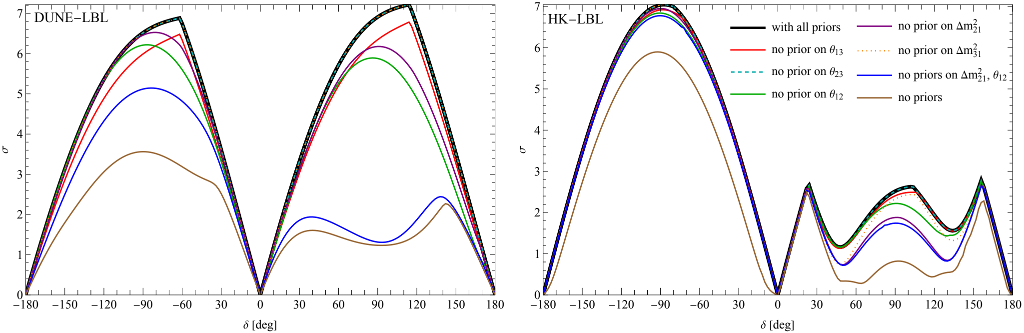
T2K does this now!

- NOvA/DUNE has modest $\cos\delta$ sensitivity; both J and δ should be reported

δ: future sensitivities

DUNE and HK will make great measurements via appearance $\nu_\mu \to \nu_e$

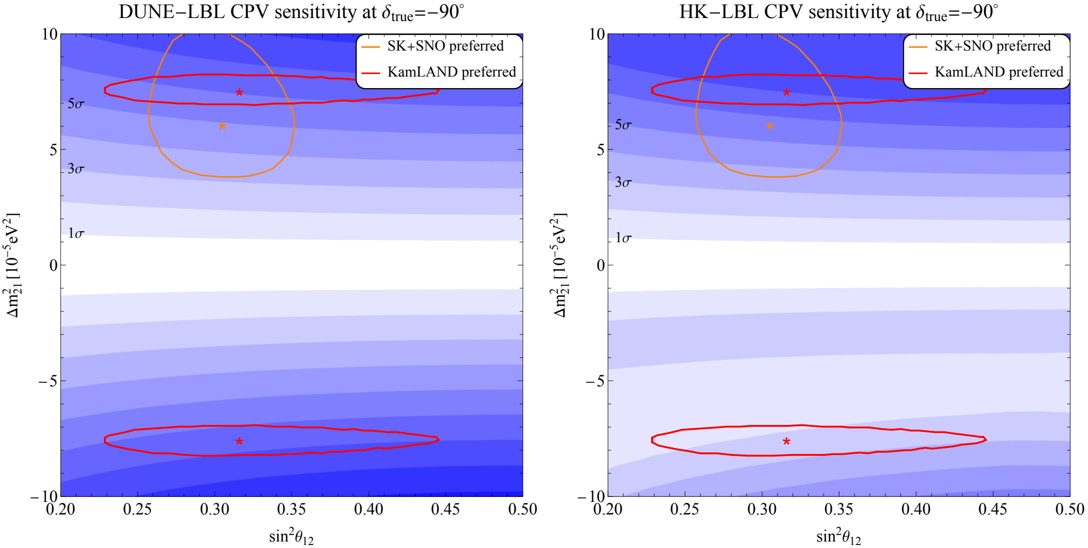
$\nu+\bar{\nu}$ helps systematics but isn't strictly necessary



Need to know solar parameters to measure δ!

Current solar knowledge: okay
Future (JUNO): excellent
PBD, J. Gehrlein 2302.08513

Impact of the true solar parameters on δ



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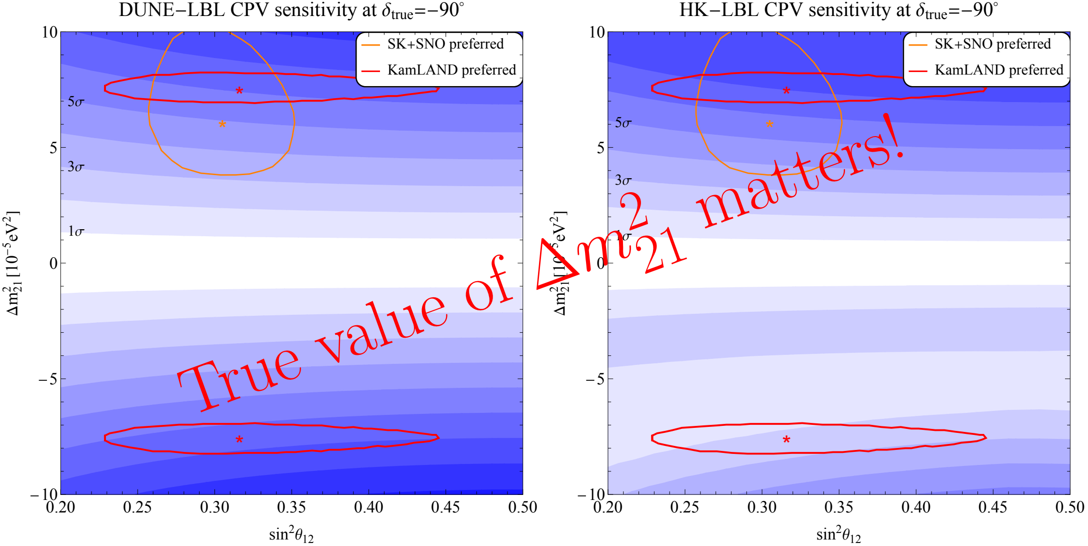


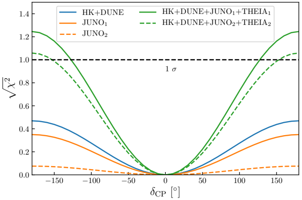
Non-standard CPV probes

1. Some information in solar due to loops in elastic scattering

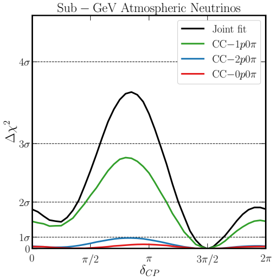
V. Brdar, X-J. Xu 2306.03160
K. Kelly, et al. 2407.03174 requires 3k Borexinos

2. Sub-GeV → sub-100 MeV atmospherics

K. Kelly, et al. 1904.02751
See also e.g. A. Suliga, J. Beacom 2306.11090



Solar (no systematics)



Atmospherics at DUNE

Non-standard CPV probes: disappearance

Possible to get at CPV with CPC processes

Disappearance probability:

$$P(\nu_\alpha \to \nu_\alpha) = 1 - 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2\Delta_{21} - 4|U_{\alpha 1}|^2|U_{\alpha 3}|^2 \sin^2\Delta_{31} - 4|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \sin^2\Delta_{32}\,,$$

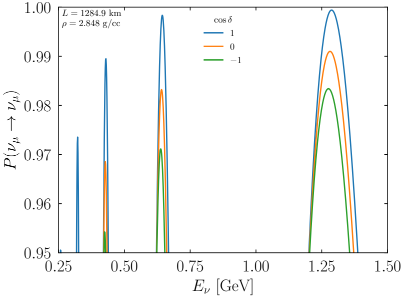
$$\Delta_{ij} = \Delta m^2_{ij} L/4E$$

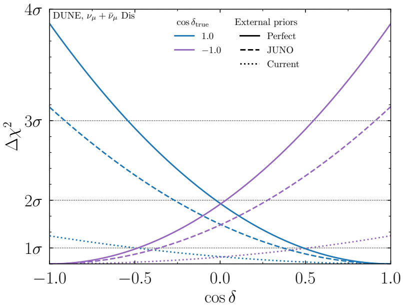
Can measure all three coeffs of each frequency $\Rightarrow$ 2 dofs

δ (and CPV) needs 4 dofs $\Rightarrow$ two dis measurements

ν_e: Daya Bay and KamLAND/JUNO

ν_μ: precision at DUNE/HK



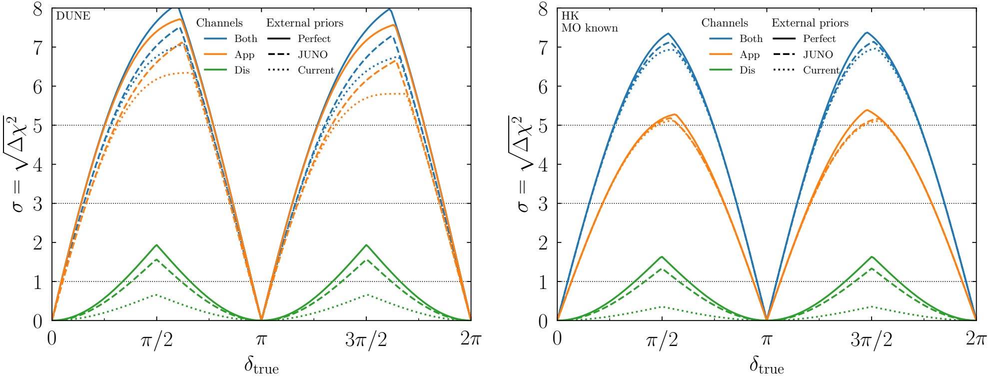


Important cross check

Different and cleaner systematics than appearance

CP violation discovery with disappearance

Need JUNO and DUNE or HK



PBD 2309.03262

How to compute neutrino oscillation probabilities quickly?

`NuFast`

github.com/PeterDenton/NuFast

Monte-Carlo estimates of statistical significances

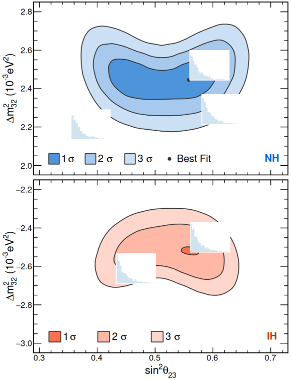
Wilks' theorem is often wrong

At each point in parameter space, simulate the experiment many times

"many" means $\gg 1/p$ for a desired p-value

This is sometimes called Feldman-Cousins

G. Feldman, R. Cousins physics/9711021



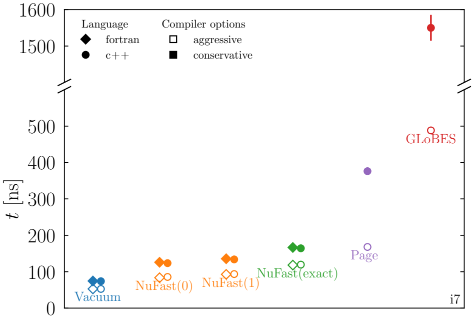
Study found most of the time was spent computing probabilities

NOvA/T2K are $\sim 3\sigma$ experiments, but DUNE/HK will be $\gtrsim 5\sigma$ experiments!

The `NuFast` approach

1. User inputs: 6 oscillation parameters, experimental details (L, E, ρ, Y_e)
2. Calculate λ_3 approximately
   - Iteratively improve with Newton's method, as needed
3. Calculate other two eigenvalues via sum rules
4. Calculate the $|V_{\alpha i}|^2$'s with the Eigenvector-Eigenvalue Identity

   PBD, S. Parke, T. Tao, X. Zhang 1908.03795
5. Get the T violating term with the NHS identity
6. Calculate the sines of the kinematic terms
7. Calculate key probabilities: P_{ee}, $P_{\mu\mu}$, & $P_{\mu e}$
8. Calculate remaining probabilities



PBD, S. Parke 2405.02400

NuFast development status

- NuFast-LBL is complete

  Relevant for accelerator and reactor

  - Default in MaCH3 and GUNDAM via NuOscillator

    Many thanks to Dan Barrow and other experimentalists for implementing and benchmarking this!

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- NuFast-Earth for atmospherics, solar, and SN neutrinos is **in progress**
  - Will contain many optimizations to reuse repeated calculations of eigenvalues/internal-eigenvectors whenever possible
  - Find me after more details

Neutrino oscillation summary

- Four known unknowns in particle physics: all neutrinos
- Mass ordering will be measured (robustness?)
- θ_{23} octant is important for flavor models
- Multiple ways to determine CP violation: key cross check given systematics/BSM
- Progress on fast code useful for experimentalists

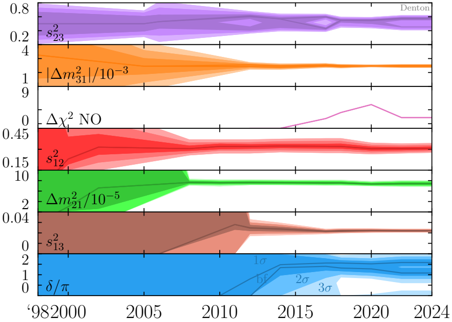
Neutrinos better fly

Nus: light, delicate
The smallest things shape the world
How do they do it?

Inspired by my trip to the Gardens on Spring Creek, Fort Collins
and Ray Bradbury's *A Sound of Thunder*

Backups

References



SK hep-ex/9807003

M. Gonzalez-Garcia, et al. hep-ph/0009350

M. Maltoni, et al. hep-ph/0207227

SK hep-ex/0501064

SK hep-ex/0604011

T. Schwetz, M. Tortola, J. Valle 0808.2016

M. Gonzalez-Garcia, M. Maltoni, J. Salvado 1001.4524

T2K 1106.2822

D. Forero, M. Tortola, J. Valle 1205.4018

D. Forero, M. Tortola, J. Valle 1405.7540

P. de Salas, et al. 1708.01186

F. Capozzi et al. 2003.08511

Do neutrinos have mass?

- Neutrinos seemed to be massless
- Direct kinematic searches have still not yielded any indication for a mass

KATRIN 2406.13516

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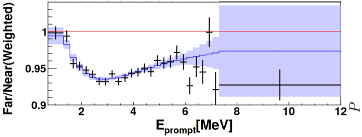
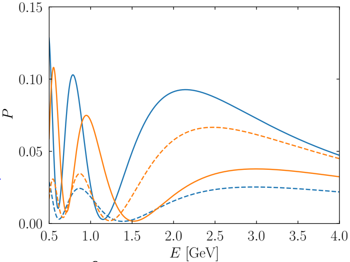


Daya Bay 1809.02261



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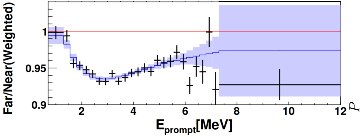

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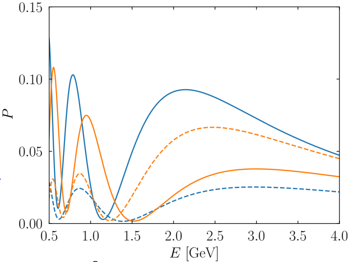

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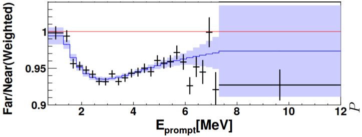

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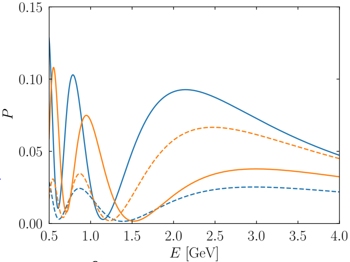

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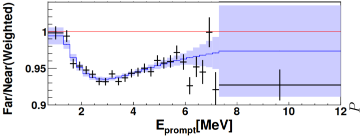

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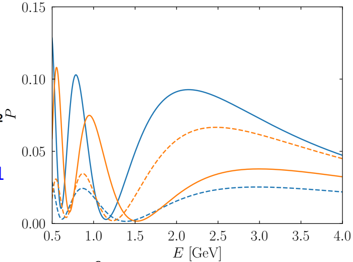

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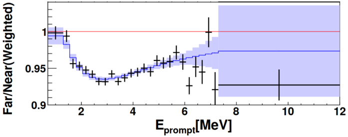

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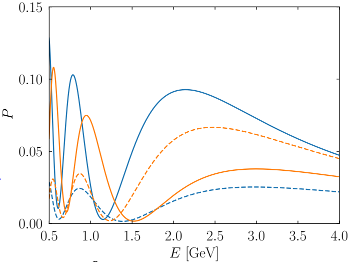

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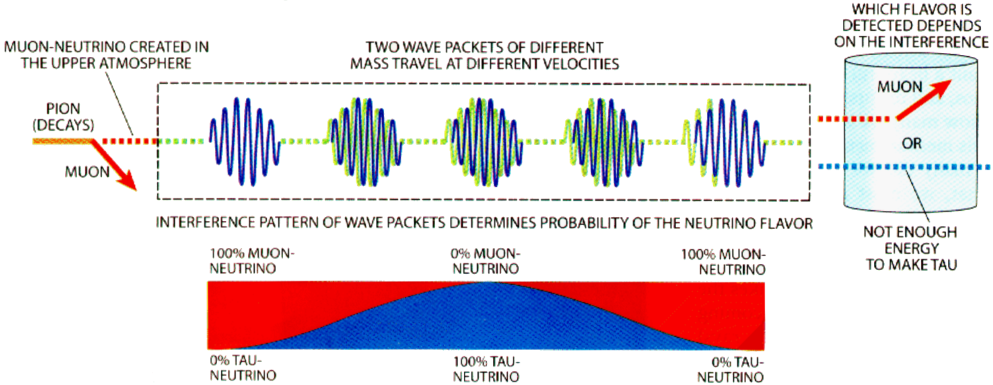
- Since they act differently at different times, they must have mass
- Masses are $> 1,000,000$ times smaller than the electron
- Their masses are still unknown!
- The mechanism by which they get mass is unknown!

How does oscillation work?

A schematic for many environments:



Some neutrino oscillation theory

Quantum mechanics exercise:

$$i\frac{\partial}{\partial t}|\nu_i\rangle = H|\nu_i\rangle$$

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$$H = \begin{pmatrix} E_1 & & \\ & E_2 & \\ & & E_3 \end{pmatrix}$$

$$|\nu_i(t)\rangle = e^{-iEt}|\nu_i(0)\rangle$$

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Subtract a common phase pt from all:

$$|\nu_i(t)\rangle = e^{-im_i^2 L/2E}|\nu_i(0)\rangle$$

Note that switching $E \simeq p$ and $x \simeq t$ is potentially problematic
Full QFT calculation gives the same answer

Some neutrino oscillation theory

Quantum mechanics exercise:

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Subtract a common phase pt from all:

$$|\nu_i(t)\rangle = e^{-im_i^2L/2E}|\nu_i(0)\rangle$$

No oscillations!

Note that switching $E \simeq p$ and $x \simeq t$ is potentially problematic
Full QFT calculation gives the same answer

Production and detection

QM superposition!

Need to produce & detect neutrinos in a different basis

In the flavor/interaction basis:

$$i\frac{\partial}{\partial t}|\nu_\alpha\rangle = H_f|\nu_\alpha\rangle$$

$\alpha = e, \mu, \tau$

$$H_f = \frac{1}{2E}U_{\rm PMNS}\begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix}U_{\rm PMNS}^\dagger$$

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$U_{\rm PMNS}$ is: complex, 3×3, unitary; 4 parameters[1]: θ_{23}, θ_{13}, θ_{12}, δ

[1]Plus two complex phases that don't affect oscillations $\propto (m_\nu/E_\nu)^2$
Many different ways to parameterize matrix:
PBD, R. Pestes 2006.09384

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PBD, R. Pestes 2006.09384

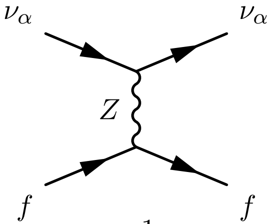
$$P(\nu_\alpha \to \nu_\beta) = \left|\sum_{i=1}^{3} U_{\alpha i}^* e^{-im_i^2 L/2E} U_{\beta i}\right|^2$$

Only $m_i^2 - m_j^2$ contribute

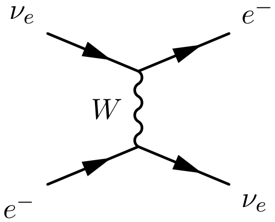
Oscillations!

Neutrinos in matter

1. Neutrinos propagate through Earth/Sun/... largely unobstructed
2. But the presence of matter modifies how they propagate!
3. Weak interaction tells neutrinos what basis to be in by modifying energy levels



$$V_{\rm NC} = \mp\frac{1}{2}\sqrt{2}G_F n_n$$



$$V_{\rm CC} = \pm\sqrt{2}G_F n_e$$

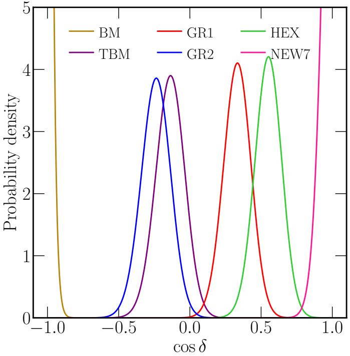
L. Wolfenstein, PRD 17 (1978)

$$H_f = \frac{1}{2E}\left[UM^2U^\dagger + \begin{pmatrix} 2EV_{\rm CC} & & \\ & 0 & \\ & & 0 \end{pmatrix}\right]$$

The importance of $\cos\delta$

- If only $\sin\delta$ is measured $\Rightarrow$ sign degeneracy: $\cos\delta = \pm\sqrt{1-\sin^2\delta}$
- Most flavor models predict $\cos\delta$

J. Gehrlein, et al. 2203.06219



L. Everett, et al. 1912.10139

δ: what is it not?

$$\delta \not\Rightarrow \text{Baryogenesis}$$

The amount of leptogenesis is a function of:

1. δ
2. the heavy mass scale
3. α, β (Majorana phases)
4. CP phases in the RH neutrinos
5. ...

C. Hagedorn, et al. 1711.02866

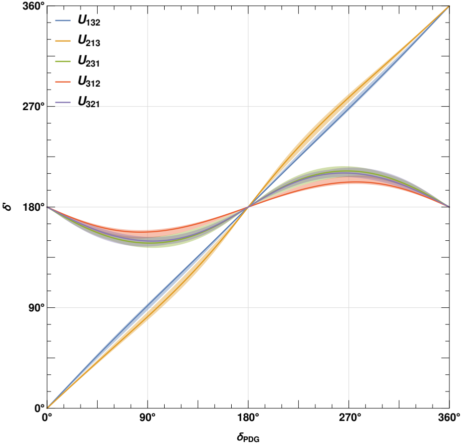
K. Moffat, et al. 1809.08251

$$\text{Measuring } \delta = 0, \pi \quad \not\Rightarrow \quad \text{no leptogenesis}$$
$$\text{Measuring } \delta \neq 0, \pi \quad \not\Rightarrow \quad \text{leptogenesis}$$

Complex phase in different parameterizations

- Can relate the complex phase in one parameterization to that in another
- U_{132} and U_{213} similar to U_{123}
- δ constrained to $\sim [150°, 210°]$ in U_{231}, U_{312}, U_{321}
- Bands indicate 3σ uncertainty on θ_{12}, θ_{13}, θ_{23}
- "50% of possible values of δ"
  $\Rightarrow$ parameterization dependent

DUNE TDR II 2002.03005



Quark mixing

From the PDG, V_{CKM} in the V_{123} parameterization is

$$\theta_{12} = 13.09^\circ \quad \theta_{13} = 0.2068^\circ \quad \theta_{23} = 2.323^\circ \quad \delta_{\mathrm{PDG}} = 68.53^\circ$$

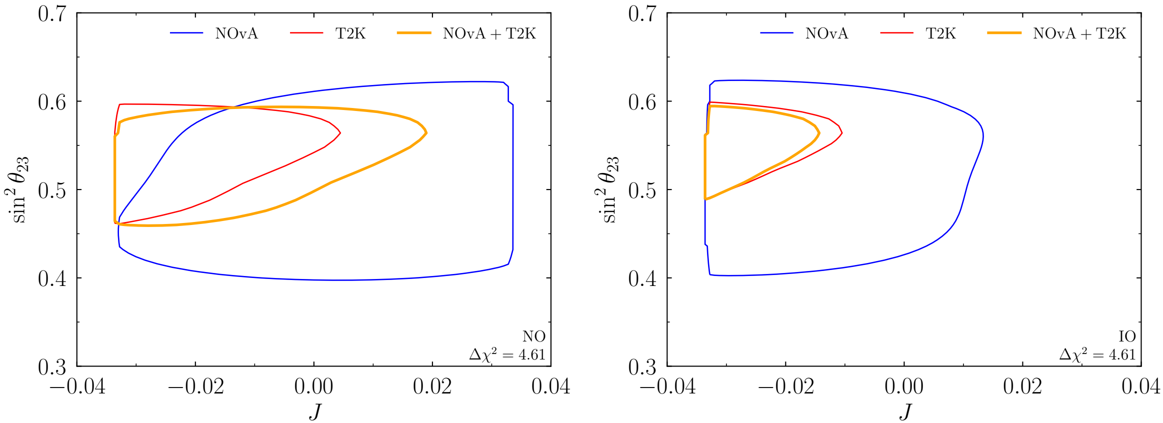
Looks like "large" CPV:

$$\sin\delta_{\mathrm{PDG}} = 0.93 \sim 1$$

yet $J_{\mathrm{CKM}}/J_{\mathrm{max}} = 3 \times 10^{-4}$.

Switch to V_{212} parameterization, $\Rightarrow \delta' = 1^\circ$ and $\sin\delta' = 0.02$.

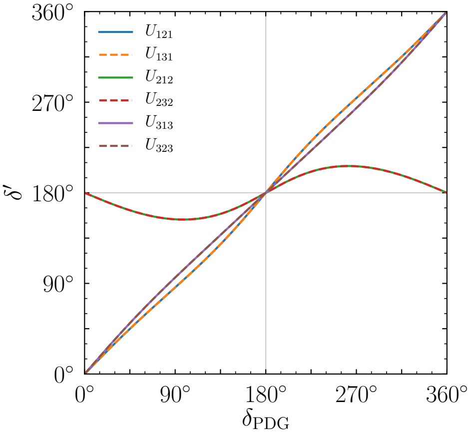
Standard oscillation parameters



Can see that the combination doesn't like the NO while it does like the IO

IO preferred over NO at $\Delta\chi^2 = 2.3$

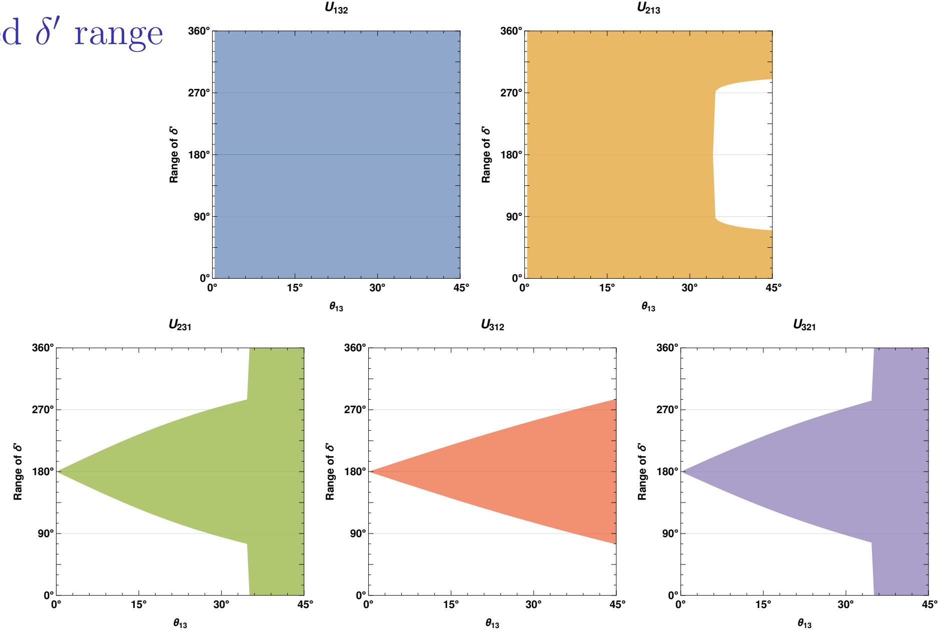
Repeated rotations



	U_{121}	U_{131}	U_{212}	U_{232}	U_{313}	U_{323}
$\lvert U_{e2}\rvert$	✓	✓	✓	✓	✗	✗
$\lvert U_{e3}\rvert$	✓	✓	✗	✗	✓	✓
$\lvert U_{\mu3}\rvert$	✗	✗	✓	✓	✓	✓

Note that $e^{i\delta}$ must be on first or third rotation

Allowed δ' range



Many interesting new physics scenarios in oscillations

1. Sterile neutrinos
2. Non-standard neutrino interactions (NSI)
   with any Lorentz structure: SPVAT
3. Non-standard neutrino self interactions
4. Neutrino decay
   with visible or invisible final states
5. Unitarity violation
6. Many others: neutrino – dark matter interactions, environmental decoherence, and Lorentz invariance or CPT violation

Many interesting new physics scenarios in oscillations

1. Sterile neutrinos

   PBD, Y. Farzan, I. Shoemaker 1811.01310
   PBD 2111.05793
   H. Davoudiasl, PBD 2301.09651

2. Non-standard neutrino interactions (NSI)

   with any Lorentz structure: SPVAT

   PBD, Y. Farzan, I. Shoemaker 1804.03660
   P. Coloma, PBD, et al. 1701.04828
   PBD, J. Gehrlein, R. Pestes 2008.01110
   PBD, J. Gehrlein 2008.06062, 2204.09060
   PBD, A. Giarnetti, D. Meloni 2210.00109, 2409.15411

3. Non-standard neutrino self interactions

   Barenboim, PBD, Oldengott 1903.02036

4. Neutrino decay

   with visible or invisible final states

   PBD, I. Tamborra 1805.05950
   PBD, A. Abdullahi 2005.07200

5. Unitarity violation

   PBD 2109.14576
   PBD, J. Gehrlein 2109.14575

6. Many others: neutrino – dark matter interactions, environmental decoherence, and Lorentz invariance or CPT violation

   See e.g. PBD, J. Gehrlein, C.-F. Kong 2502.14027

Shape-shifting sterile neutrinos

How to evade constraints?

Suppose:

1. Sterile neutrinos talk to dark matter

   DM is ultralight boson

2. Dark matter talks to baryons

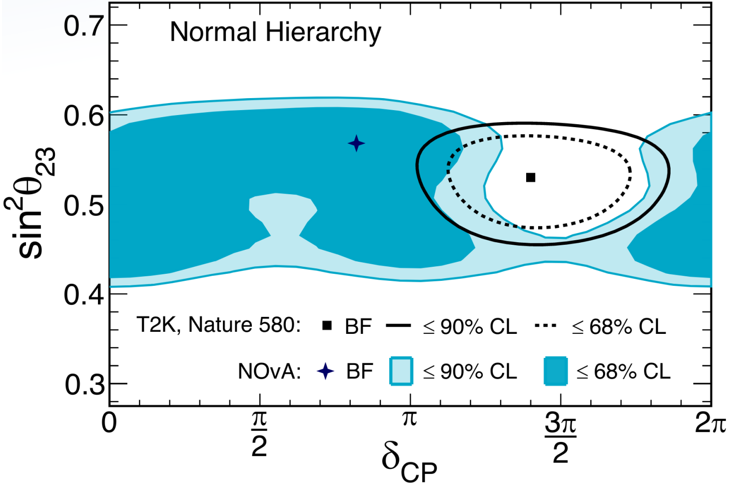
Then:

1. Sterile neutrinos aren't abundantly produced in the early universe
2. Mixing angle in the Sun is suppressed
3. Reactor constraints still exist

H. Davoudiasl, PBD 2301.09651
PBD 2301.11106

CP violation at NOvA and T2K?

Excitement at the Neutrino conference!



A. Himmel for NOvA 10.5281/zenodo.3959581

NSI review

$$\mathcal{L}_{\rm NSI} = -2\sqrt{2}G_F \sum_{\alpha,\beta,f,P} \epsilon^{f,P}_{\alpha\beta}(\bar\nu_\alpha\gamma^\mu\nu_\beta)(\bar f\gamma_\mu f)$$

Models with large NSIs consistent with CLFV:

Y. Farzan, I. Shoemaker 1512.09147   Y. Farzan, J. Heeck 1607.07616   D. Forero and W. Huang 1608.04719
K. Babu, A. Friedland, P. Machado, I. Mocioiu 1705.01822   PBD, Y. Farzan, I. Shoemaker 1804.03660
U. Dey, N. Nath, S. Sadhukhan 1804.05808   Y. Farzan 1912.09408   N. Bernal, Y. Farzan 2211.15686
S. Abbaslu, Y. Farzan 2407.13834

Affects oscillations via new matter effect

$$H = \frac{1}{2E}\left[UM^2U^\dagger + a\begin{pmatrix}1+\epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau}\\ \epsilon^*_{e\mu} & \epsilon_{\mu\mu} & \epsilon_{\mu\tau}\\ \epsilon^*_{e\tau} & \epsilon^*_{\mu\tau} & \epsilon_{\tau\tau}\end{pmatrix}\right]$$

Matter potential $a \propto G_F\rho E$

B. Dev, K. Babu, PBD, P. Machado, et al. 1907.00991

Estimate size of effect: magnitude

$$|\epsilon_{e\beta}| \approx \frac{s_{12}c_{12}c_{23}\pi\Delta m^2_{21}}{2s_{23}w_\beta}\left|\frac{\sin\delta_{\rm T2K}-\sin\delta_{\rm NOvA}}{a_{\rm NOvA}-a_{\rm T2K}}\right| \approx \begin{cases}0.22 & \text{for } \beta=\mu\\ 0.24 & \text{for } \beta=\tau\end{cases}$$

$a \propto \rho E$

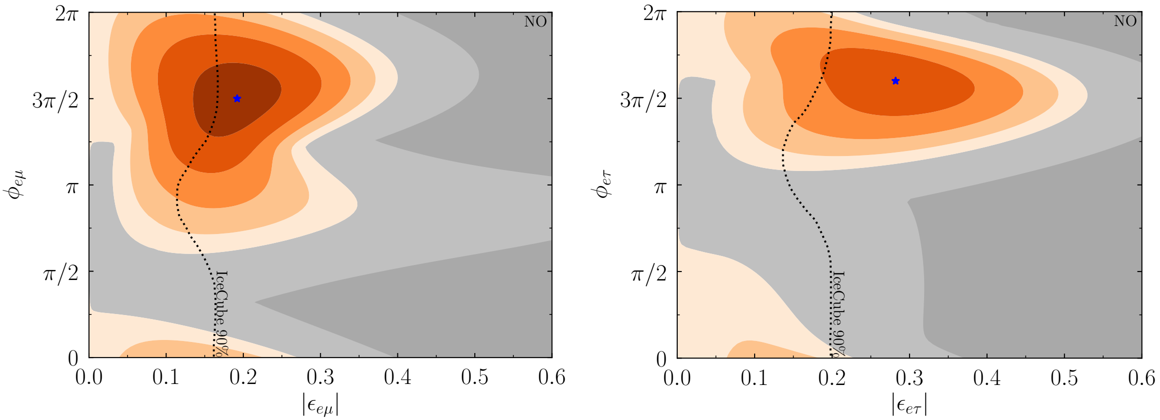
$w_\beta = s_{23},\ c_{23}$ for $\beta=\mu,\tau$

Assumed upper octant $\theta_{23} > 45^\circ$

Consistency checks:

- $\sin\delta_{\rm NOvA} = \sin\delta_{\rm T2K} \Rightarrow |\epsilon| = 0$
- $\sin\delta_{\rm NOvA} \neq \sin\delta_{\rm T2K}$ and $a_{\rm NOvA} = a_{\rm T2K} \Rightarrow |\epsilon| \to \infty$
- Octant:
  1. LBL is governed by ν_3
  2. Upper octant $\Rightarrow \nu_3$ is more ν_μ
  3. More $\nu_\mu \Rightarrow$ need less new physics coupling to ν_μ to produce a given effect

NSI parameters



Orange is preferred over SM at integer values of $\Delta\chi^2$, dark gray is disfavored at 4.61

T. Ehrhardt, IceCube PPNT (2019)

$\epsilon_{\mu\tau}$, IO in backups

Other CP violating NSI constraints

NSI effects grow with energy, density, and distance

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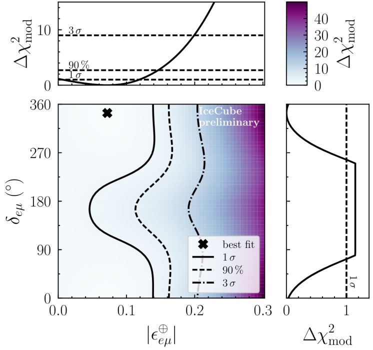
Best probes:

- $\epsilon_{\mu\tau}$: atmospheric
- $\epsilon_{e\mu}$, $\epsilon_{e\tau}$: LBL appearance, atmospheric
- IceCube
  - Constraint is at LBL best fit with 3 yrs

    10 yrs of data in the bank
  - Prefers non-zero $|\epsilon_{e\mu}|$ at $\sim 1\sigma$



T. Ehrhardt, IceCube PPNT (2019)

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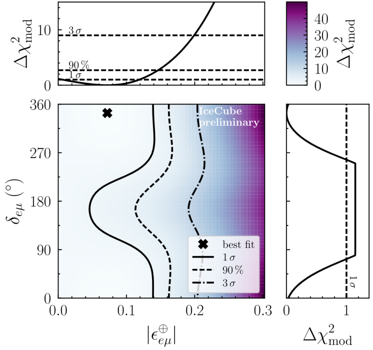

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T. Ehrhardt, IceCube PPNT (2019)

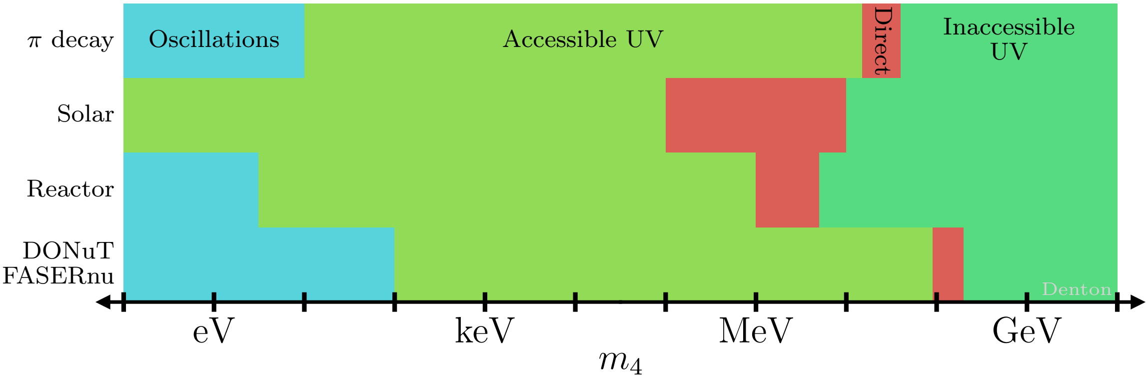
- Super-K
  - Only consider real NSI
  - Comparable sensitivity as IceCube

Super-K 1109.1889

- COHERENT
  - Only applies to NSI models with $M_{Z'} \gtrsim 10$ MeV
  - NSI u, d, e configuration matters
  - Comparable constraints

COHERENT 1708.01294
PBD, Y. Farzan, I. Shoemaker 1804.03660
PBD, J. Gehrlein 2008.06062

Unitarity violation: a tale of two regimes



*Details depends on the specific experiment/channel

Unitarity violation: how to calculate

Kinematically **accessible** states

1. Unitary calculation of full $n \times n$ matrix
2. Oscillation averaged:

$$\sin^2 \frac{\Delta m_{41}^2 L}{4E} \to \frac{1}{2}$$

$$\sin \frac{\Delta m_{41}^2 L}{4E} \to 0$$

3. No matter effect:

$$H^{\text{mat}} = \text{diag}(V_{\text{CC}} + V_{\text{NC}}, V_{\text{NC}}, V_{\text{NC}}, 0, \dots)$$

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Kinematically **inaccessible** states

1. Nonunitary calculation of $m \times m$ matrix

   m = number of kinematically accessible states

2. Rescale probability:

$$P_{\alpha\beta} = \frac{|\sum_{i=1}^{\rm acc} U_{\alpha i}^* e^{iP_iL} U_{\beta i}|}{(\sum_{i=1}^{\rm acc} U_{\alpha i}^* U_{\alpha i})(\sum_{i=1}^{\rm acc} U_{\beta i}^* U_{\beta i})}$$

3. Cannot subtract multiples of $\mathbb{1}$
4. Rescale cross section/flux as appropriate
5. Rescale G_F in matter effect

Unitarity violation status from oscillations

3σ maximal deviations from unitarity

Leptons

	Hu+	Ellis+
ν_e row	0.003	0.05
ν_μ row	0.02	0.04
ν_τ row	0.2	0.82
ν_1 col	0.06	0.22
ν_2 col	0.09	0.27
ν_3 col	0.12	0.40

Quarks

u row	0.0015	$\sim 2.2\sigma$ tension
c row	0.06	
t row	-	
d col	0.005	
s col	0.06	
b col	-	

Lepton constraints don't include anomalies

Care is required

S. Ellis, K. Kelly, S. Li 2008.01088

Z. Hu, et al. 2008.09730

S. Parke, M. Ross-Lonergan 1508.05095

PDG

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PDG

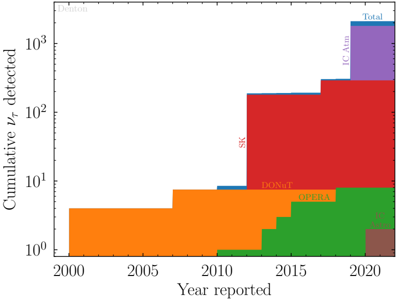
Vastly different mixing angle hierarchy

$\Rightarrow$

Like comparing apples and steak

Leptons: tau row is the weakest

1. Existing global analyses use OPERA and SNO
2. More data from atmospheric ν_τ appearance!



Also astrophysical ν_τ appearance; weak but distinct!

Atmospheric works because τ is in direct region

PBD, et al. 2203.05591

Unitarity violation: tau row

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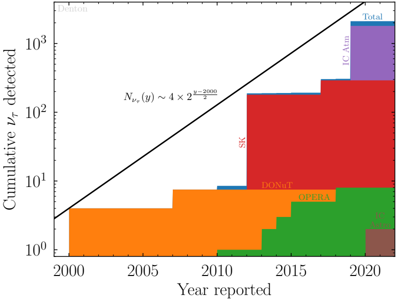

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Tau neutrino data set doubles every two years!

PBD, et al. 2203.05591

Unitarity violation

Consistency of the three-flavor oscillation picture?

and/or

Searches for unitarity violation?

Unitarity violation

Consistency of the three-flavor oscillation picture?

and/or

Searches for unitarity violation?

Not the same!

Lots of models to test standard three-flavor picture:
Sterile, unitarity violation, NSI, neutrino decay, decoherence, …

Unitarity violation: what is it?

Our 3×3 matrix isn't unitary:

$$U_3 U_3^\dagger \neq \mathbb{1}$$

Addition of new flavor states $\nu_a, \nu_b, \nu_c, \ldots$ and new mass states ν_4, ν_5, ν_6

$$U \to \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & \cdots \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} & \cdots \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} & \cdots \\ U_{a1} & U_{a2} & U_{a3} & U_{a4} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Unitarity Violation $\Rightarrow$

New mass states not directly accessible by oscillations or decay

Thus check if U_3 is what it should be

Unitarity constraints

Unitary violation: the study of how $U_{3\times3}$ is not unitary independent of m_4, m_5, …
Constraints vary considerably in the literature:

$$1-|U_{e1}|^2-|U_{e2}|^2-|U_{e3}|^2<\begin{cases}0.05\\0.001\end{cases}\quad\text{at }2\sigma$$

S. Parke, M. Ross-Lonergan 1508.05095

Z. Hu, et al. 2008.09730

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All analyses *assume* unitarity
Throw out LSND, MiniBooNE, RAA, gallium, etc.

S. Parke, M. Ross-Lonergan 1508.05095

Z. Hu, et al. 2008.09730

Unitarity violation

- Could conceivably differentiate: 2 new states from 1, but not 3+ from 2
- Zero distance effect ⇒ near detector **with flux prediction**

E.g. RAA, Gallium

- Numerous parameterizations: α matrix, η matrix, submatrix & Cauchy-Schwartz

All apply to the inaccessible cases only

- There is an approximate correspondence to sterile and NSI

$$\alpha_{ee} \approx \frac{1}{2}(s_{14}^2 + s_{15}^2 + s_{16}^2) \approx -\epsilon_{ee}\,, \quad \ldots$$

M. Blennow, et al. 1609.08637

Applies one experiment at a time

- Additional EW precision information: W, Z, π, μ, τ decays

Care is required

S. Antush, et al. hep-ph/0607020

S. Antusch, O. Fischer 1407.6607

Unitarity violation: mass ranges for tau neutrinos

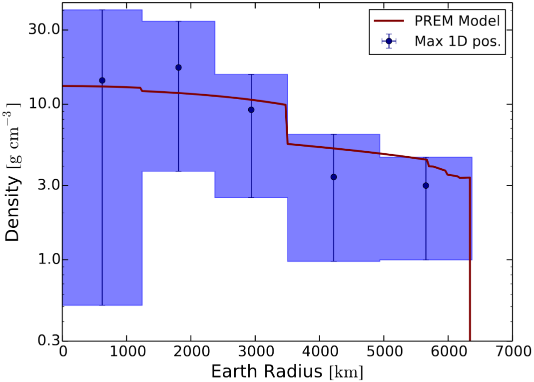
experiment	(4,4) (m_4)	(5,3) (m_4)
atmospheric ν_μ disappearance	$\in [10\text{ eV}, 15\text{ MeV}]$	$\gtrsim 40$ MeV
atmospheric ν_τ appearance	$\in [10\text{ eV}, 15\text{ MeV}]$	$\gtrsim 40$ MeV
astrophysical ν_τ appearance	$\lesssim 15$ MeV	$\gtrsim 40$ MeV
solar ^{8}B	$\lesssim 5$ MeV	$\gtrsim 20$ MeV
DONuT/FASERnu	$\in$ [100 eV, 90 MeV]	$\gtrsim 200$ MeV
LBL ν_τ appearance (OPERA)	$\in$ [1 eV, 15 MeV]	$\gtrsim 40$ MeV
LBL ν_τ appearance (DUNE)	$\in$ [0.1 eV, 15 MeV]	$\gtrsim 40$ MeV
LBL ν_μ disappearance (DUNE)	$\in$ [0.1 eV, 15 MeV]	$\gtrsim 40$ MeV
CEvNS	$\in$ [10 eV, 15 MeV]	$\gtrsim 40$ MeV

PBD, J. Gehrlein 2109.14575

Terrestrial tomography

HE neutrinos are absorbed in the Earth

Assumes isotropic flux



A. Donini, S. Palomares-Ruiz, J. Salvado 1803.05901

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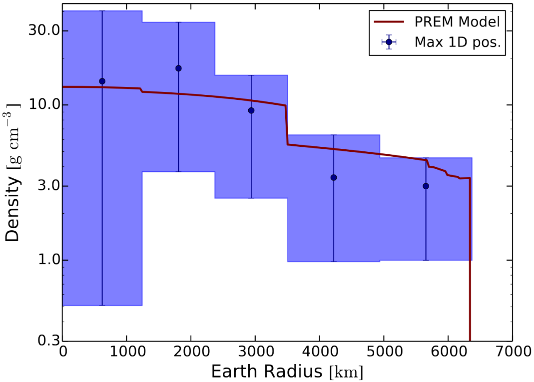
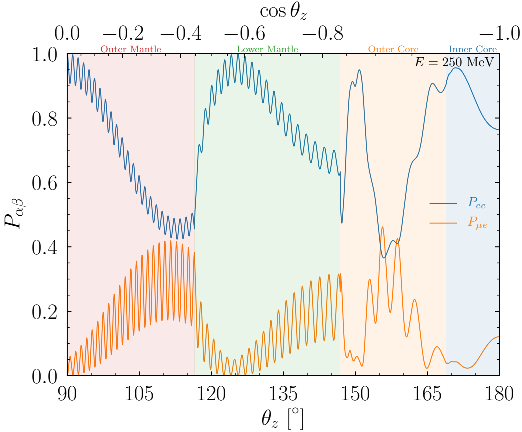


A. Donini, S. Palomares-Ruiz, J. Salvado 1803.05901

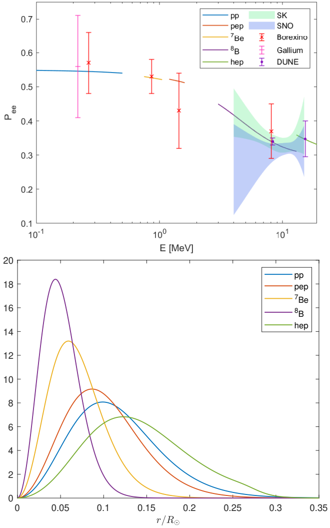
Can also do this with oscillations



DUNE-atm can measure $r^{\oplus}_{\rm core}$ to 9%

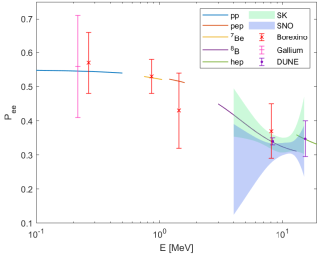
See also K. Kelly, et al. 2110.00003

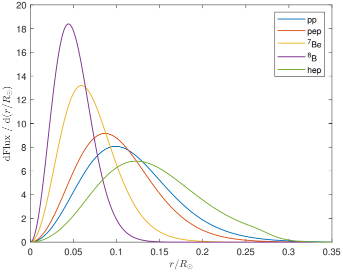
Solar tomography



Solar neutrinos constrain Sun's density at different radii

Solar tomography

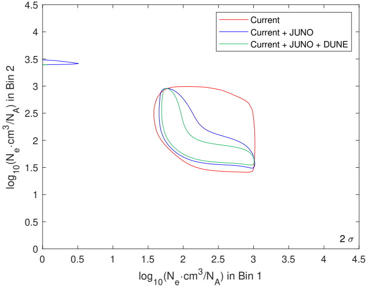




Solar neutrinos constrain Sun's density at different radii

Bin 1: $\frac{r}{R_\odot} \in [0, 0.05]$

Bin 2: $\frac{r}{R_\odot} \in [0.05, 0.1]$



$$\log_{10} N_e = Ar + B$$